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**Građevinarstvo**

Modul:

Konstrukcije

Godina/Semestar:

**III godina / V semestar**

Naziv predmeta (šifra):

**Teorija betonskih konstrukcija 1  
(B3O3B1)**

Nastavnik:

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Naslov vežbi:

**Moment nosivosti preseka.  
Centrični pritisak. Centrično zatezanje.**

Datum :

26.10.2024.

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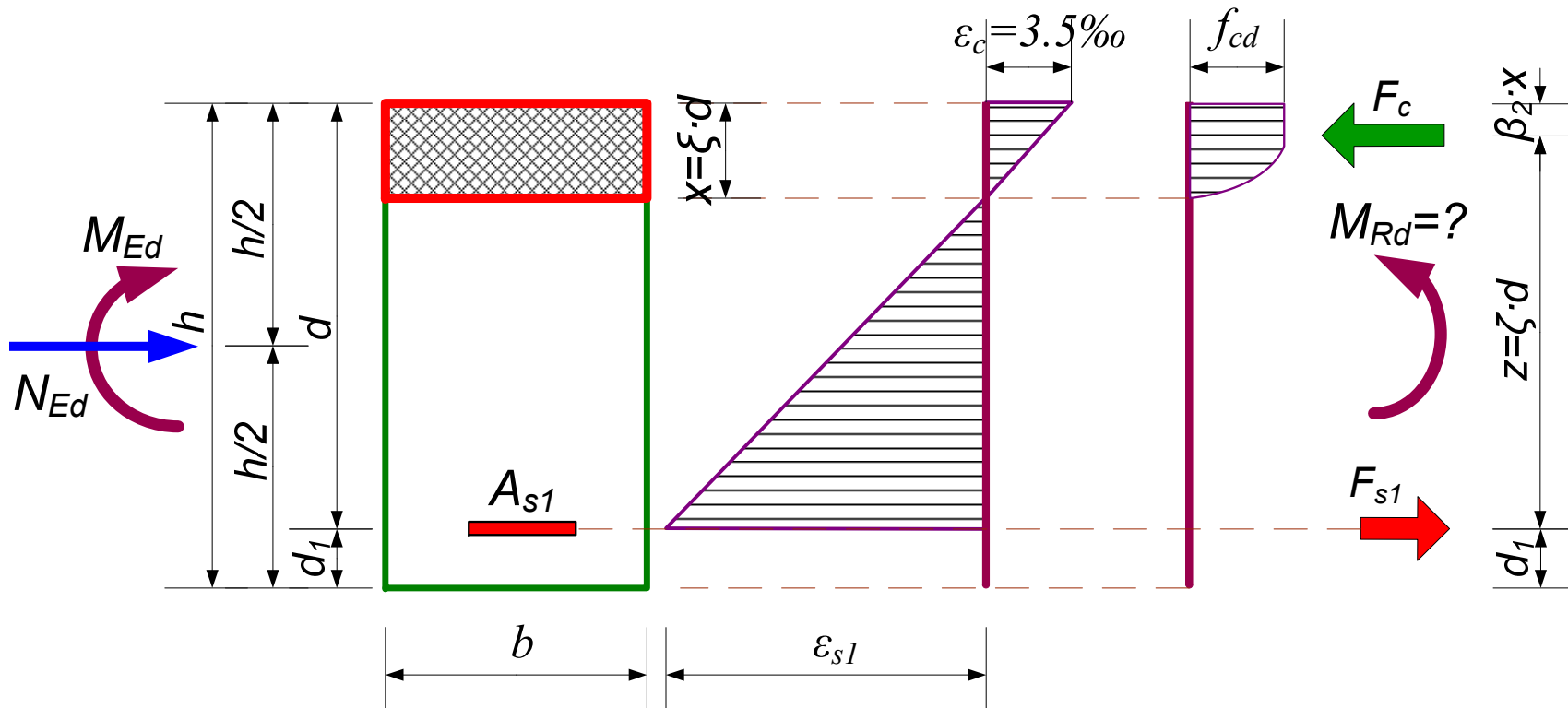
*Beograd, 2023.*

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# Određivanje MOMENTA NOSIVOSTI PRESEKA

Određivanje  $M_{Rd}$  - bez uzimanja u obzir nosivosti  $A_{s2}$



$$\sum N = 0$$



$$F_c - F_{s1} = N_{Ed}$$

$$\sum M_s = 0$$



$$M_{Rds} = F_c z = M_{Eds} = M_{Ed} + N_{Ed} \cdot \left( \frac{h}{2} - d_1 \right)$$

# Određivanje $M_{Rd}$ - bez uzimanja u obzir nosivosti $A_{s2}$

$$\sum N = 0$$

$$F_{s1} = F_c - N_{Ed}$$

$$A_{s1} = \beta_1 \xi \frac{f_{cd}}{f_{yd}} bd - \frac{N_{Ed}}{f_{yd}} = 0.810 \xi \frac{f_{cd}}{f_{yd}} bd - \frac{N_{Ed}}{f_{yd}} = \omega_1 \frac{f_{cd}}{f_{yd}} bd - \frac{N_{Ed}}{f_{yd}}$$

$$\omega_1 = 0.810 \xi = \frac{A_{s1} \cdot f_{yd} + N_{Ed}}{b \cdot d \cdot f_{cd}}$$

