



Univerzitet u Beogradu – Građevinski fakultet
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Studijski program:

GRAĐEVINARSTVO

Modul:

PŽA, HVE, MTI

Godina/Semestar:

III godina / V semestar

Naziv predmeta (šifra):

Betonske konstrukcije 1

(B2S3BK, B2H3BK, B2M3BK, B1S3BK)

Nastavnik:

Jelena Dragaš

Naslov predavanja:

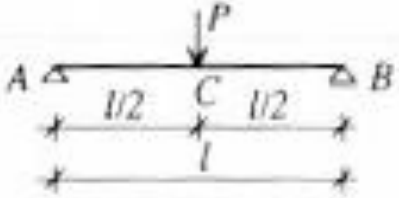
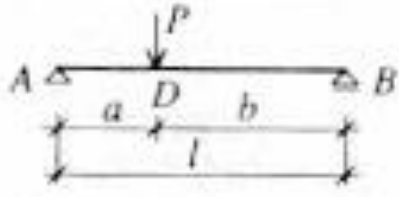
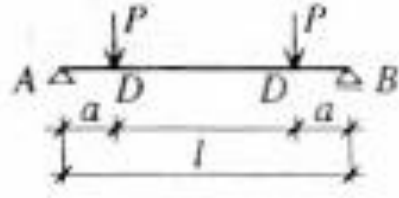
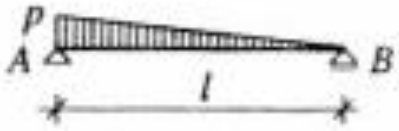
Priprema za kolokvijum

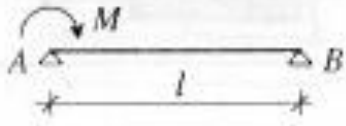
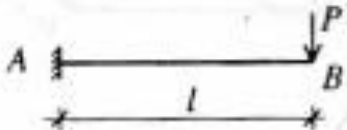
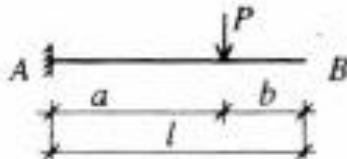
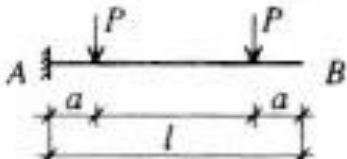
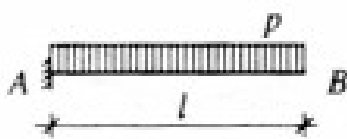
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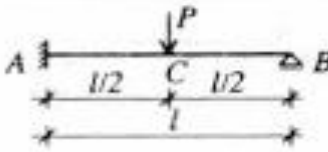
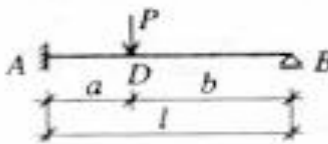
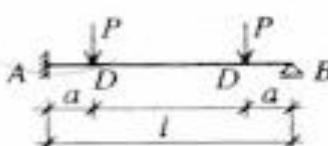
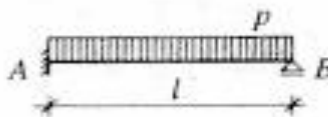
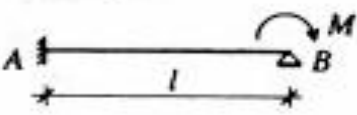
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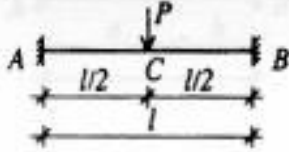
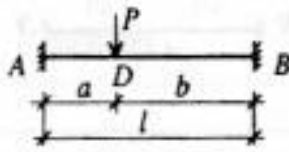
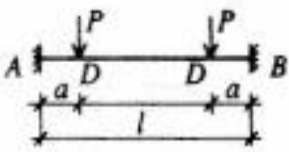

Beograd, 2021.

Sva autorska prava autora prezentacije i/ili video snimaka su zaštićena. Snimak ili prezentacija se mogu koristiti samo za nastavu na daljinu studenta Građevinskog fakulteta Univerziteta u Beogradu u školskoj 2021/2022. i ne mogu se koristiti za druge svrhe bez pismene saglasnosti autora materijala.

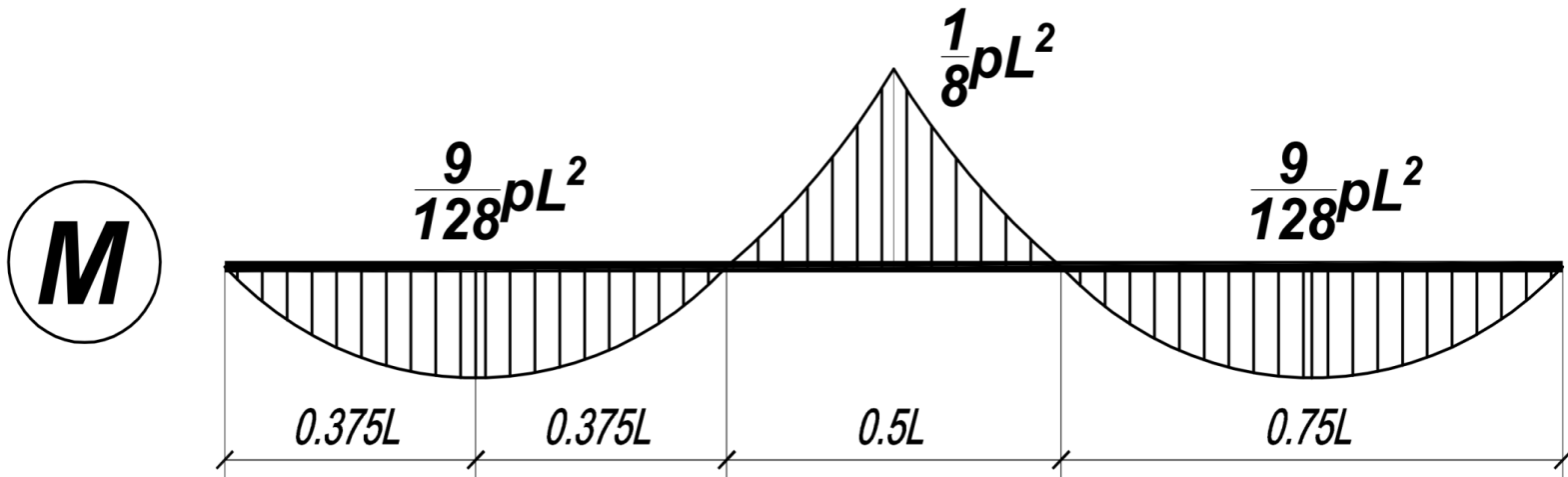
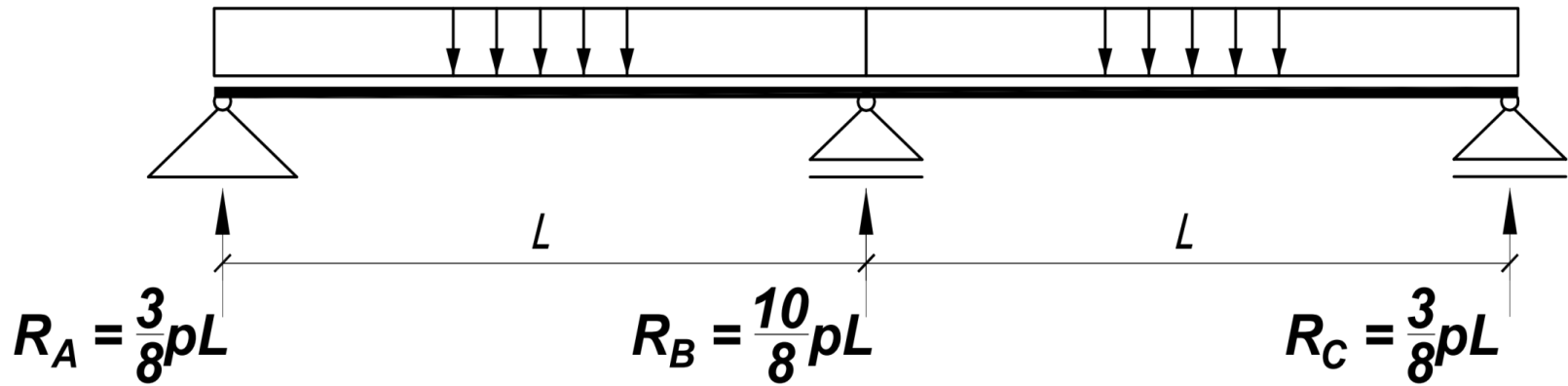
	Oslovačke reakcije R	Ekstremne vrednosti momenata M_{max}
	$A = B = \frac{P}{2}$	$M_c = \frac{Pl}{4}$
	$A = P \frac{b}{l}$ $B = P \frac{a}{l}$	$M_D = P \frac{ab}{l}$
	$A = B = P$	$M_D = Pa$
	$A = \frac{pl}{3}$ $B = \frac{pl}{6}$	$M_D = \frac{pl^2}{(9\sqrt{3})}$ $x_D = l - \frac{l}{\sqrt{3}}$

	R	M_{max}
	$A = -\frac{M}{l}$ $B = \frac{M}{l}$	$M_A = M$
	$R = A$	$M_{max} = M_A$
	P	$-Pl$
	P	$-Pa$
	$2P$	$-Pl$
	pl	$-\frac{pl^2}{2}$

