



Univerzitet u Beogradu – Građevinski fakultet  
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Studijski program: **Građevinarstvo**  
Modul: Konstrukcije  
Godina/Semestar: **III godina / V semestar**

Naziv predmeta (šifra): **Teorija betonskih konstrukcija 1  
(b2k3b1)**

Nastavnik: **Ivan Ignjatović**

Naslov vežbi: **Kombinacije opterećenja**

Datum : 03.11.2022.

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Beograd, 2020.

*Sva autorska prava autora prezentacije i/ili video snimaka su zaštićena. Snimak ili prezentacija se mogu koristiti samo za nastavu na daljinu studenta Građevinskog fakulteta Univerziteta u Beogradu u školskoj 2020/2021 i ne mogu se koristiti za druge svrhe bez pismene saglasnosti autora*



## Zadatak 20 – KOMBINOVANJE OPTEREĆENJA

*Odrediti potrebnu površinu armature za stub poznatih dimenzija, pravougaonog poprečnog preseka, koji je opterećen momentima savijanja usled stalnog ( $M_G$ ) i opterećenja vetrom ( $M_w$ ). Podaci za proračun:*

$$M_G = 100 \text{ kNm}$$

$$M_w = \pm 200 \text{ kNm}$$

$$b = 25 \text{ cm}$$

$$h = 65 \text{ cm}$$

*C25/30*

*B500B*



### **a. zategnuta spoljašnja ivica stuba**

$$M_{Ed} = 1.35 \times 100 + 1.5 \times 200 = 435 \text{ kNm}$$

$$\text{pretp. } d_1 = 7 \text{ cm}$$

$$d = 65 - 7 = 58 \text{ cm}$$

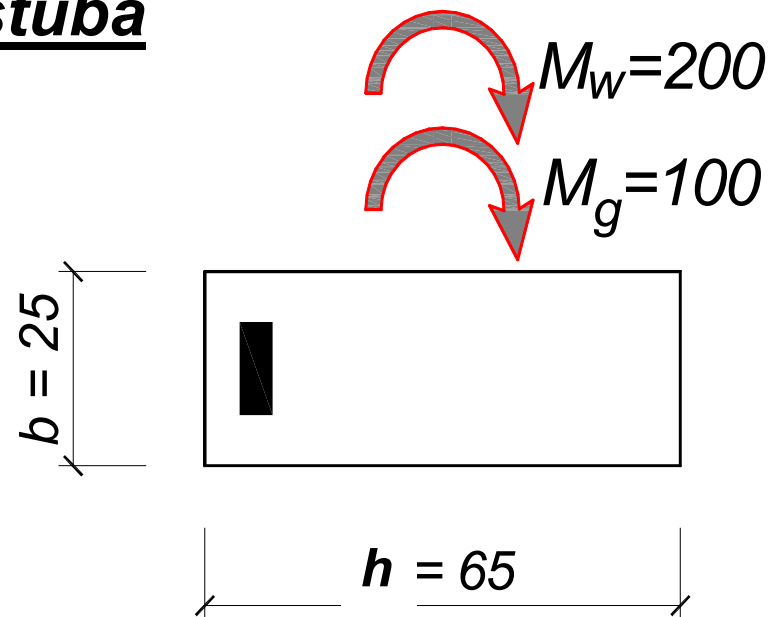
$$C25/30 \Rightarrow f_{cd} = 14.2 \text{ MPa}$$

$$k = \frac{58}{\sqrt{\frac{435 \times 10^2}{25 \times 1.42}}} = 1.657$$

$$\varepsilon_c / \varepsilon_{s1} = 3.5 / 2.321\text{‰} \Rightarrow \omega_1 = 48.673\%$$

$$A_{s1} = 48.673 \times \frac{25 \times 58}{100} \times \frac{1.42}{43.5} = 23.04 \text{ cm}^2$$