TABLICE



**k**

$$\sum M_s = 0$$

$$k = \frac{d}{\sqrt{\frac{M_{Eds}}{bf_{cd}}}}$$

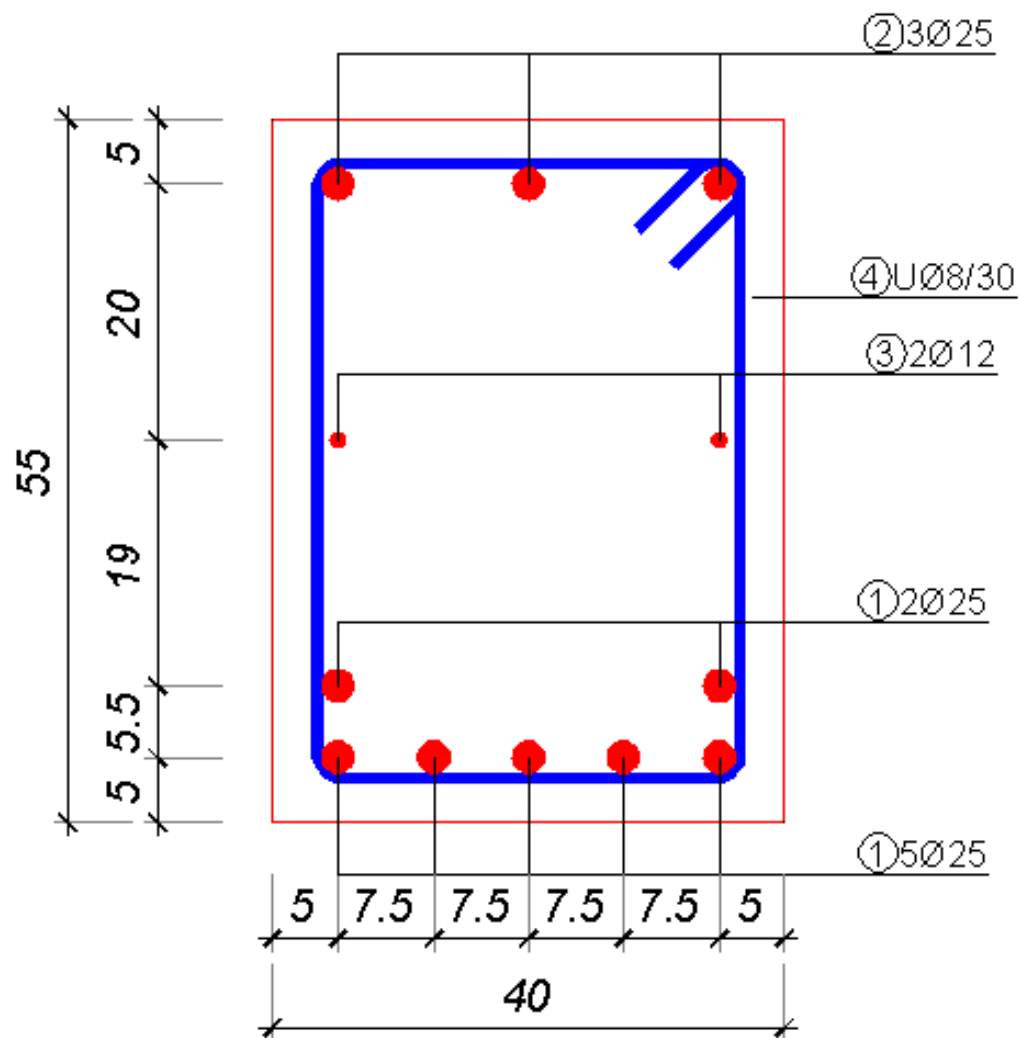


$$M_{Rds} = M_{Eds} = \left( \frac{d}{k} \right)^2 \cdot b \cdot f_{cd}$$

$$M_{Rd} = M_{Rds} - N_{Ed} \cdot \left( \frac{h}{2} - d_1 \right)$$



## Zadatak 14a – Moment nosivosti preseka bez $A_{s2}$



C30/37

B500B

$N_{Ed} = -360$  kN (zatezanje)

$d^I = 5.0$  cm

$d^{II} = 10.5$  cm

$d_1 = (5 \times 5.5 + 2 \times 10.5) / 7$

$d_1 = 6.93$  cm

$d = 55 - 6.93 = 48.07$  cm

$A_{s1} = 34.37$  cm<sup>2</sup> (7RØ25)

# Zadatak 14a – Moment nosivosti preseka bez $A_{s2}$

C30/37  $f_{cd} = 0,85 \cdot 30 / 1.5 = 17 \text{ MPa} = 1.7 \text{ kN/cm}^2$

B500 B  $f_{yd} = 500 / 1.15 = 435 \text{ MPa} = 43.5 \text{ kN/cm}^2$

$$\omega_1 = \frac{A_{s1} \cdot f_{yd} + N_{Ed}}{b \cdot d \cdot f_{cd}} = \frac{34.37 \cdot 43.5 + (-360)}{40 \cdot 48.07 \cdot 1.7} = 0.347 = 37.725\%$$