	R	M_{max}
	$A = \frac{11}{16} P$ $B = \frac{5}{16} P$	$M_A = -\frac{3}{16} Pl$ $M_C = \frac{5}{32} Pl$
	$A = \frac{Pb}{2l^3} (3l^2 - b^2)$ $B = \frac{Pa^2}{2l^3} (3l - a)$	$M_A = -\frac{Pab}{2l^2} (l + b)$ $M_D = \frac{Pa^2b}{2l^3} (2l + b)$
	$A = \frac{P}{2l^2} (2l^2 + 3al - 3a^2)$ $B = \frac{P}{2l^2} (2l^2 - 3al + 3a^2)$	$M_A = -\frac{3Pa}{2l} (l - a)$ $M_{D2} = \frac{Pa}{2l^2} k$ $k = 2l^2 - 3al + 3a^2$
	$A = \frac{5}{8} pl$ $B = \frac{3}{8} pl$	$M_A = -\frac{pl^2}{8}$ $M_D = \frac{9}{128} pl^2$ $x_D = \frac{5}{8} l$
	$A = -\frac{3M}{2l}$ $B = \frac{3M}{2l}$	$M_A = \frac{M}{2}$ $M_B = -M$

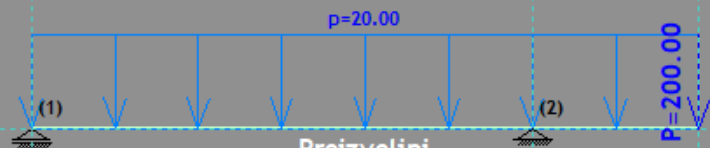
	R	M_{max}
	$A = B = \frac{P}{2}$	$M_A = M_B = -\frac{Pl}{8}$ $M_C = \frac{Pl}{8}$
	$A = \frac{Pb^2}{l^3} (l + 2a)$ $B = \frac{Pa^2}{l^3} (l + 2b)$	$M_A = -P \frac{ab^2}{l^2}$ $M_B = -P \frac{a^2b}{l^2}$ $M_D = 2P \frac{a^2b^2}{l^3}$
	$A = B = P$	$M_A = M_B = -\frac{Pa}{l} (l - a)$ $M_C = \frac{Pa^2}{l}$
	$A = B = \frac{Pl}{2}$	$M_A = M_B = -\frac{Pl^2}{12}$ $M_C = \frac{Pl^2}{24}$ $x_C = \frac{l}{2}$

$$p = 1.35g + 1.5q \text{ (kN/m')}$$





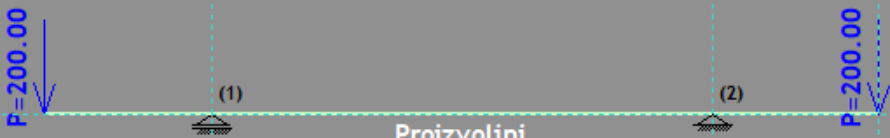
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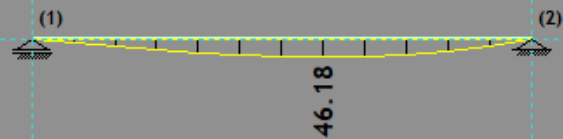
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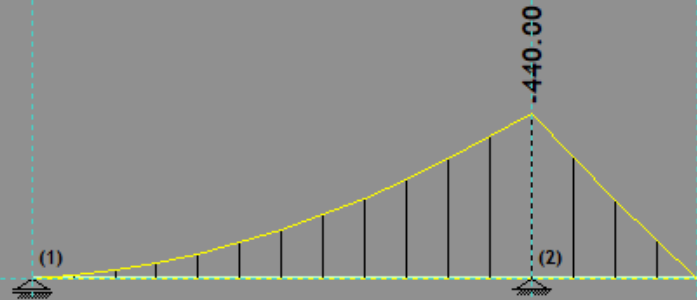
Proizvoljni