*usvojeno:*                      **5Ø25 (24.55 cm<sup>2</sup>)**



$$\varepsilon_{s1} < 2.5\text{‰}, \text{ ali}$$

$$\varepsilon_{s1} > \varepsilon_{yd} (= f_{yd} / E_s)$$

## **b. zategnuta unutrašnja ivica stuba**

$$M_{Ed} = 1.0 \times (-100) + 1.5 \times 200 = 200 \text{ kNm}$$

$$\text{pretp. } d_1 = 5 \text{ cm}$$

$$d = 65 - 5 = 60 \text{ cm}$$

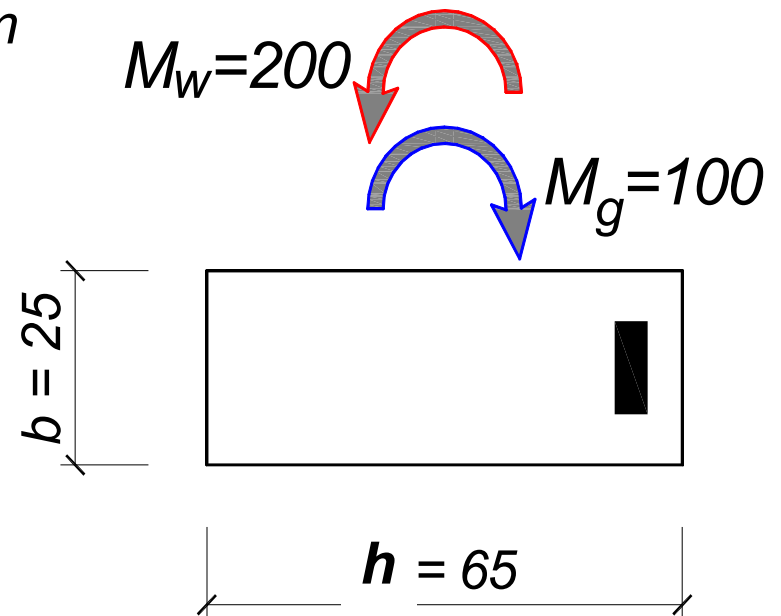
$$C25/30 \Rightarrow f_{cd} = 14.2 \text{ MPa}$$

$$k = \frac{60}{\sqrt{\frac{200 \times 10^2}{25 \times 1.42}}} = 2.528$$

$$\varepsilon_c / \varepsilon_{s1} = 3.5 / 13.18\% \Rightarrow \omega_1 = 16.982\%$$

$$A_{s1} = 16.982 \times \frac{25 \times 60}{100} \times \frac{1.42}{43.5} = 8.31 \text{ cm}^2$$

*usvojeno:*                      **2Ø25** (9.82 cm<sup>2</sup>)



## Zadatak 21 – KOMBINOVANJE OPTEREĆENJA

*Odrediti potrebnu površinu armature za stub poznatih dimenzija, pravougaonog poprečnog preseka, opterećen zadatim uticajima. Podaci za proračun:*

$$M_G = 100 \text{ kNm}$$

$$N_G = 500 \text{ kN}$$

$$M_w = \pm 200 \text{ kNm}$$

$$b = 25 \text{ cm}$$

$$h = 65 \text{ cm}$$

C25/30

B500 B



### **a. zategnuta spoljašnja ivica stuba**

$$M_{Ed} = 1.35 \times 100 + 1.5 \times 200 = 435 \text{ kNm}$$

$$N_{Ed} = 1.35 \times 500 = 675 \text{ kN}$$

$$\text{pretp. } d_1 = 7 \text{ cm}$$

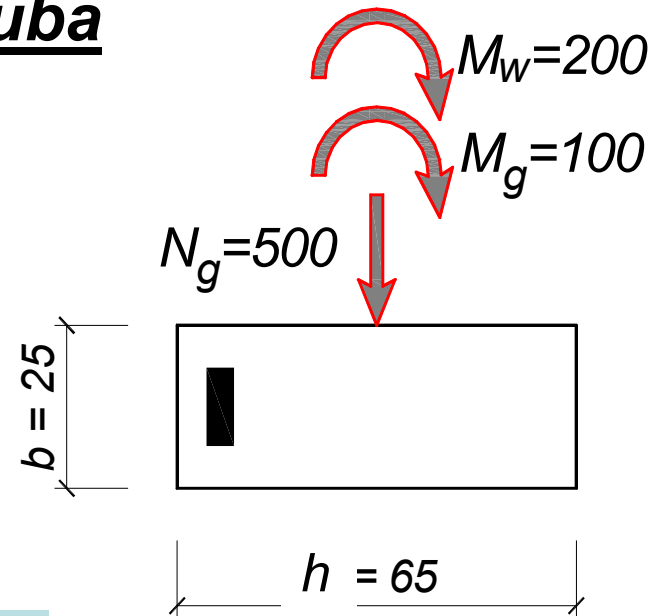
$$d = 65 - 7 = 58 \text{ cm}$$

$$C25/30 \Rightarrow f_{cd} = 14.2 \text{ MPa}$$

$$M_{Eds} = 435 + 675 \times \left( \frac{0.65}{2} - 0.07 \right) = 607.1 \text{ kNm}$$

$$k = \frac{58}{\sqrt{\frac{607.1 \times 10^2}{25 \times 1.42}}} = 1.402 \Rightarrow \varepsilon_{s1} < 2.5\text{‰}$$