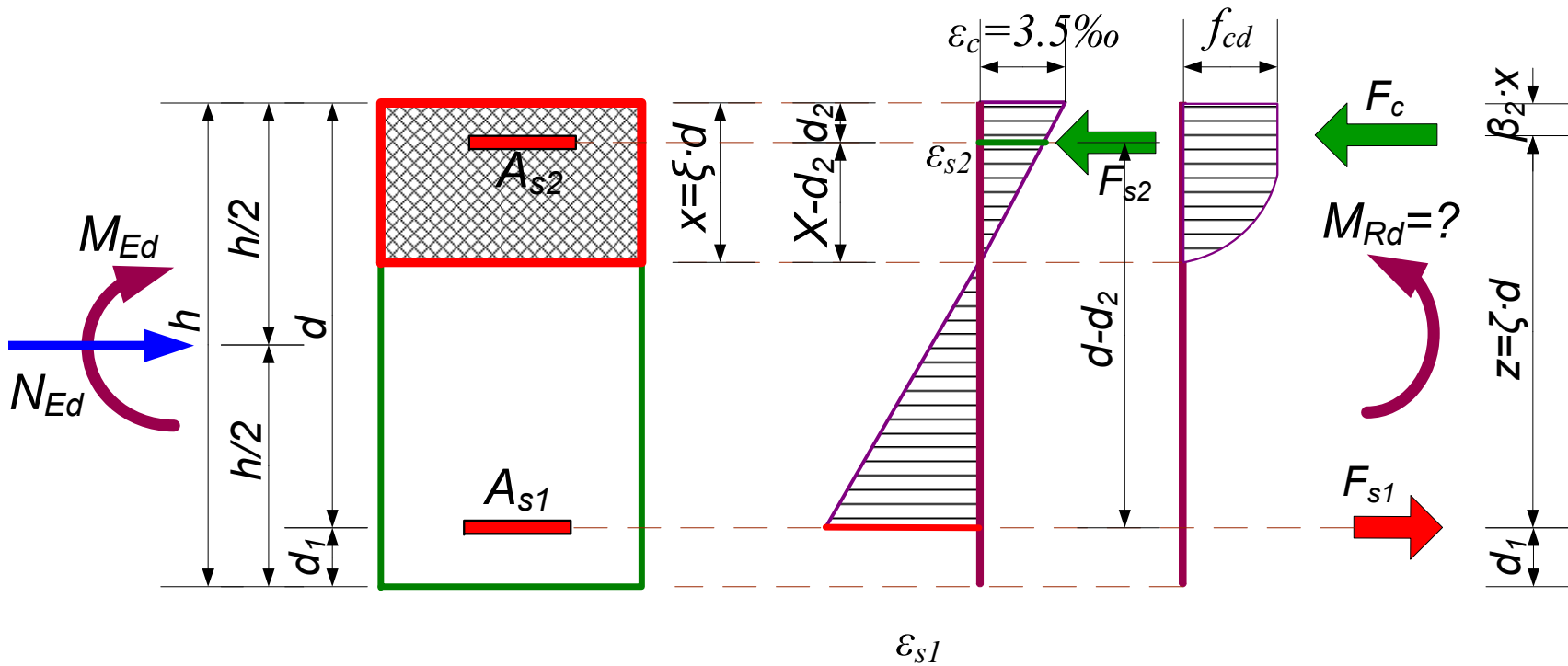
$\varepsilon_c$ (‰)	$\varepsilon_{s1}$ (‰)	$\xi$	$\zeta$	$\omega_1$ (%)	$k$	$\mu$
3.50	4.70	0.427	0.822	34.553	1.876	0.284

$$M_{Rds} = M_{Eds} = \left(\frac{d}{k}\right)^2 \cdot b \cdot f_{cd} = \left(\frac{48.07}{1.876}\right)^2 \cdot 40 \cdot 1.7 = 44647 \text{ kNcm} = 446.47 \text{ kNm}$$

$$M_{Rd} = M_{Rds} - (-N_{Ed}) \cdot \left(\frac{h}{2} - d_1\right) = 446.47 + 360 \left(\frac{0.55}{2} - 0.0693\right) = 520.5 \text{ kNm}$$