Proizvoljni



46.18



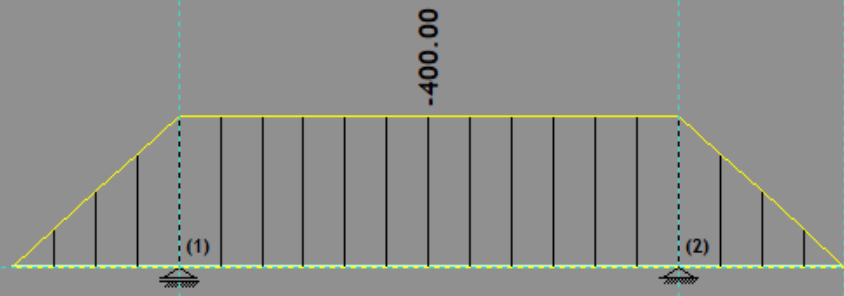
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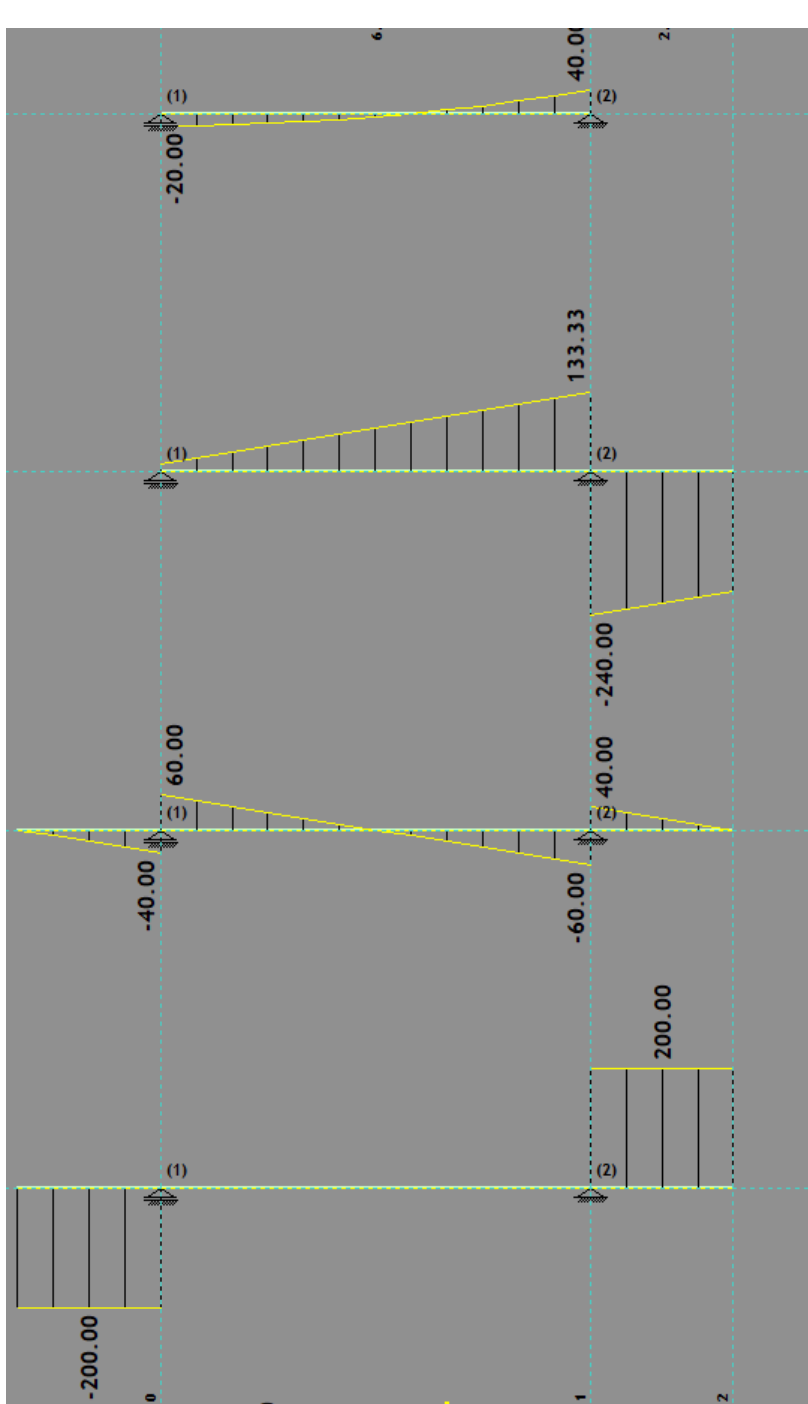
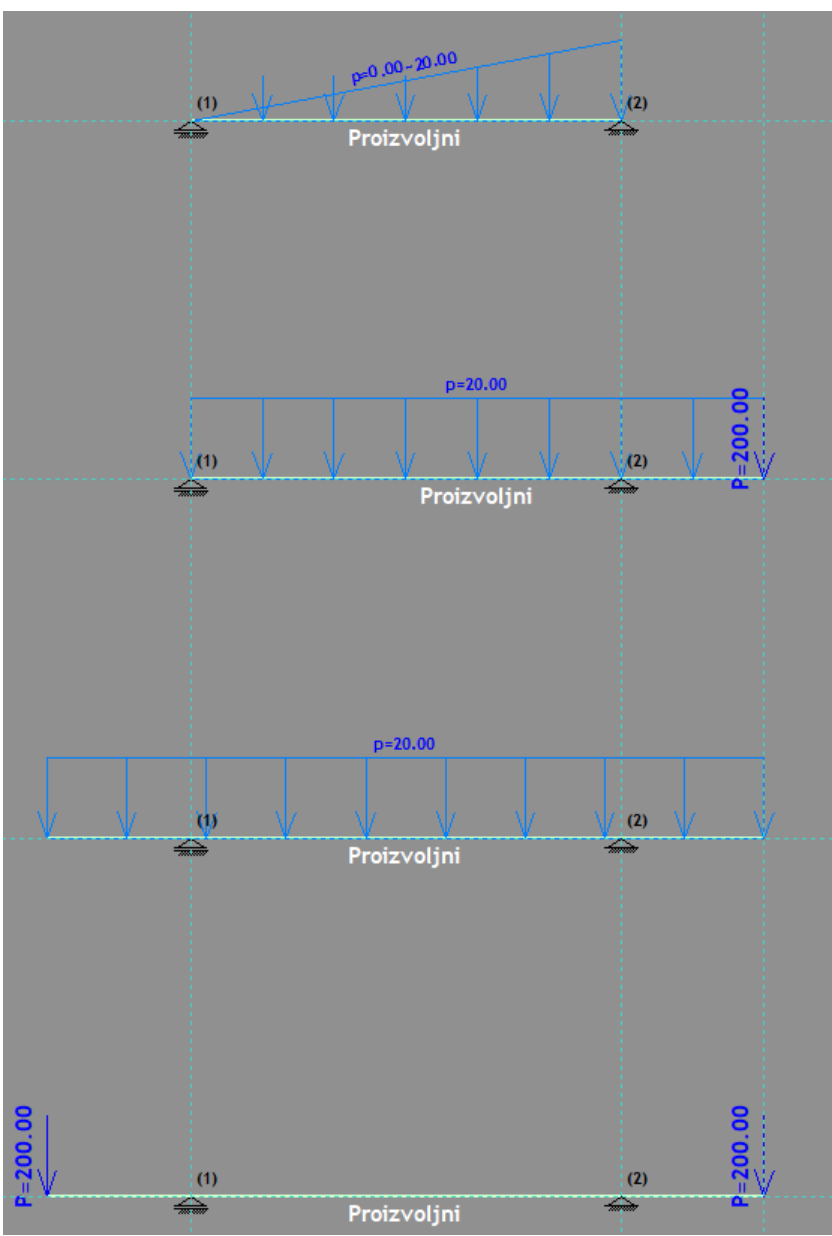
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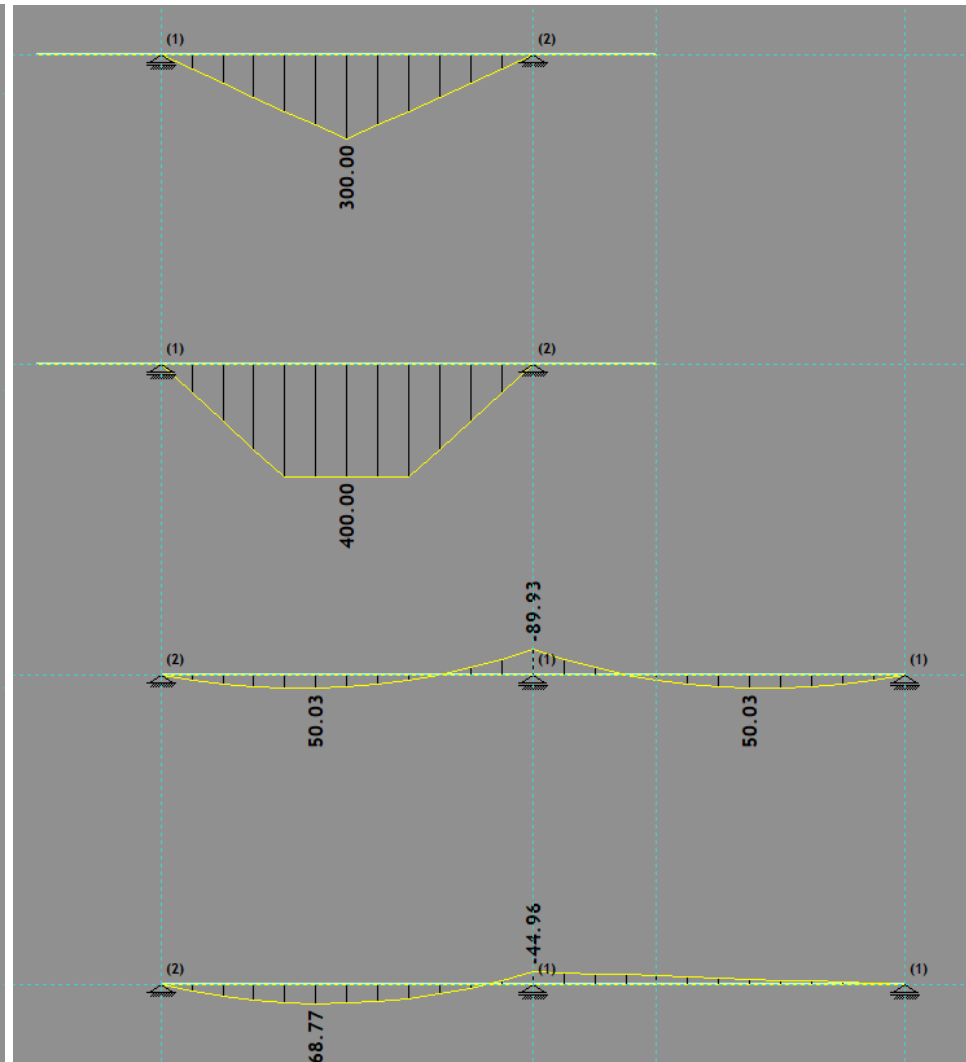
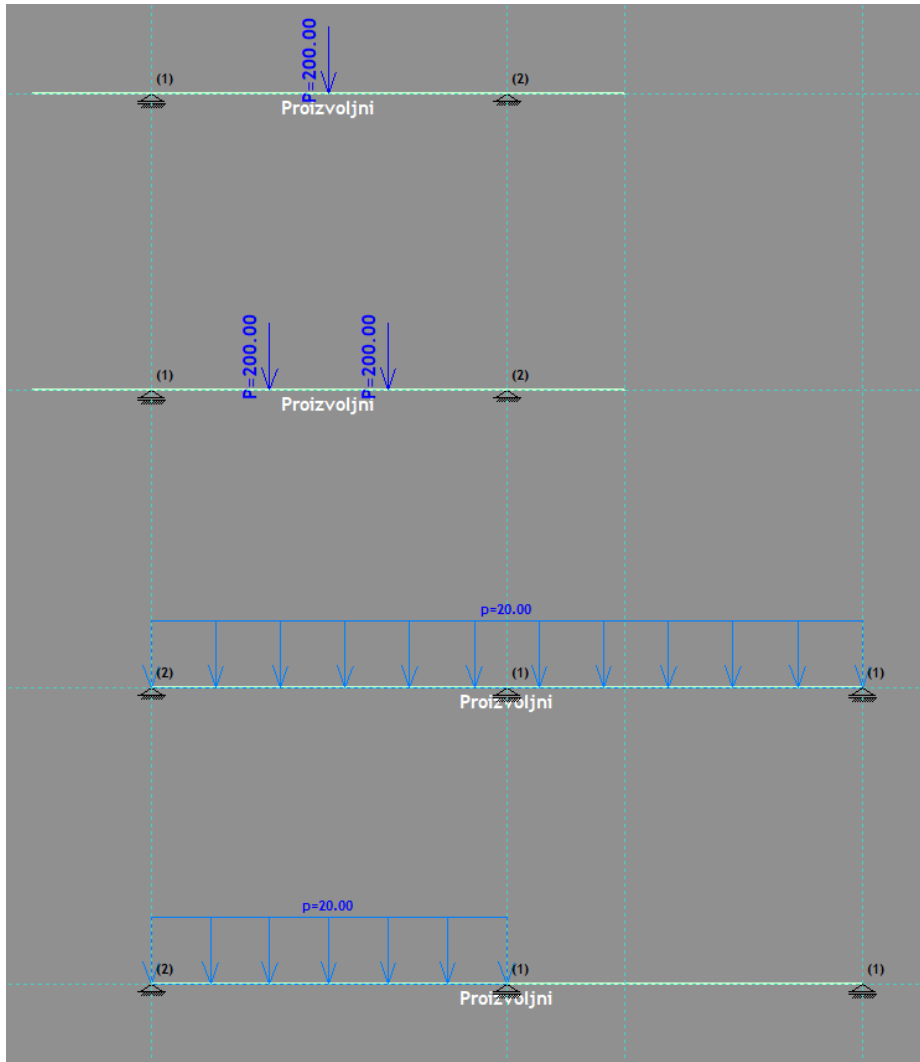
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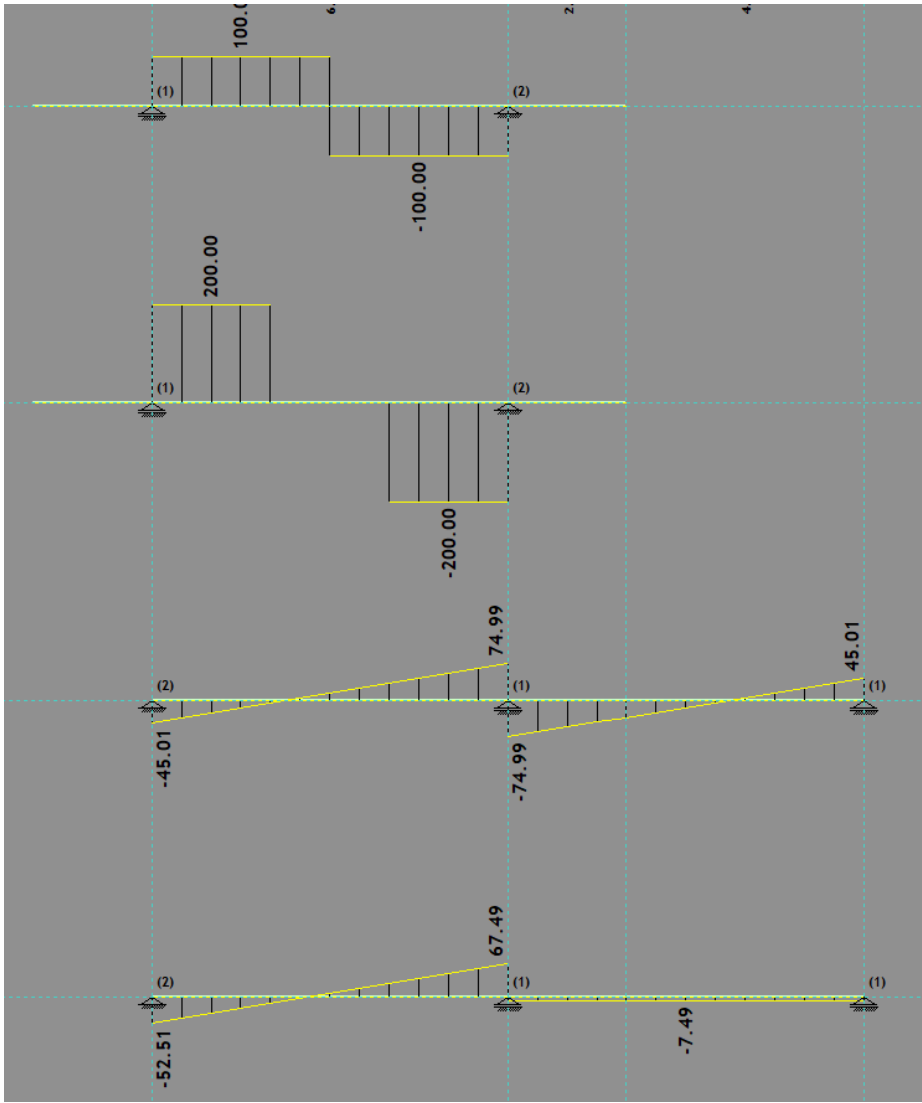
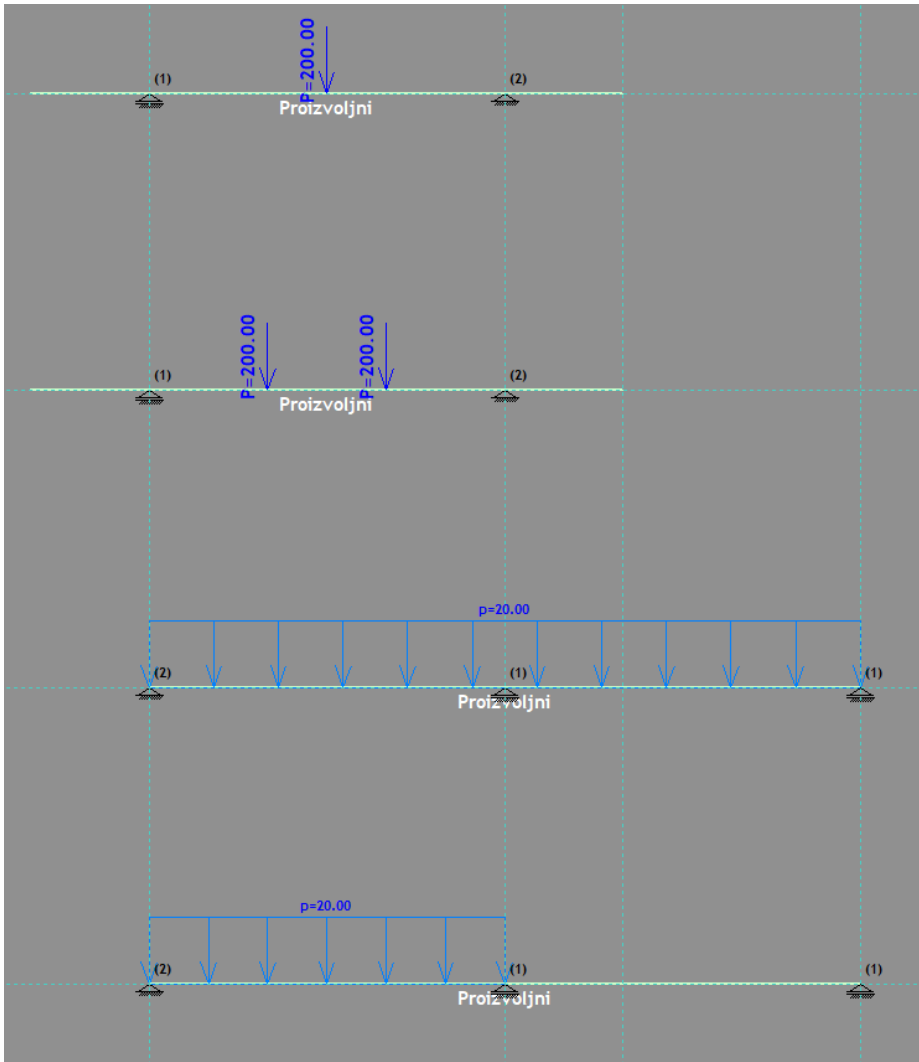
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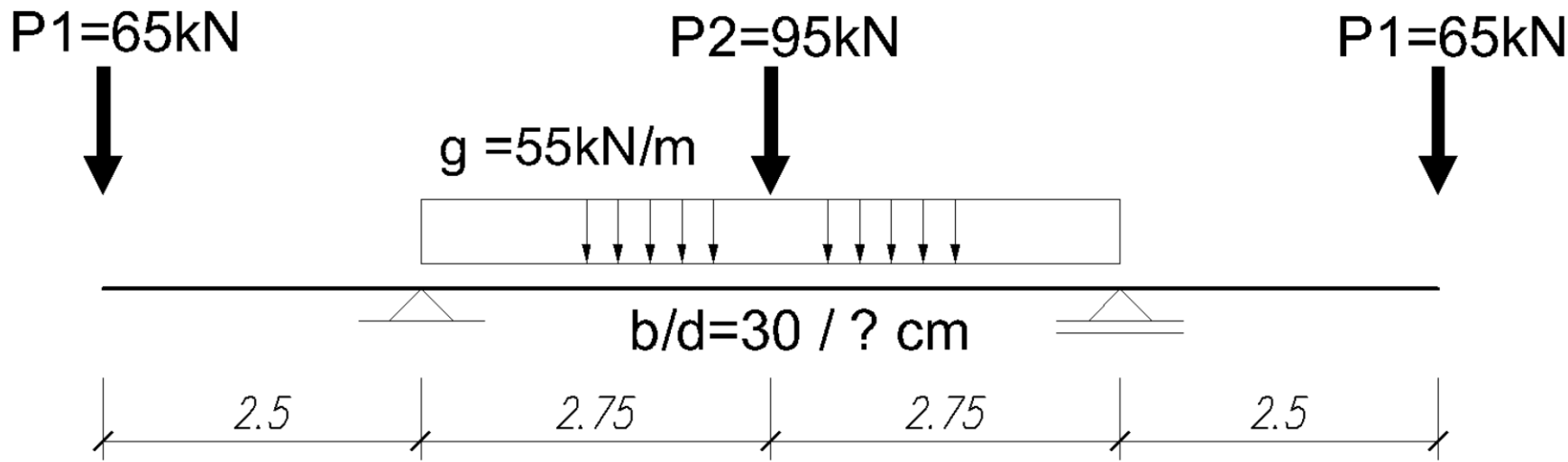