**Kako je  $\varepsilon_{s1} < 2.5\text{‰}$ , presek se **OBOSTRANO ARMIRA**.**



**usvojeno**  $\varepsilon_{s1,lim} = 2.5\text{‰} \Rightarrow k_{lim} = 1.672, \omega_{Rd,lim} = 47.222\%$

$$M_{Rd,lim} = \left( \frac{58}{1.672} \right)^2 \times 25 \times 1.42 \times 10^{-2} = 427.2 \text{ kNm}$$

$$\Delta M = 607.1 - 427.2 = 179.9 \text{ kNm}$$

$$\text{pretp. } d_2 = 5 \text{ cm} \Rightarrow A_{s2} = \frac{179.9 \times 10^2}{(58 - 5) \times 43.5} = 7.8 \text{ cm}^2$$

$$A_{s1} = 47.222 \times \frac{25 \times 58}{100} \times \frac{1.42}{43.5} - \frac{675}{43.5} + 7.8 = 14.63 \text{ cm}^2$$



## **b. zategnuta unutrašnja ivica stuba**

$$M_{Ed} = 1.0 \times (-100) + 1.5 \times 200 = 200 \text{ kNm}$$

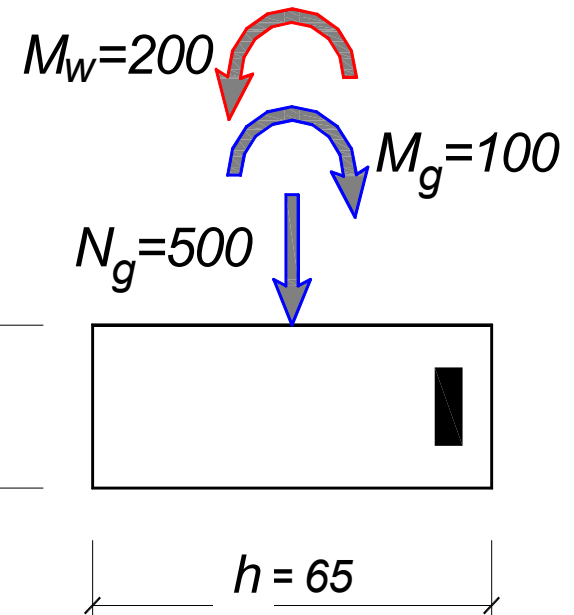
$$N_{Ed} = 1.0 \times 500 = 500 \text{ kN}$$

$$\text{pretp. } d_1 = 5 \text{ cm}$$

$$d = 65 - 5 = 60 \text{ cm}$$

$$C25/30 \Rightarrow f_{cd} = 14.2 \text{ MPa}$$

**“povoljno”  
dejstvo stalnog  
opterećenja**

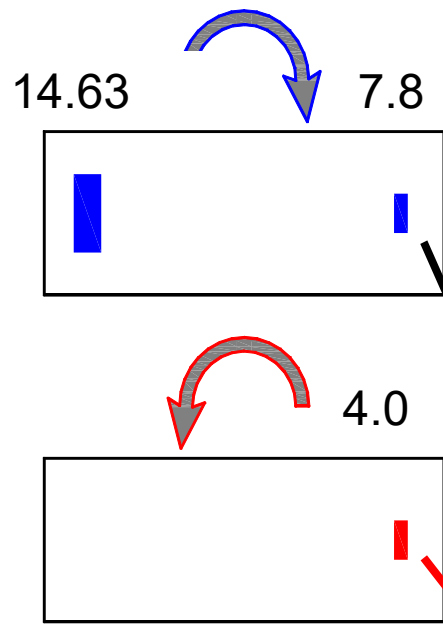


$$M_{Eds} = 200 + 500 \times \left( \frac{0.65}{2} - 0.05 \right) = 337.5 \text{ kNm}$$

$$k = \frac{60}{\sqrt{\frac{337.5 \times 10^2}{25 \times 1.42}}} = 1.946 \Rightarrow \begin{aligned} \varepsilon_c / \varepsilon_{s1} &= 3.5 / 5.435\text{‰} \\ \omega_1 &= 31.646\% \end{aligned}$$

$$A_{s1} = 31.646 \times \frac{25 \times 60}{100} \times \frac{1.42}{43.5} - \frac{500}{43.5} = 4.0 \text{ cm}^2$$