# Određivanje $M_{Rd}$ - uzimanje u obzir nosivosti $A_{s2}$



$$\sum N = 0$$



$$F_c + F_{s2} - F_{s1} = N_{Ed}$$



$\xi$

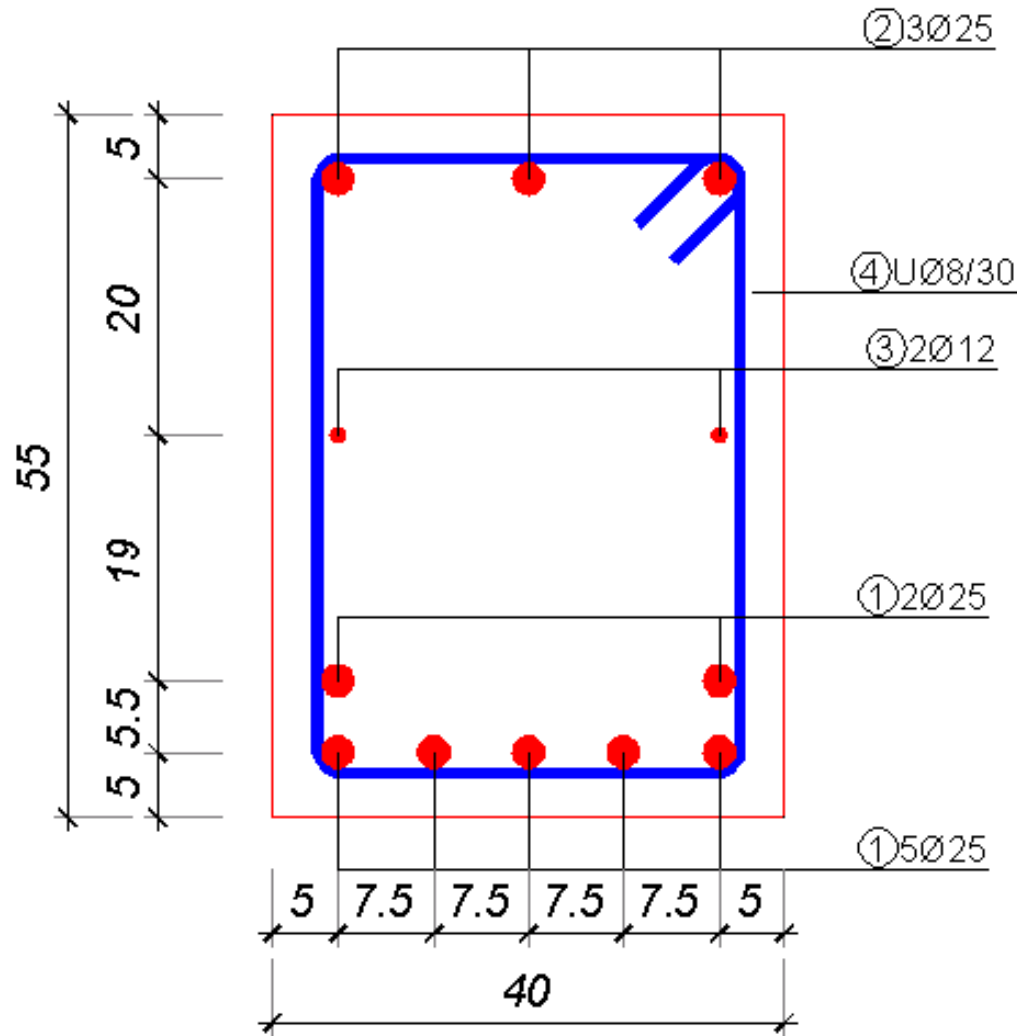
$$\sum M_s = 0$$



$$M_{Rds} = F_c \cdot z + F_{s2} \cdot (d - d_2) = M_{Eds} = M_{Ed} + N_{Ed} \cdot \left( \frac{h}{2} - d_1 \right)$$



# Zadatak 14b – Moment nosivosti preseka sa $A_{s2}$



C30/37

B500B

$N_{Ed} = -360$  kN

$d^I = 5.0$  cm

$d^{II} = 10.5$  cm

$d_1 = (5 \times 5.5 + 2 \times 10.5) / 7$

$d_1 = 6.93$  cm

$d = 55 - 6.93 = 48.07$  cm

$A_{s1} = 34.37$  cm<sup>2</sup> (7R $\text{Ø}25$ )

$A_{s2} = 14.73$  cm<sup>2</sup> (3R $\text{Ø}25$ )



# Zadatak 14b – Moment nosivosti preseka sa $A_{s2}$

## 1. ITERACIJA

1. Pretpostavka:  $\xi=0,259$

$\varepsilon_c$ (‰)	$\varepsilon_{s1}$ (‰)	$\xi$	$\zeta$	$\omega_1$ (%)	$k$	$\mu$
3.50	10.00	0.259	0.892	20.988	2.311	0.187

2. Sračunavanje unutrašnjih sila u preseku:

$$F_c = 0.810\xi dbf_{cd} = 0.81 \cdot 0.259 \cdot 48.07 \cdot 40 \cdot 1.7 = 685.75kN$$