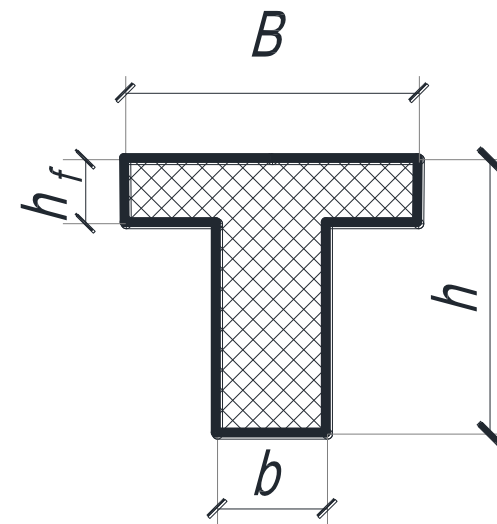
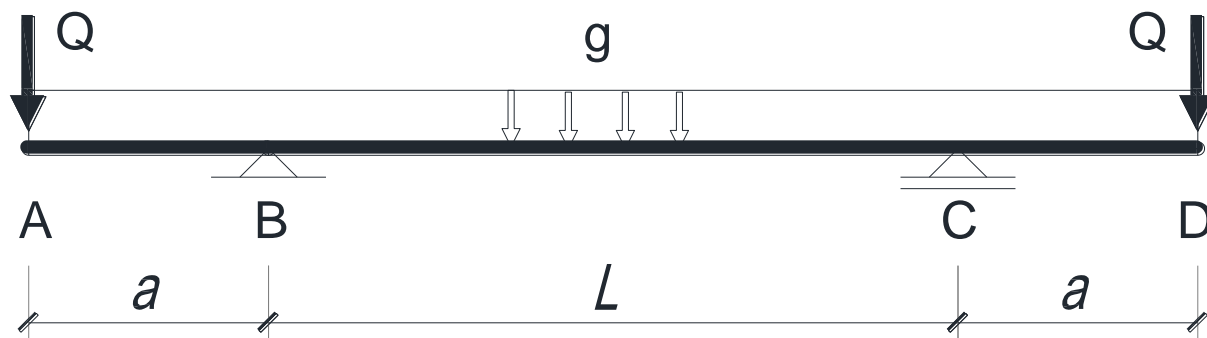
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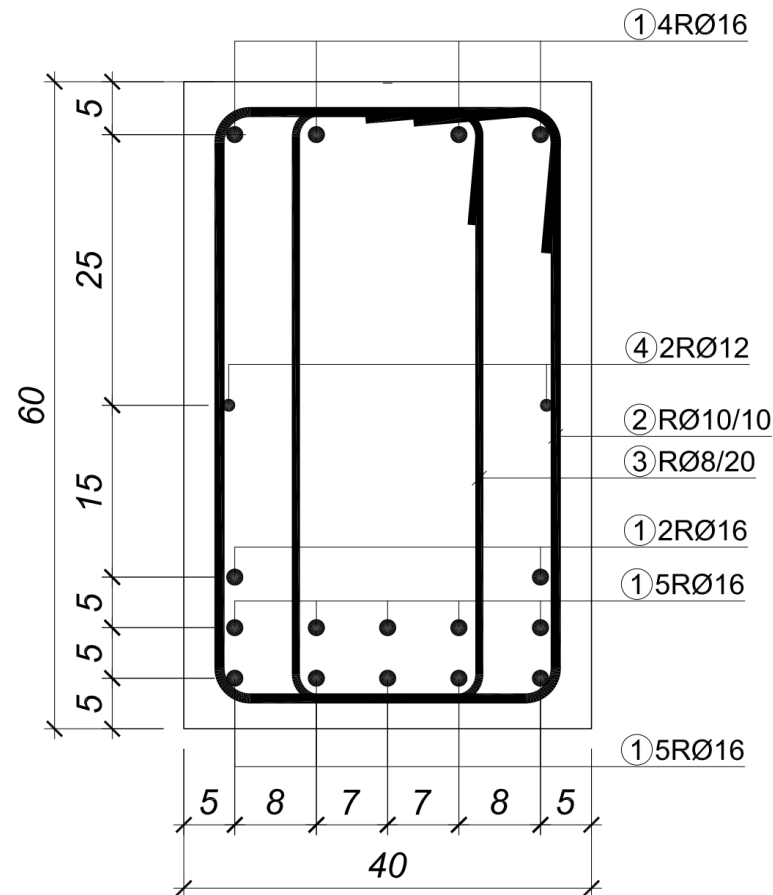