*POTREBNO:*



*unutra:*  $A_{s,potr.} = \max. \left\{ \begin{matrix} 7.8 \\ 4.0 \end{matrix} \right\} = 7.8 \text{ cm}^2$

*usvojeno: 3Ø20 (9.42 cm<sup>2</sup>)*

*spolja:*  $A_{s,potr.} = 14.63 \text{ cm}^2$

*usvojeno: 5Ø20 (15.7 cm<sup>2</sup>)*

## Zadatak 22 – KOMBINOVANJE OPTEREĆENJA

*Dimenzionisati stub poznatih dimenzija, pravougaonog poprečnog preseka, opterećen zadatim uticajima. Opterećenja  $q$  i  $w$  su povremena i NE MORAJU delovati istovremeno. Podaci za proračun:*

$$M_G = 100 \text{ kNm}$$

$$N_Q = 500 \text{ kN}$$

$$M_w = \pm 200 \text{ kNm}$$

$$b = 25 \text{ cm}$$

$$h = 65 \text{ cm}$$

C25/30

B500B



## a. zategnuta spoljašnja ivica stuba

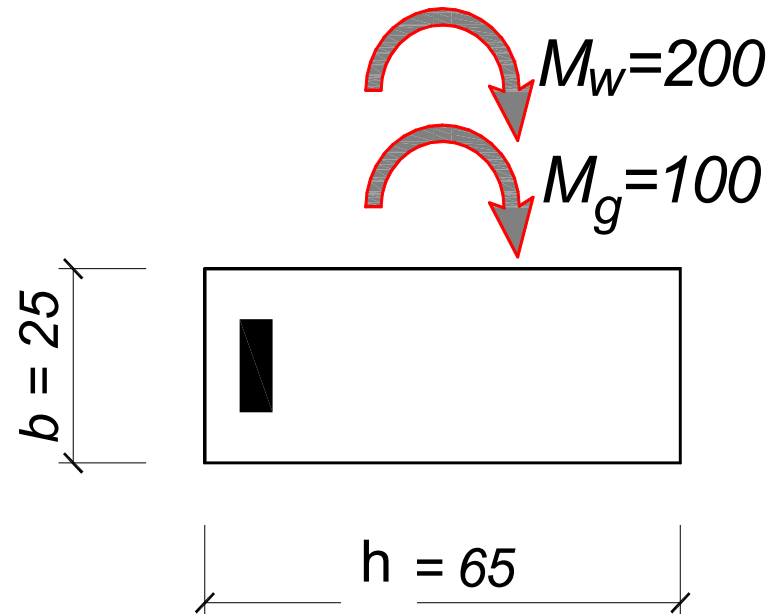
### a.1 MINIMALNA sila pritiska

$$M_{Ed} = 1.35 \times 100 + 1.5 \times 200 = 435 \text{ kNm}$$

$$N_{Ed} = 0$$

$$\text{pretp. } d_1 = 7 \text{ cm} \Rightarrow d = 65 - 7 = 58 \text{ cm}$$

$$C25/30 \Rightarrow f_{cd} = 14.2 \text{ MPa}$$



$$k = \frac{58}{\sqrt{\frac{435 \times 10^2}{25 \times 1.42}}} = 1.657$$

$$\varepsilon_c / \varepsilon_{s1} = 3.5 / 2.321\text{‰} \Rightarrow \omega_1 = 48.673\%$$

$$A_{s1} = 48.673 \times \frac{25 \times 58}{100} \times \frac{1.42}{43.5} = 23.04 \text{ cm}^2$$