$$F_{s1} = A_{s1} \cdot \sigma_{s1} = 34.36 \cdot 43.5 = 1495kN$$



# Zadatak 14b – Moment nosivosti preseka sa $A_{s2}$

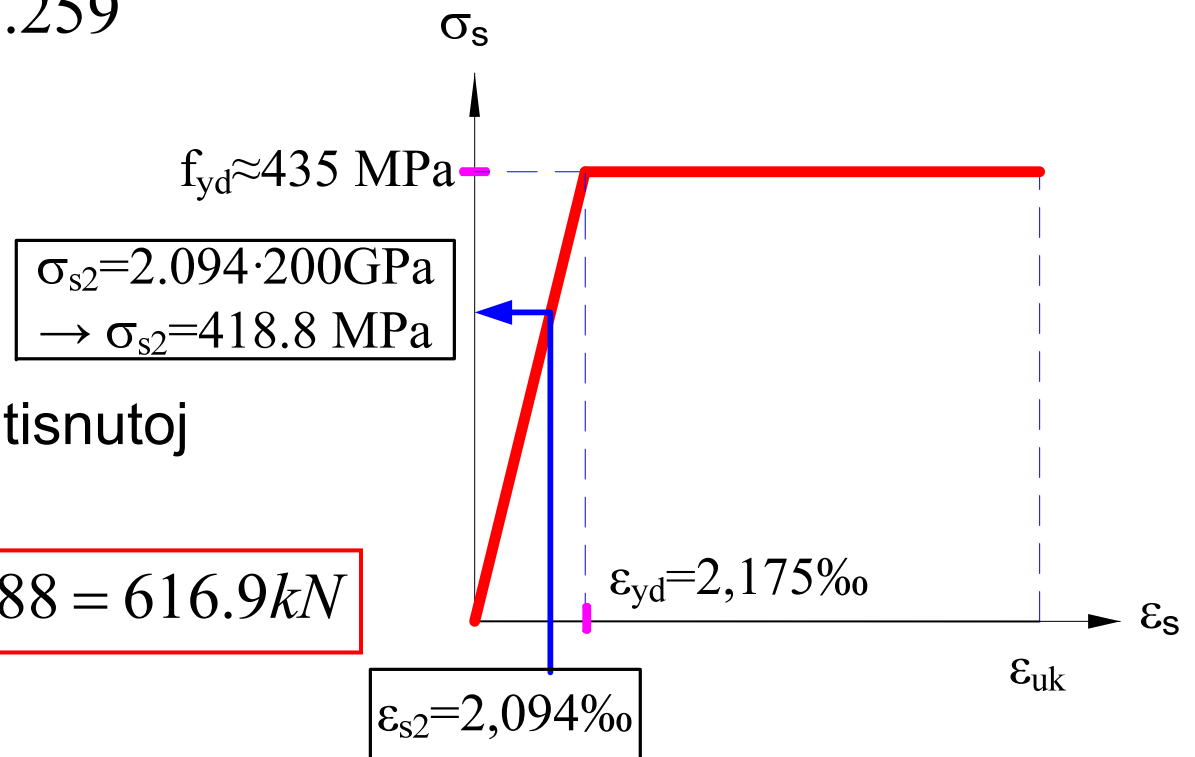
## 1. ITERACIJA

3. Sračunavanje dilatacije u pritisnutoj armaturi:

$$\varepsilon_{s2} = \frac{\xi - \frac{d_2}{d}}{\xi} \varepsilon_{cu2} = \frac{0.259 - \frac{5}{48.07}}{0.259} \cdot 3.5 = 2.094\text{‰} < 2.175\text{‰}$$

4. Sračunavanje sile u pritisnutoj armaturi:

$$F_{s2} = A_{s2} \cdot \sigma_{s2} = 14.73 \cdot 41.88 = 616.9 \text{ kN}$$



## Zadatak 14b – Moment nosivosti preseka sa $A_{s2}$

### 1. ITERACIJA

5. Suma normalnih sila:  $F_c + F_{s2} - F_{s1} = N_{Ed}$

$$685.8 + 616.9 - 1495 - (-360) = 167.7 \text{ kN}$$

**Smanjiti silu u pritisnutom betonu, tj. podići neutralnu liniju, tj. smanjiti  $\xi$  !!!**

### 2. ITERACIJA

1. Pretpostavka:  $\xi=0,226$

$\varepsilon_c$ (‰)	$\varepsilon_{s1}$ (‰)	$\xi$	$\zeta$	$\omega_1$ (%)	$k$	$\mu$
3.50	12.00	0.226	0.906	18.280	2.457	0.166

2. Sračunavanje unutrašnjih sila u preseku:

$$F_c = 0.810 \xi d b f_{cd} = 0.81 \cdot 0.226 \cdot 48.07 \cdot 40 \cdot 1.7 = 598.4 \text{ kN}$$

$$F_{s1} = A_{s1} \cdot \sigma_{s1} = 34.36 \cdot 43.5 = 1495 \text{ kN}$$



## Zadatak 14b – Moment nosivosti preseka sa $A_{s2}$

### 2. ITERACIJA

3. Sračunavanje dilatacije u pritisnutoj armaturi:

$$\varepsilon_{s2} = \frac{\xi^{II} - \frac{d_2}{d}}{\xi^{II}} \varepsilon_{cu2} = \frac{0.226 - \frac{5}{48.07}}{0.226} \cdot 3.5 = 1.889\text{‰} < 2.175\text{‰}$$

$$\sigma_{s2} = \varepsilon_{s2} \cdot E_s = 1.889 \cdot 200\text{GPa} = 377.8\text{MPa}$$

4. Sračunavanje sile u pritisnutoj armaturi:

$$F_{s2} = A_{s2} \cdot \sigma_{s2} = 14.73 \cdot 37.78 = 556.5\text{kN}$$

5. Suma sila:

$$598.4 + 556.5 - 1495 - (-360) = 19.9\text{kN}$$



**smanjiti  $\xi$  !**

# Zadatak 14b – Moment nosivosti preseka sa $A_{s2}$

## 3. ITERACIJA

1. Pretpostavka:  $\xi=0,219$

$\varepsilon_c$ (‰)	$\varepsilon_{s1}$ (‰)	$\xi$	$\zeta$	$\omega_1$ (%)	$k$	$\mu$
3.50	12.50	0.219	0.909	17.708	2.492	0.161
3.50	12.00	0.226	0.906	18.280	2.457	0.166