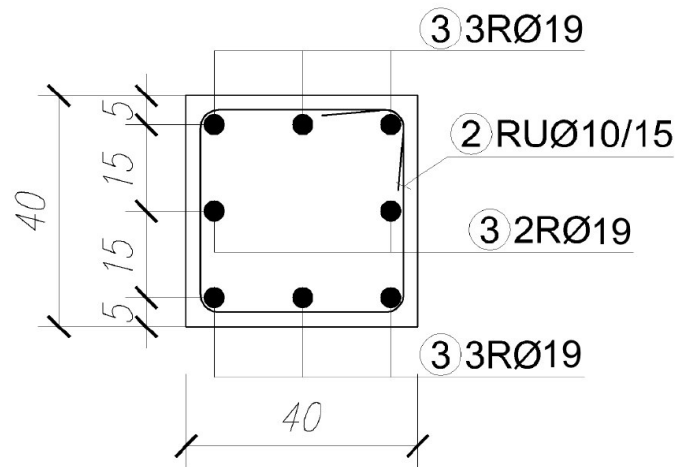


1. Za armiranobetonski element konstantnog poprečnog preseka, armiran prema skici desno potrebno je odrediti:

1. Granični moment savijanja oko jače ose koji presek može da prihvati ne uzimajući u obzir doprinos pritisnute armature.
2. Graničnu transverzalnu silu $V_{u,x}$ koju presek može da prihvati. Usvojiti da je $\theta=35^\circ$.



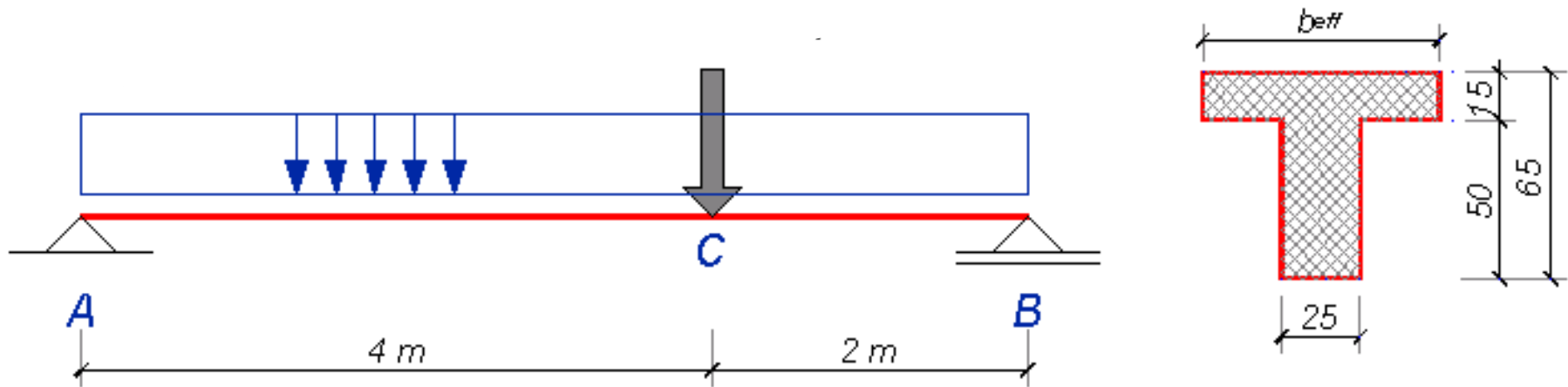
3. AB presek, armiran prema skici, centrično je napregnut silom pritiska $N_g = 600 \text{ kN}$ usled stalnog opterećenja. Uz zadovoljenje propisanih koeficijenata sigurnosti odrediti maksimalne vrednosti sila koje mogu delovati na presek:
- maksimalnu centričnu silu pritiska usled povremenog opterećenja N_p
 - maksimalnu centričnu silu zatezanja usled povremenog opterećenja Z_p (10 poena)



Zadatak 22

Dimenzionisati nosač sistema proste grede, čiji su uticaji i poprečni presek prikazani na skici.

C25/30 B500B XC2



C25/30

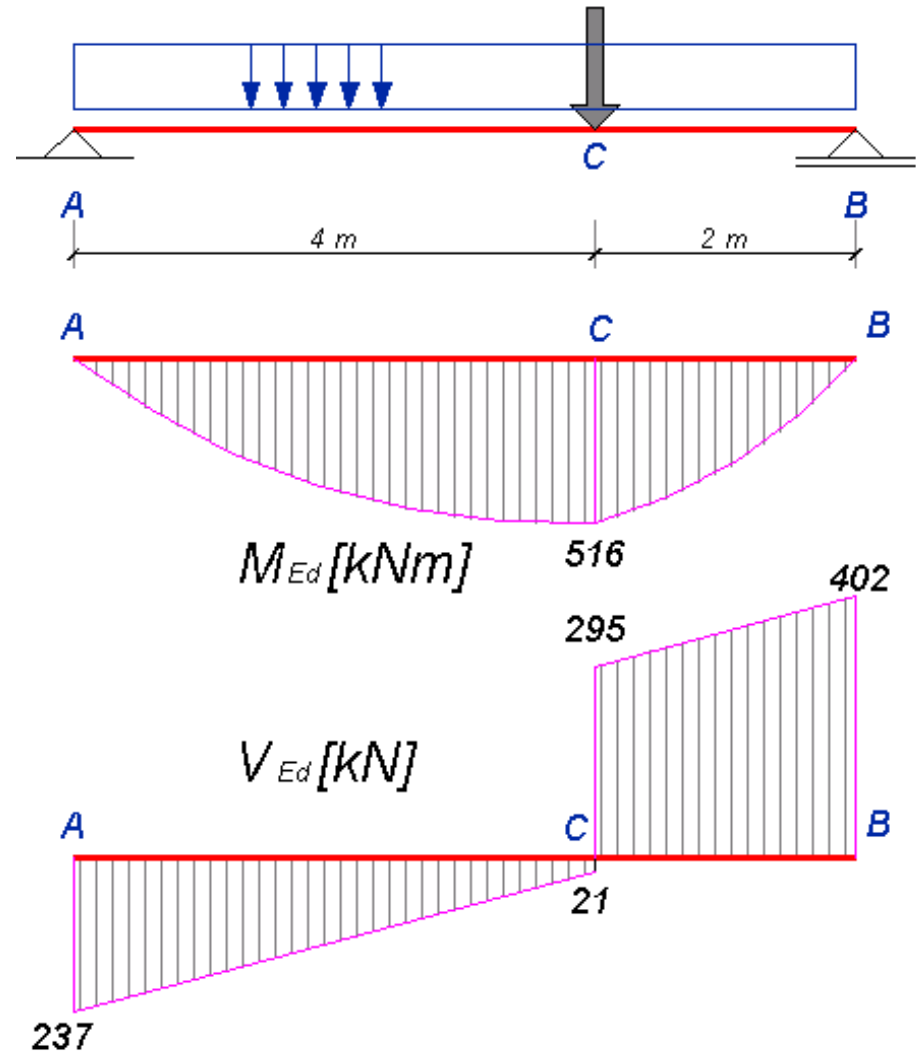
$$f_{cd} = 0.85 \cdot 25 / 1.5 = 14.2 \text{ MPa} = 1.42 \text{ kN/cm}^2$$