*usvojeno:* **5Ø25 (24.55 cm<sup>2</sup>)**



## a. zategnuta spoljašnja ivica stuba

### a.2 MAKSIMALNA sila pritiska

#### a.2.1 dominantno promenljivo – VETAR

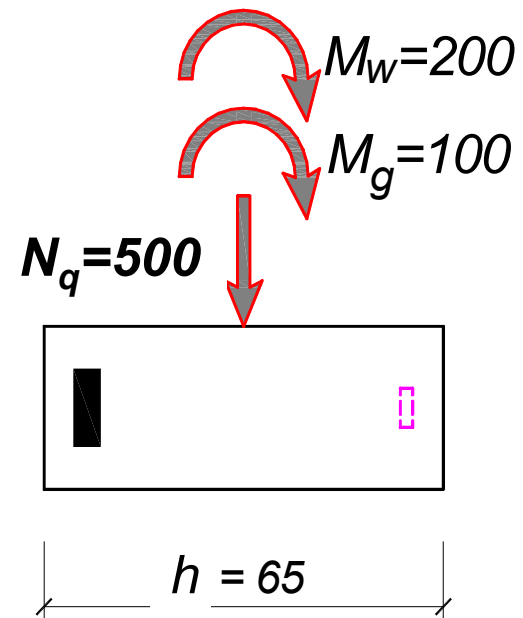
$$1,35 \cdot G_k + 1,5 \cdot (F_{k,w} + 0,5 \cdot Q_{k,n} + 0,7 \cdot Q_{k,es}) = 1,35 \cdot G_k + 1,5 \cdot F_{k,w} + 0,75 \cdot Q_{k,n} + 1,05 \cdot Q_{k,es}$$

$$M_{Ed} = 1,35 \times 100 + 1,5 \times 200 = 435 \text{ kNm}$$

$$N_{Ed} = 1,5 \times 0,7 \times 500 = 525 \text{ kN}$$

$$M_{Eds} = 435 + 525 \times \left( \frac{0,65}{2} - 0,07 \right) = 568,9 \text{ kNm}$$

$$k = \frac{58}{\sqrt{\frac{568,9 \times 10^2}{25 \times 1,42}}} = 1,449 \Rightarrow \varepsilon_{s1} < 2,5\text{‰}$$



**Kako je  $\varepsilon_{s1} < 2,5\text{‰}$ , presek se **OBOSTRANO ARMIRA.****

**usvojeno**  $\varepsilon_{s1,lim} = 2.5\text{‰} \Rightarrow k_{lim} = 1.672, \omega_{Rd,lim} = 47.222\%$

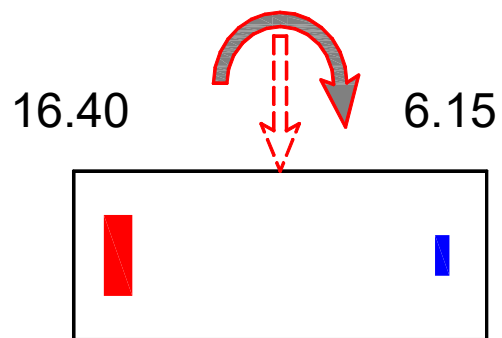
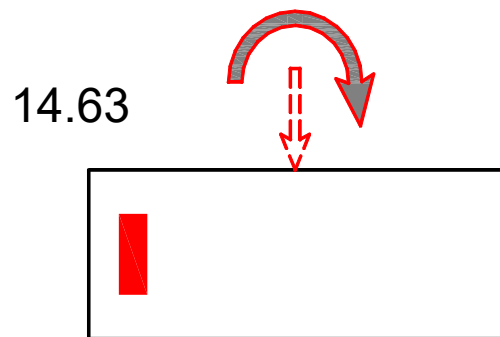
$$M_{Rd,lim} = \left( \frac{58}{1.672} \right)^2 \times 25 \times 1.42 \times 10^{-2} = 427.2 \text{ kNm}$$

$$\Delta M = 568.9 - 427.2 = 141.7 \text{ kNm}$$

$$\text{pretp. } d_2 = 5 \text{ cm} \Rightarrow A_{s2} = \frac{141.7 \times 10^2}{(58 - 5) \times 43.5} = 6.15 \text{ cm}^2$$

$$A_{s1} = 47.222 \times \frac{25 \times 58}{100} \times \frac{1.42}{43.5} - \frac{525}{43.5} + 6.15 = 16.4 \text{ cm}^2$$





**POTREBNO:**

23.04

6.15



$$\textit{spolja: } A_{s,potr.} = \max. \left\{ \begin{array}{l} 23.04 \\ 16.40 \end{array} \right\} = 23.04 \text{ cm}^2$$

$$\textit{unutra: } A_{s,potr.} = \max. \left\{ \begin{array}{l} 0 \\ 6.15 \end{array} \right\} = 6.15 \text{ cm}^2$$