2. Sračunavanje unutrašnjih sila u preseku:

$$F_c = 0.810\xi dbf_{cd} = 0.81 \cdot 0.219 \cdot 48.07 \cdot 40 \cdot 1.7 = 579.8kN$$

$$F_{s1} = A_{s1} \cdot \sigma_{s1} = 34.36 \cdot 43.5 = 1495kN$$

3. Sračunavanje dilatacije u pritisnutoj armaturi:

$$\varepsilon_{s2} = \frac{\xi^{III} - \frac{d_2}{d}}{\xi^{III}} \varepsilon_{cu2} = \frac{0.219 - \frac{5}{48.07}}{0.219} \cdot 3.5 = 1.838\text{‰} < 2.175\text{‰}$$



## Zadatak 14b – Moment nosivosti preseka sa $A_{s2}$

### 3. ITERACIJA

$$\sigma_{s2} = \varepsilon_{s2} \cdot E_s = 1.838 \cdot 200 \text{ GPa} = 367.7 \text{ MPa}$$

4. Sračunavanje sile u pritisnutoj armaturi:

$$F_{s2} = A_{s2} \cdot \sigma_{s2} = 14.73 \cdot 36.76 = 541.5 \text{ kN}$$

5. Suma sila:

$$579.8 + 541.5 - 1495 - (-360) = -13.7 \text{ kN}$$

**Za dve uzastopne vrednosti  $\xi$  iz tabele suma N sila menja znak !  
PREKIDA SE SA ITERACIJAMA (može li da se nastavi ?)**



tačna vrednost  $\xi$  je između 0,226 i 0,219, tačna vrednost dilatacije  $12,0\text{‰} < \varepsilon_{s1} < 12,5\text{‰}$

## Zadatak 14b – Moment nosivosti preseka sa $A_{s2}$

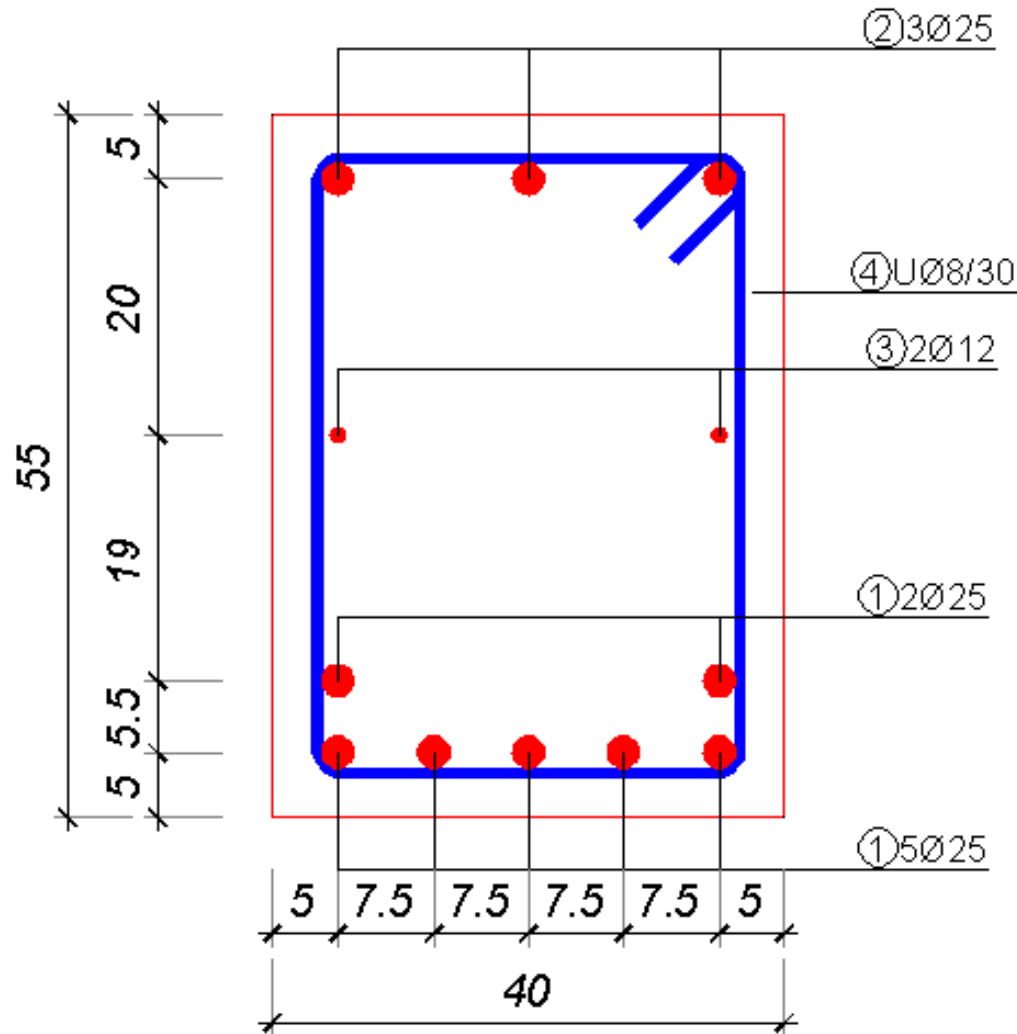
6. Sračunavanje sume momenata unutrašnjih sila u preseku oko težišta zategnute armature:

$$\begin{aligned} M_{Rds} &= F_c \cdot z + F_{s2} \cdot (d - d_2) = \\ &= 579.8 \cdot 0.909 \cdot 48.07 + 541.5 \cdot (48.07 - 5) = 486.57 \text{ kNm} \end{aligned}$$