B500 B

$$f_{yd} = 500 / 1.15 = 435 \text{ Mpa} = 43.5 \text{ kN/cm}^2$$

Zadatak 22

1. Statički uticaji



Zadatak 22

2. Dimenzionisanje prema momentima savijanja

1. $M_{Ed}=516 \text{ kNm}$

2. pretp. $d_1 = 7 \text{ cm}$

$$d = h - d_1 = 65 - 7 = 58 \text{ cm}$$

3. Efektivna širina: $b_{eff}=0.4 \cdot l_0 + b_w = 0.4 \cdot 600 + 25 = 265 \text{ cm}$

4. Računa se: $k = \frac{d}{\sqrt{\frac{M_{Ed}}{b_{eff} \cdot f_{cd}}}} = \frac{58}{\sqrt{\frac{516 \cdot 10^2}{265 \cdot 1.42}}} = 4.953$ $\varepsilon_c / \varepsilon_{s1} = 3.5/65\text{‰}$

$$A_{s1,lim} = \omega_{1,lim} b_{eff} d \frac{f_{cd}}{f_{yd}} = 4.086 \cdot \frac{265 \cdot 58}{100} \cdot \frac{1.42}{43.5} = 20.5 \text{ cm}^2$$

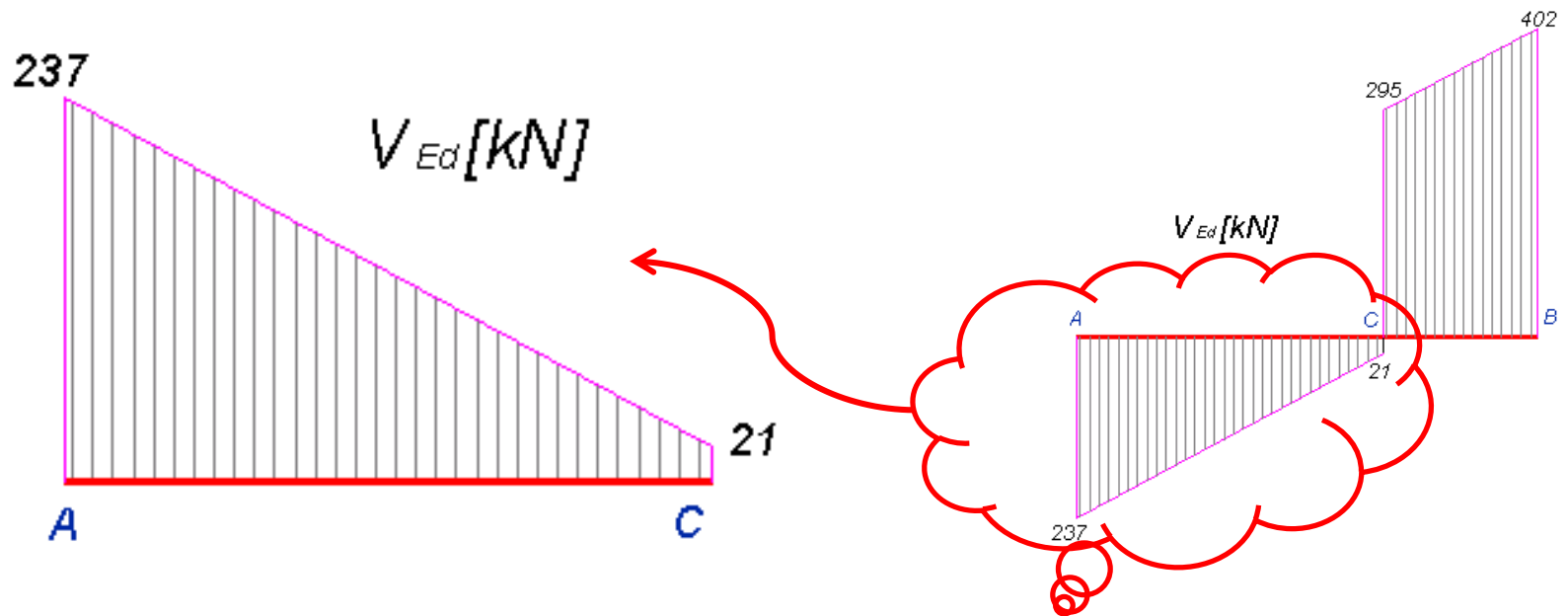
Usvojeno: **5Ø25** (24.55 cm²)



Zadatak 22

3. Dimenzionisanje prema smičućim silama: **DEO A-C**

1. Proračunske vrednosti uticaja na posmatranom segmentu:



Zadatak 22

2. Proračun nosivosti betona na smicanje:

$$V_{Rd,c} = \left[C_{Rd,c} \cdot k \cdot (100 \rho_i f_{ck})^{1/3} + k_1 \cdot \sigma_{cp} \right] \cdot b_w \cdot d$$

$$C_{Rd,c} = \frac{0.18}{\gamma_c} = \frac{0.18}{1.5} = 0.12$$

$$k = 1 + \sqrt{\frac{200}{d}} = 1 + \sqrt{\frac{200}{580}} = 1.587$$

$$\rho_i = \frac{A_{sl}}{b_w \cdot d} = \frac{2 \cdot 4.91}{30 \cdot 58} = 0.0056 < 0.02$$

Vidi plan armature!

$$V_{Rd,c} = \left[0.12 \cdot 1.587 \cdot (100 \cdot 0.0056 \cdot 25)^{1/3} \right] \cdot \frac{25 \cdot 58}{10} = 66.5 \text{ kN}$$

Zadatak 22

3. Minimalna nosivost betona na smicanje:

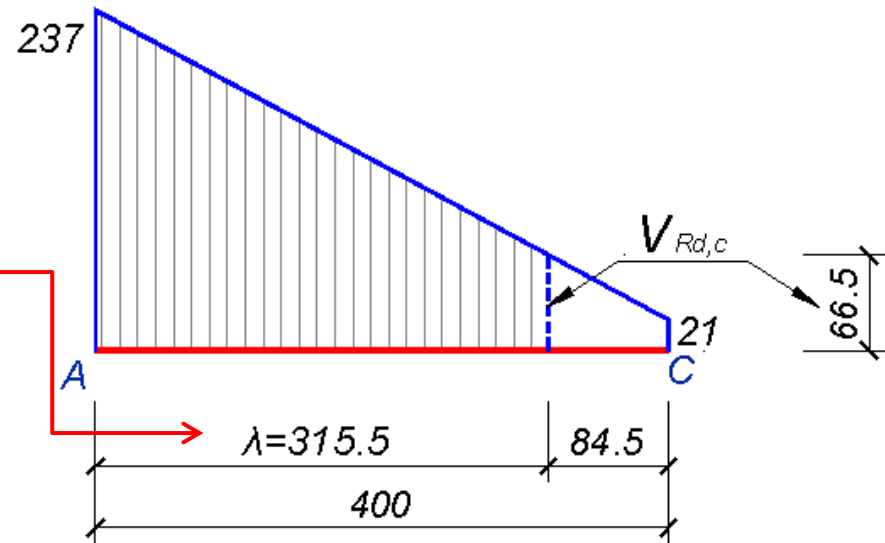
$$V_{\min} = \left[v_{\min} + k_1 \cdot \sigma_{cp} \right] \cdot b_w \cdot d = \left[0.0035 \cdot k^{3/2} \cdot f_{ck}^{1/2} \right] \cdot b_w \cdot d$$

$$V_{\min} = \left[0.0035 \cdot 1.587^{3/2} \cdot 25^{1/2} \right] \cdot 25 \cdot 58 = 50.7 \text{ kN}$$

4. Nosivost betona na smicanje:

$$V_{Rd,c} = \max \left\{ \begin{array}{l} 66.5 \text{ kN} \\ 50.7 \text{ kN} \end{array} \right\} = 66.5 \text{ kN}$$

Na delu nosača – **dužini osiguranja (λ)** potrebno osiguranje poprečnom armaturom!