## a. zategnuta spoljašnja ivica stuba

### a.2 MAKSIMALNA sila pritiska

#### a.2.2 dominantno promenljivo – **KORISNO**

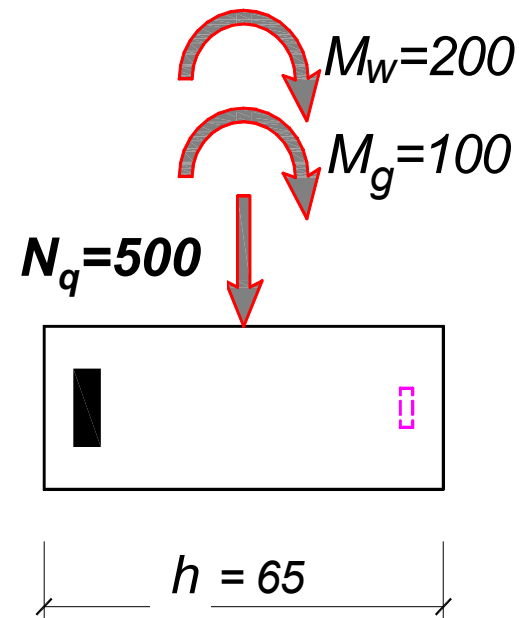
$$1,35 \cdot G_k + 1,5 \cdot (Q_{k,es} + 0,5 \cdot Q_{k,n} + 0,6 \cdot F_{k,w}) = 1,35 \cdot G_k + 1,5 \cdot Q_{k,es} + 0,75 \cdot Q_{k,n} + 0,9 \cdot F_{k,w}$$

$$M_{Ed} = 1,35 \times 100 + 1,5 \times 0,6 \times 200 = 315 \text{ kNm}$$

$$N_{Ed} = 1,5 \times 500 = 750 \text{ kN}$$

$$M_{Eds} = 315 + 750 \times \left( \frac{0,65}{2} - 0,07 \right) = 506,3 \text{ kNm}$$

$$k = \frac{58}{\sqrt{\frac{506,3 \times 10^2}{25 \times 1,42}}} = 1,536 \Rightarrow \varepsilon_{s1} < 2,5\text{‰}$$



**Kako je  $\varepsilon_{s1} < 2,5\text{‰}$ , presek se **OBOSTRANO** ARMIRA.**

**usvojeno**  $\varepsilon_{s1,lim} = 2.5\text{‰} \Rightarrow k_{lim} = 1.672, \omega_{Rd,lim} = 47.222\%$

$$M_{Rd,lim} = \left( \frac{58}{1.672} \right)^2 \times 25 \times 1.42 \times 10^{-2} = 427.2 \text{ kNm}$$

$$\Delta M = 506.3 - 427.2 = 79.1 \text{ kNm}$$

$$\text{pretp. } d_2 = 5 \text{ cm} \Rightarrow A_{s2} = \frac{79.1 \times 10^2}{(58 - 5) \times 43.5} = 3.43 \text{ cm}^2$$

$$A_{s1} = 47.222 \times \frac{25 \times 58}{100} \times \frac{1.42}{43.5} - \frac{750}{43.5} + 3.43 = 8.54 \text{ cm}^2$$



## **b. zategnuta unutrašnja ivica stuba**

### **b.1 MINIMALNA sila pritiska**

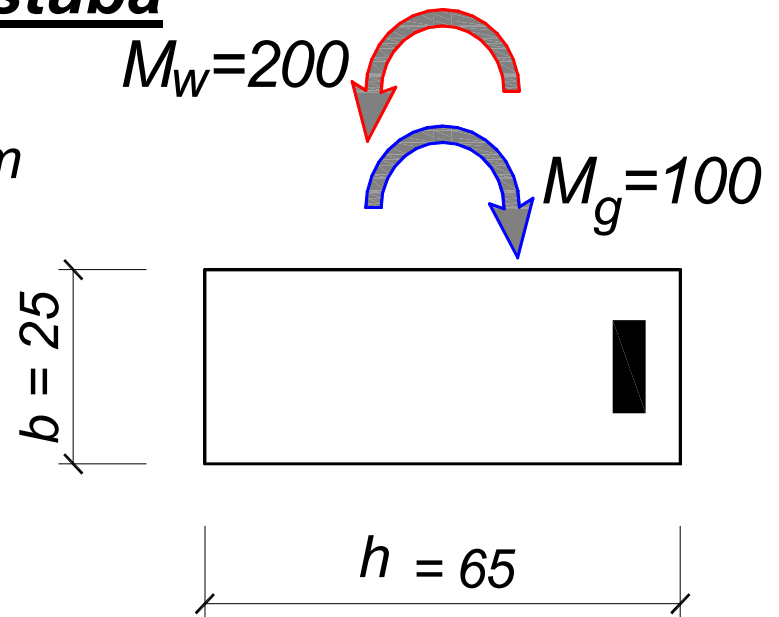
$$M_{Ed} = 1.0 \times (-100) + 1.5 \times 200 = 200 \text{ kNm}$$

$$N_{Ed} = 0$$

(videti primer 1b)

pretp.  $d_1 = 5 \text{ cm} \Rightarrow d = 65 - 5 = 60 \text{ cm}$

C25/30  $\Rightarrow f_{cd} = 14.2 \text{ MPa}$



$$k = \frac{60}{\sqrt{\frac{200 \times 10^2}{25 \times 1.42}}} = 2.528$$

$$\varepsilon_c / \varepsilon_{s1} = 3.5 / 13.18\% \Rightarrow \omega_1 = 16.982\%$$

$$A_{s1} = 16.982 \times \frac{25 \times 60}{100} \times \frac{1.42}{43.5} = 8.31 \text{ cm}^2$$

usvojeno:

**2Ø25** (9.82 cm<sup>2</sup>)

## **b. zategnuta spoljašnja ivica stuba**

b.2 **MAKSIMALNA** sila pritiska

**dominantno promenljivo – VETAR**

$$1,35 \cdot G_k + 1,5 \cdot (F_{k,w} + 0,5 \cdot Q_{k,n} + 0,7 \cdot Q_{k,es}) = 1,35 \cdot G_k + 1,5 \cdot F_{k,w} + 0,75 \cdot Q_{k,n} + 1,05 \cdot Q_{k,es}$$

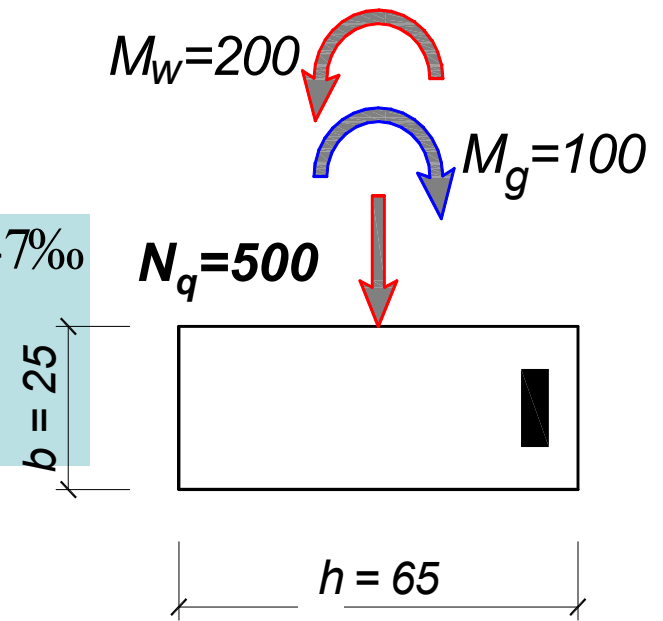
$$M_{Ed} = 1,0 \times (-100) + 1,5 \times 200 = 200 \text{ kNm}$$

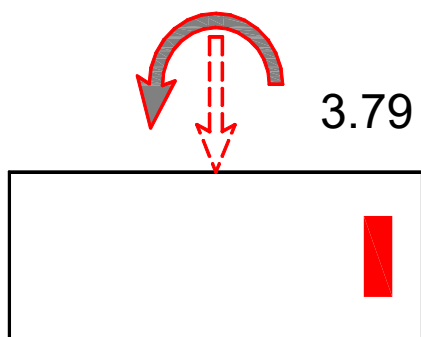
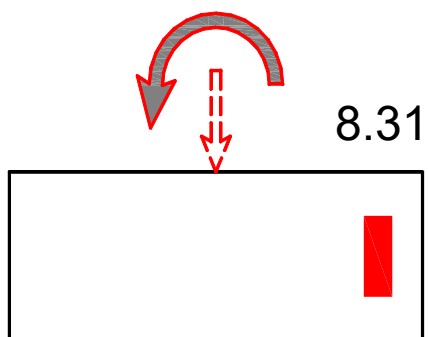
$$N_{Ed} = 1,5 \times 0,7 \times 500 = 525 \text{ kN}$$

$$M_{Eds} = 200 + 525 \times \left( \frac{0,65}{2} - 0,05 \right) = 443,4 \text{ kNm}$$

$$k = \frac{60}{\sqrt{\frac{344,4 \times 10^2}{25 \times 1,42}}} = 1,926 \Rightarrow \begin{aligned} \varepsilon_c / \varepsilon_{s1} &= 3,5 / 5,247\text{‰} \\ \omega &= 32,391\% \end{aligned}$$

$$A_{a1} = 32,391 \times \frac{25 \times 60}{100} \times \frac{1,42}{43,5} - \frac{525}{43,5} = 3,79 \text{ cm}^2$$