Zadatak 22

5. Određivanje potrebne armature za smicanje

Na dužini osiguranja:

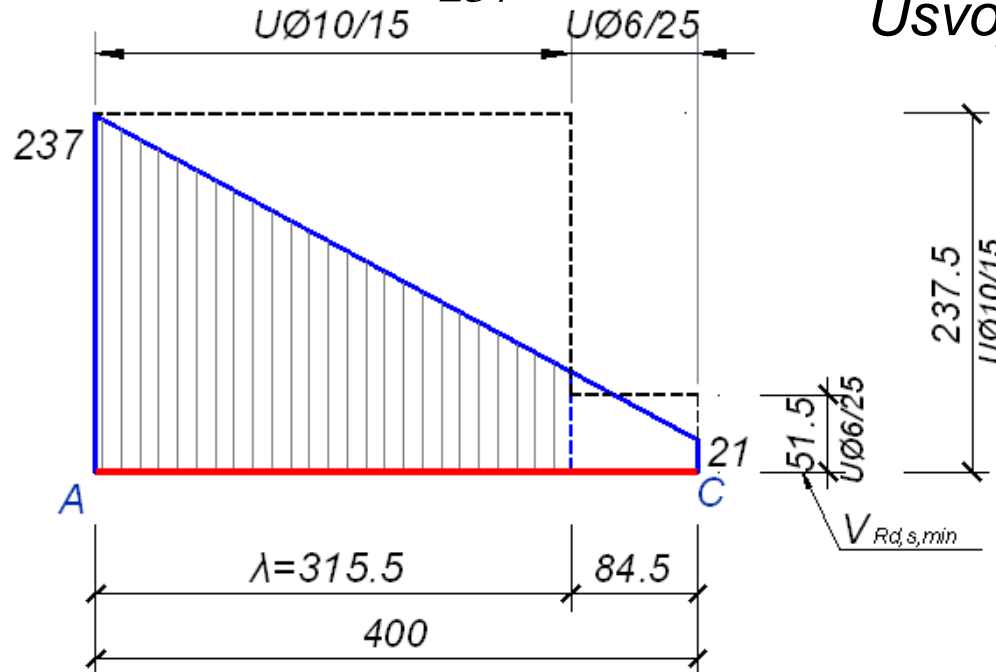
$$V_{Ed} = V_{Rd,s}$$

$$V_{Rd,s} = \frac{m \cdot a_{sw}^{(1)}}{s} \cdot z \cdot f_{ywd} \cdot ctg\theta = V_{Ed}$$

Usvojeno: $\theta=45^0$, $ctg\theta=1$, $m=2$, $\text{Ø}10$, $a_{sw}^{(1)}=0.785 \text{ cm}^2$

$$s_{rač} \leq \frac{m \cdot a_{sw}^{(1)}}{V_{Ed}} \cdot z \cdot f_{ywd} \cdot ctg\theta = \frac{2 \cdot 0.785}{237} \cdot 0.9 \cdot 58 \cdot 43.5 \cdot 1.0 = 15.04 \text{ cm}$$

Usvojeno: **UØ10/15**



Zadatak 22

6. Kontrola minimalne armature za smicanje

$$\rho_{w,\min} = 0.08 \cdot \frac{\sqrt{f_{ck}}}{f_{yk}} = 0.08 \cdot \frac{\sqrt{25}}{500} = 0.0008 = 0.08\%$$

$$\rho_w = \frac{m \cdot a_{sw}^{(1)}}{s \cdot b_w} \quad \longrightarrow \quad s_{\rho,\min} = \frac{m \cdot a_{sw}^{(1)}}{\rho_{w,\min} \cdot b_w} = \frac{2 \cdot 0.785}{0.0008 \cdot 25} = 78.5 \text{ cm}$$

Usvojeno: **UØ6/25**

$$s_{\rho,\min} = \frac{m \cdot a_{sw}^{(1)}}{\rho_{w,\min} \cdot b_w} = \frac{2 \cdot 0.283}{0.0008 \cdot 25} = 28.3 \text{ cm}$$

7. Kontrola maksimalne smičuće nosivosti:

$$V_{Rd,\max} = \frac{\alpha_{cw} \cdot b_w \cdot z \cdot v_1 \cdot f_{cd}}{\text{ctg}\theta + \text{tg}\theta} = \frac{1.0 \cdot 25 \cdot 0.9 \cdot 58 \cdot 0.54 \cdot 1.42}{1+1} = 500.3 \text{ kN}$$

Zadatak 22

8. Maksimalno podužno rastojanje armature s_{\max}

Табела 1 – Највеће подужно растојање између елемената арматуре за смицање $s_{l,\max}$

	Прорачунска вредност силе смицања V_{Ed}^*	Класе чврстоће бетона	
		$\leq C 50/60$	$> C 50/60$
1.	$V_{Ed} \leq 0,3V_{Rd,\max}$	$0,75 d^{**}) \leq 300 \text{ mm}$	$0,75 d \leq 200 \text{ mm}$
2.	$0,3V_{Rd,\max} \leq V_{Ed} \leq 0,6V_{Rd,\max}$	$0,55 d \leq 300 \text{ mm}$	$0,55 d \leq 200 \text{ mm}$
3.	$V_{Ed} > 0,6V_{Rd,\max}$	$0,3 d \leq 200 \text{ mm}$	

* $V_{Rd,\max}$ може да се одреди поједностављено са $\theta = 40^\circ$ ($\cot \theta = 1,2$).

** За греде код којих је $h < 200 \text{ mm}$ и $V_{Ed} \leq V_{Rd}$, растојање не мора да буде мање од 150 mm .

$$0.3 \cdot V_{Rd,\max} = 0.3 \cdot 500.3 = 150.1 \text{ kN} < V_{Ed} < 0.6 \cdot V_{Rd,\max} = 0.6 \cdot 500.3 = 300.2 \text{ kN}$$

$$s_{\max} = \min \{0.55d; 30\text{cm}\} = \min \{0.55 \cdot 58 = 31.9; 30\text{cm}\} = 30\text{cm}$$

Zadatak 22

9. Usvajanje armature za smicanje

Usvojeni prečnik: $\text{Ø}10$

Usvojeno rastojanje:

$$s = \min \left\{ \begin{array}{l} s_{rač} \\ s_{\rho, \min} \\ s_{\max} \end{array} \right\} = \left\{ \begin{array}{l} 15.04 \\ 78.5 \\ 30.0 \end{array} \right\} = 15.04 \text{ cm}$$

Usvojeno: $\text{UØ}10/15$

10. Nosivost usvojene armature za smicanje:

$$V_{Rd,s} = \frac{2 \cdot 0.785}{15} \cdot 0.9 \cdot 58 \cdot 43.5 \cdot 1.0 = 237.7 \text{ kN}$$

$$V_{Rd,s} = 237.7 \text{ kN} \left\{ \begin{array}{l} > 237 \text{ kN} = V_{Ed} \\ < 500.3 \text{ kN} = V_{Rd, \max} \end{array} \right. \begin{array}{l} \longrightarrow \text{Nosivost uzengija veća od proračunske sile smicanja} \\ \longrightarrow \text{Osigurano od loma pritisnute betonske dijagonale} \end{array}$$

Zadatak 22– SMICANJE

11. Dužina osiguranja: dužina $\lambda=315.5 \text{ cm}$

12. Dodatna zategnuta armatura

$$\Delta F_{td} = \left\{ \begin{array}{l} V_{Ed}, V_{Ed} < V_{Rd,c} \\ 0.5 \cdot V_{Ed} \cdot \text{ctg} \theta, V_{Ed} > V_{Rd,c} \end{array} \right\} = 0.5 \cdot 237 \cdot 1.0 = 118.5 \text{ kN}$$

$$\Delta A_{s1} = \frac{\Delta F_{td}}{f_{yd}} = \frac{118.8}{43.5} = 2.72 \text{ cm}^2 < 2 \cdot 4.91 = 9.82 \text{ cm}^2$$

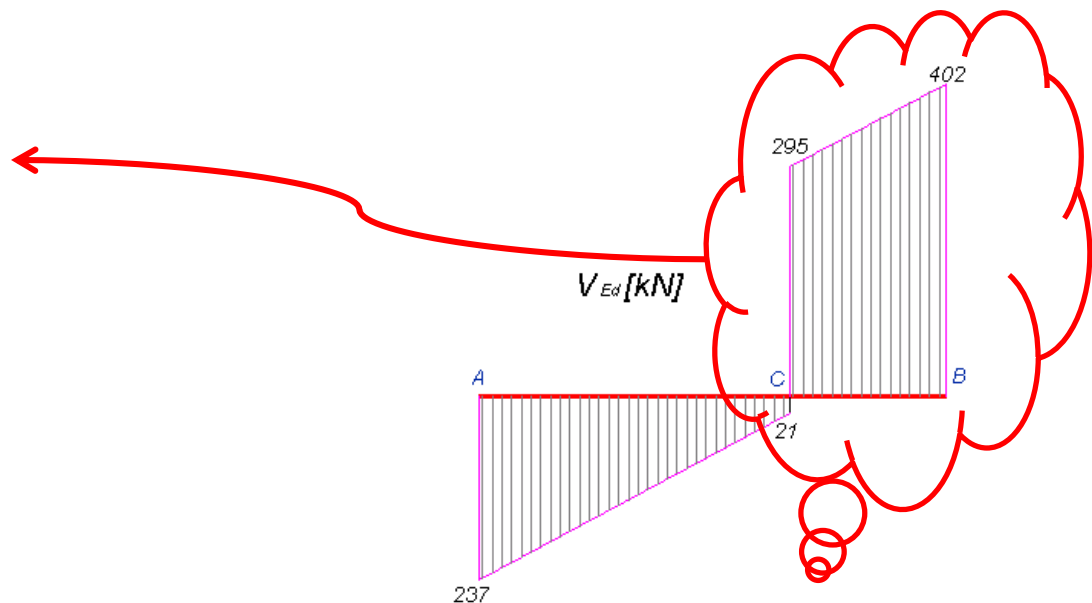
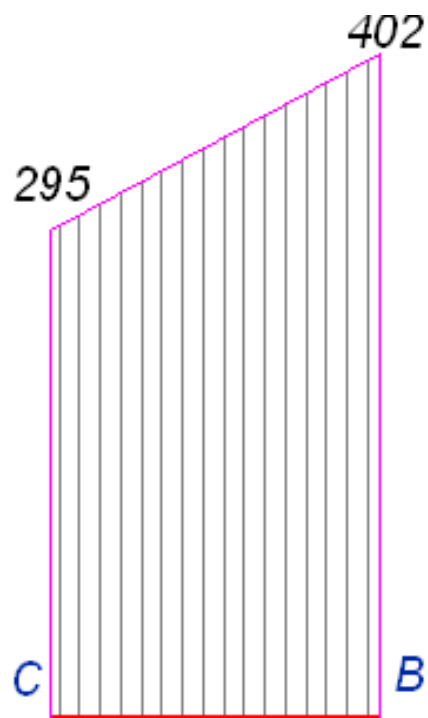
Armatura koju treba obezbediti u donjoj zoni na mestu slobodnog oslonca A