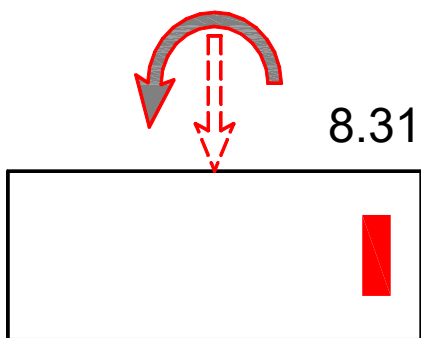
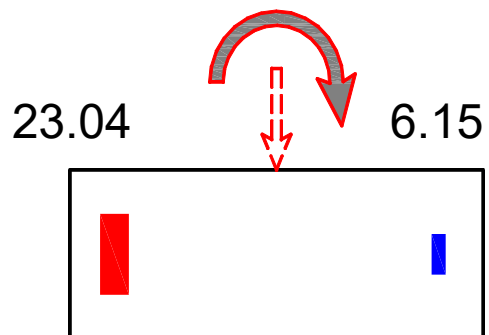
*POTREBNO:*

8.31



*spolja:*  $A_{s,potr.} = 0$

*unutra:*  $A_{s,potr.} = \max. \left\{ \begin{matrix} 8.31 \\ 3.79 \end{matrix} \right\} = 8.31 \text{ cm}^2$



*POTREBNO:*

23.04                      8.31



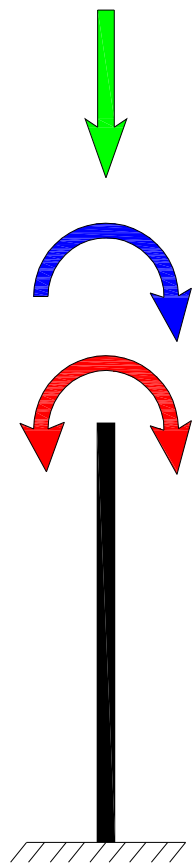
$$spolja: A_{s,potr.} = \max. \begin{Bmatrix} 23.04 \\ 0 \end{Bmatrix} = 23.04 cm^2$$

**5Ø25 (24.55 cm<sup>2</sup>)**

$$unutra: A_{s,potr.} = \max. \begin{Bmatrix} 6.15 \\ 8.31 \end{Bmatrix} = 8.31 cm^2$$

**2Ø25 (9.82 cm<sup>2</sup>)**

# Zadatak 23 – KOMBINOVANJE OPTEREĆENJA



$$N_Q = 800 \text{ kN}$$

$$M_G = 500 \text{ kNm}$$

$$M_W = \pm 200 \text{ kNm}$$

$$M_{Ed} = 1.35 \cdot 100 + 1.5 \cdot 200 = 435 \text{ kNm}$$

$$N_{Ed} = 0$$

$$\Rightarrow A_{s1} = 24.8 \text{ cm}^2 \quad A_{s2} = 0.3 \text{ cm}^2$$

$$M_{Ed} = 1.35 \cdot 100 + 1.5 \cdot 200 = 435 \text{ kNm}$$

$$N_{Ed} = 1.5 \cdot 0.7 \cdot 800 = 840 \text{ kN}$$

$$\Rightarrow A_{s1} = 14.3 \text{ cm}^2 \quad A_{s2} = 9.14 \text{ cm}^2$$

$$M_{Ed} = 1.35 \cdot 100 + 1.5 \cdot 0.6 \cdot 200 = 315 \text{ kNm}$$

$$N_{Ed} = 1.5 \cdot 800 = 1200 \text{ kN}$$

$$\Rightarrow A_{s1} = 4.7 \text{ cm}^2 \quad A_{s2} = 7.8 \text{ cm}^2$$

$$M_{Ed} = 1.0 \cdot (-100) + 1.5 \cdot 200 = 200 \text{ kNm}$$

$$N_{Ed} = 0$$

$$\Rightarrow A_{s2} = 9.77 \text{ cm}^2$$