Armatura iz dimenzionisanja prema momentima savijanja – vidi plan armature

Zadatak 22 – SMICANJE

3. Dimenzionisanje prema smičućim silama: **DEO B-C**

1. Proračunske vrednosti uticaja na posmatranom segmentu:



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2. Proračun nosivosti betona na smicanje:

$$V_{Rd,c} = \left[C_{Rd,c} \cdot k \cdot (100 \rho_i f_{ck})^{1/3} + k_1 \cdot \sigma_{cp} \right] \cdot b_w \cdot d$$

$$C_{Rd,c} = \frac{0.18}{\gamma_c} = \frac{0.18}{1.5} = 0.12$$

$$k = 1 + \sqrt{\frac{200}{d}} = 1 + \sqrt{\frac{200}{580}} = 1.587$$

$$\rho_i = \frac{A_{sl}}{b_w \cdot d} = \frac{2 \cdot 4.91}{30 \cdot 58} = 0.0056 < 0.02$$

Vidi plan armature!

$$V_{Rd,c} = \left[0.12 \cdot 1.587 \cdot (100 \cdot 0.0056 \cdot 25)^{1/3} \right] \cdot \frac{25 \cdot 58}{10} = 66.5 \text{ kN}$$

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3. Minimalna nosivost betona na smicanje:

$$V_{\min} = [v_{\min} + k_1 \cdot \sigma_{cp}] \cdot b_w \cdot d = [0.0035 \cdot k^{3/2} \cdot f_{ck}^{1/2}] \cdot b_w \cdot d$$

$$V_{\min} = [0.0035 \cdot 1.587^{3/2} \cdot 25^{1/2}] \cdot 25 \cdot 58 = 50.7 \text{ kN}$$

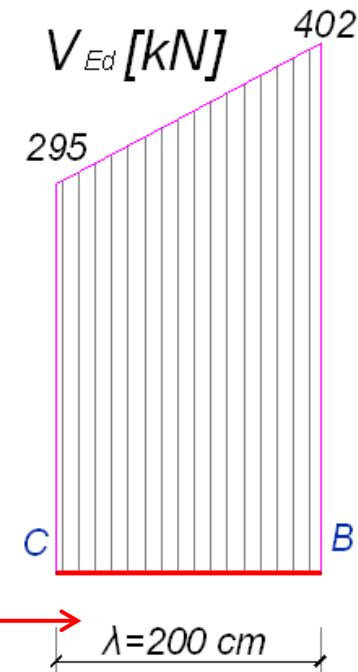
4. Nosivost betona na smicanje:

$$V_{Rd,c} = \max \left\{ \begin{array}{l} 66.5 \text{ kN} \\ 50.7 \text{ kN} \end{array} \right\} 66.5 \text{ kN} < 295 \text{ kN}$$

Ceo segment nosača B-C potrebno je osigurati poprečnom armaturom.



Dužina osiguranja (λ) jednaka je dužini segmenta B-C



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5. Određivanje potrebne armature za smicanje

Na dužini osiguranja:

$$V_{Ed} = V_{Rd,s}$$

$$V_{Rd,s} = \frac{m \cdot a_{sw}^{(1)}}{s} \cdot z \cdot f_{ywd} \cdot ctg\theta = V_{Ed}$$

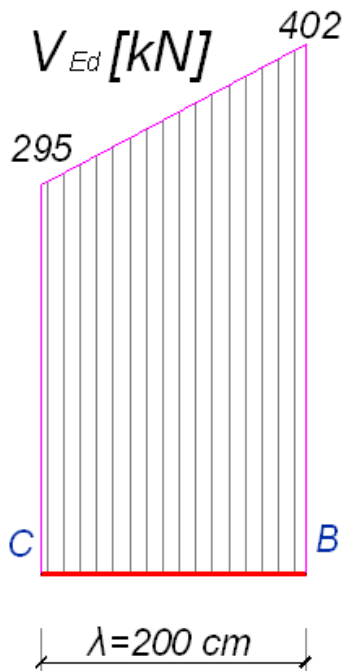
Usvojeno: $\theta=45^0$, $ctg\theta=1$, $m=2$, $\text{Ø}10$, $a_{sw}^{(1)}=0.785 \text{ cm}^2$

$$s_{rač} \leq \frac{m \cdot a_{sw}^{(1)}}{V_{Ed}} \cdot z \cdot f_{ywd} \cdot ctg\theta = \frac{2 \cdot 0.785}{402} \cdot 0.9 \cdot 58 \cdot 43.5 \cdot 1.0 = 8.9 \text{ cm}$$

Usvojeno: $\theta=45^0$, $ctg\theta=1$, $m=2$, $\text{Ø}12$, $a_{sw}^{(1)}=1.13 \text{ cm}^2$

$$s_{rač} \leq \frac{m \cdot a_{sw}^{(1)}}{V_{Ed}} \cdot z \cdot f_{ywd} \cdot ctg\theta = \frac{2 \cdot 1.13}{402} \cdot 0.9 \cdot 58 \cdot 43.5 \cdot 1.0 = 12.7 \text{ cm}$$

Usvojeno: **UØ12/12.5**



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5. Nosivost usvojenih uzengija:

Usvojeno: $\theta=45^\circ$, $\text{ctg}\theta=1$, $m=2$, $\text{Ø}12$, $a_{sw}^{(1)}=0.785 \text{ cm}^2$ **$s=12.5 \text{ cm}$**

$$V_{Rd,s} = \frac{m \cdot a_{sw}^{(1)}}{s} \cdot z \cdot f_{ywd} \cdot \text{ctg}\theta$$

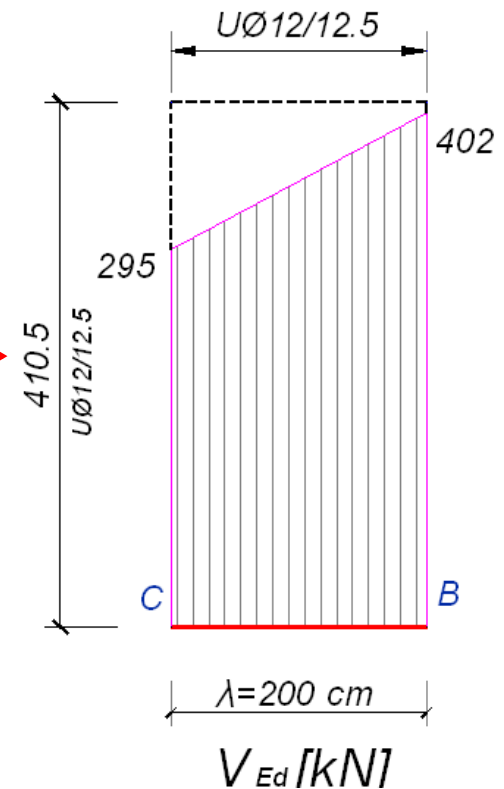
$$V_{Rd,s} = \frac{2 \cdot 1.13}{12.5} \cdot 0.9 \cdot 58 \cdot 43.5 \cdot 1.0 = 410.5 \text{ kN} \left\{ \begin{array}{l} > 51.4 = V_{Rd,s,\min} \\ < 500.3 = V_{Rd,\max} \end{array} \right.$$

6. Nosivost minimalne armature za smicanje

$$V_{Rd,s,\min} = \frac{2 \cdot 0.283}{25} \cdot 0.9 \cdot 58 \cdot 43.5 \cdot 1.0 = 51.4 \text{ kN}$$

7. Kontrola maksimalne smičuće nosivosti:

$$V_{Rd,\max} = \frac{\alpha_{cw} \cdot b_w \cdot z \cdot v_1 \cdot f_{cd}}{\text{ctg}\theta + \text{tg}\theta} = \frac{1.0 \cdot 25 \cdot 0.9 \cdot 58 \cdot 0.54 \cdot 1.42}{1+1} = 500.3 \text{ kN}$$



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11. Dužina osiguranja: dužina $\lambda=200$ cm

12. Dodatna zategnuta armatura

$$\Delta F_{td} = \left\{ \begin{array}{l} V_{Ed}, V_{Ed} < V_{Rd,c} \\ 0.5 \cdot V_{Ed} \cdot \text{ctg} \theta, V_{Ed} > V_{Rd,c} \end{array} \right\} = 0.5 \cdot 402 \cdot 1.0 = 201 \text{ kN}$$

$$\Delta A_{s1} = \frac{\Delta F_{td}}{f_{yd}} = \frac{201}{43.5} = 4.62 \text{ cm}^2 < 2 \cdot 4.91 = 9.82 \text{ cm}^2$$

Armatura koju treba obezbediti u donjoj zoni na mestu slobodnog oslonca B

Armatura iz dimenzionisanja prema momentima savijanja – vidi plan armature