7. Sračunavanje momenta nosivosti preseka za zadatu normalnu silu u preseku:

$$\begin{aligned} M_{Ed} &= M_{Rds} - N_{Ed} \cdot \left( \frac{h}{2} - d_1 \right) = \\ &= 486.57 - (-360) \cdot \left( \frac{0.55}{2} - 0.0693 \right) = 560.6 \text{ kNm} \end{aligned}$$

# Zadatak 14 - rezime



C30/37

B500B

$N_{Ed} = -360 \text{ kN}$

Zadatak 14a (bez  $A_{s2}$ ):

$M_{Rd} = 520.5 \text{ kNm}$

Zadatak 14b (sa  $A_{s2}$ ):

$M_{Rd} = 560.6 \text{ kNm}$

# Određivanje $M_{Rd}$ - uzimanje u obzir nosivosti $A_{s2}$

1. Pretpostavka položaja neutralne linije,  $\xi$

2. Sračunavanje unutrašnjih sila u preseku:

$$F_c = 0.810 \xi d b f_{cd}$$

$$F_{s1} = A_{s1} \cdot \sigma_{s1}$$

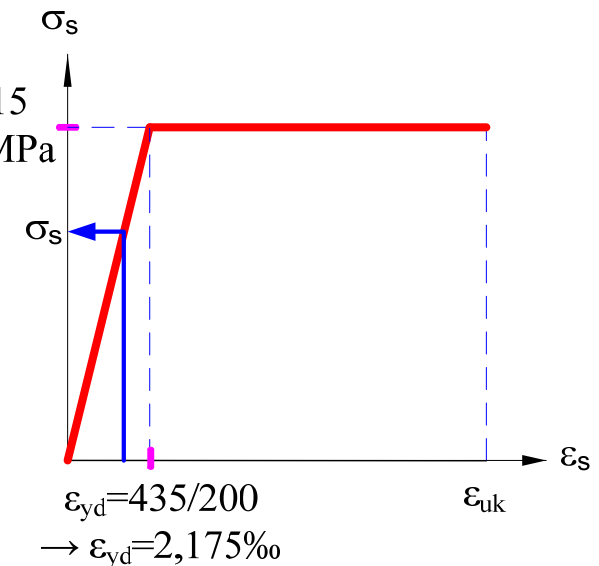
3. Sračunavanje dilatacije u pritisnutoj armaturi:

$$\varepsilon_{s2} = \frac{\xi - \frac{d_2}{d}}{\xi} \varepsilon_{cu2}$$



$$f_{yd} = 500 / 1,15$$

→  $f_{yd} \approx 435 \text{ MPa}$



4. Sračunavanje sila u pritisnutoj armaturi:

$$F_{s2} = A_{s2} \cdot \sigma_{s2}$$





## Određivanje $M_{Rd}$ - uzimanje u obzir nosivosti $A_{s2}$

5. Variranje položaja neutralne linije,  $\xi$ , dok se ne zadovolji uslov ravnoteže:

$$F_c + F_{s1} - F_{s2} = N_{Ed}$$

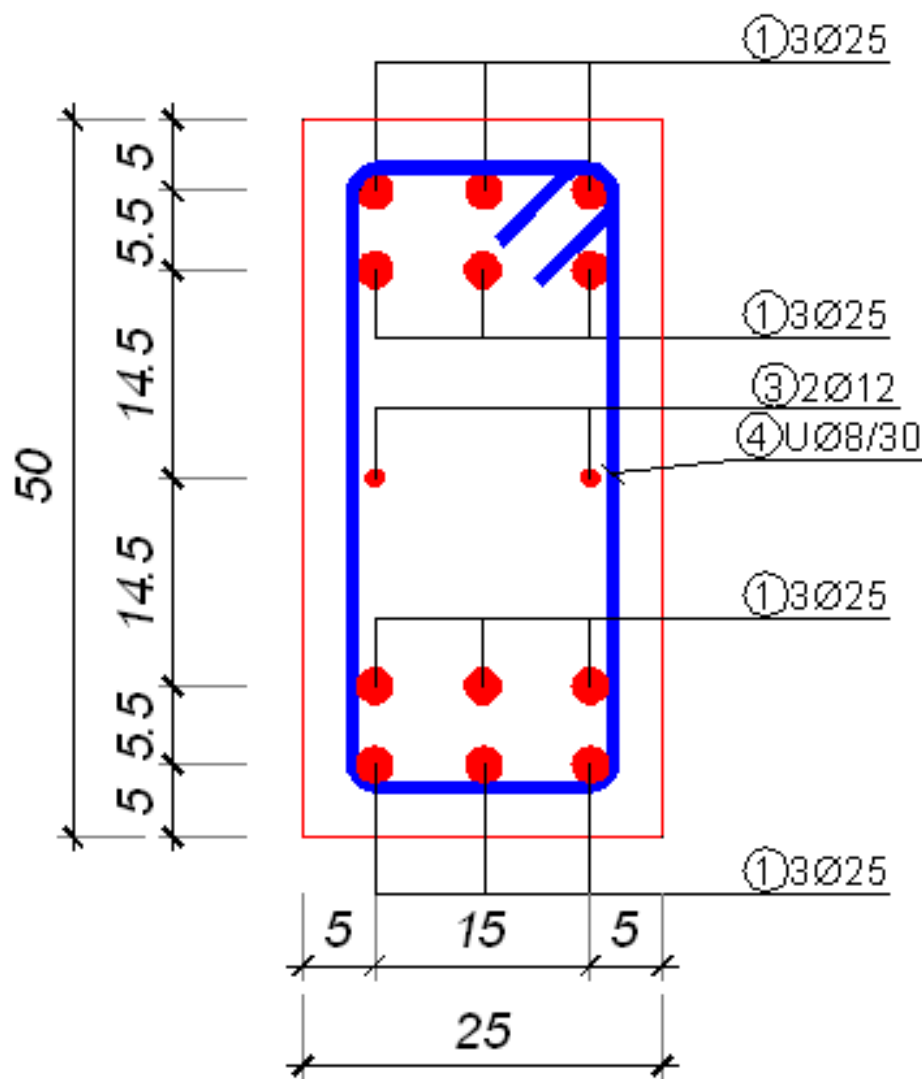
6. Sračunavanje sume momenata unutrašnjih sila u preseku oko težišta zategnute armature:

$$M_{Rds} = F_c \cdot z + F_{s2} \cdot (d - d_2)$$

7. Sračunavanje momenta nosivosti preseka za zadatu normalnu silu u preseku:

$$M_{Ed} = M_{Rds} - N_{Ed} \cdot \left( \frac{h}{2} - d_1 \right)$$

## Zadatak 15a – Moment nosivosti preseka bez $A_{s2}$



C40/50

B500B

$N_{Ed} = 325$  kN

$d^I = 5.0$  cm

$d^{II} = 10.5$  cm

$d_1 = (3 \times 5 + 3 \times 10.5) / 6$

$d_1 = 7.75$  cm

$d = 50 - 7.75 = 42.25$  cm

$A_{s1} = 29.45$  cm<sup>2</sup> (6Ø25)

# Zadatak 15a – Moment nosivosti preseka bez $A_{s2}$

C40/50  $f_{cd} = 0,85 \cdot 40 / 1.5 = 22.67 \text{ MPa} = 2.27 \text{ kN/cm}^2$

B500 B  $f_{yd} = 500 / 1.15 = 435 \text{ MPa} = 43.5 \text{ kN/cm}^2$

$$\omega_1 = \frac{A_{s1} \cdot f_{yd} + N_{Ed}}{b \cdot d \cdot f_{cd}} = \frac{29.45 \cdot 43.5 + 325}{25 \cdot 42.25 \cdot 2.27} = 0.6698 = 66.984\%$$

$\varepsilon_c$ (‰)	$\mu$	$\omega_1$ (%)	$\xi$	$\zeta$	$\varepsilon_{s1}$ (‰)	$K$
3.50	0.440	67.214	0.830	0.655	0.715	1.508

Postupak je iterativan, menja se napon u armaturi

$$\sigma_{s1} = 0.715 \cdot 200 \text{ GPa} = 143 \text{ MPa}$$

## 2. ITERACIJA

$$\omega_1 = \frac{A_{s1} \cdot \sigma_{s1} + N_{Ed}}{b \cdot d \cdot f_{cd}} = \frac{29.45 \cdot 14.3 + 325}{25 \cdot 42.25 \cdot 2.27} = 0.31119 = 31.119\%$$



# Zadatak 15a – Moment nosivosti preseka bez $A_{s2}$

$$\omega_1 = \frac{A_{s1} \cdot \sigma_{s1} + N_{Ed}}{b \cdot d \cdot f_{cd}} = \frac{29.45 \cdot 14.3 + 325}{25 \cdot 42.25 \cdot 2.27} = 0.31119 = 31.119\%$$

$\varepsilon_c$ (‰)	$\mu$	$\omega_1$ (%)	$\xi$	$\zeta$	$\varepsilon_{s1}$ (‰)	$\kappa$
3.50	0.260	30.909	0.382	0.841	5.667	1.961

$\varepsilon_{s1} > 2.5 \text{ ‰}$  , tj.  $\sigma_{s1} = f_{yd}$

 Izraz za  $\omega_1$  postaje isti kao u 1. iteraciji, beskonačan ciklus

 Problem rešavamo postavljanjem uslova ravnoteže,  $\sum N = 0$

$\varepsilon_{c2}$ (‰)	$\varepsilon_{s1}$ (‰)	$\xi$ (-)	$\sigma_{s1}$ (kN/cm <sup>2</sup> )	$F_c$ (kN)	$F_{s1}$ (kN)	$N_{Ed}$ (kN)	$\sum N_u$ (kN)
3.5	2.500	0.583	43.50	1132.0	1281.1	325	-474.1 !
3.5	1.750	0.667	35.00	1310.7	1030.8	325	-45.0
3.5	1.642	0.681	32.84	1338.2	967.1	325	46.1



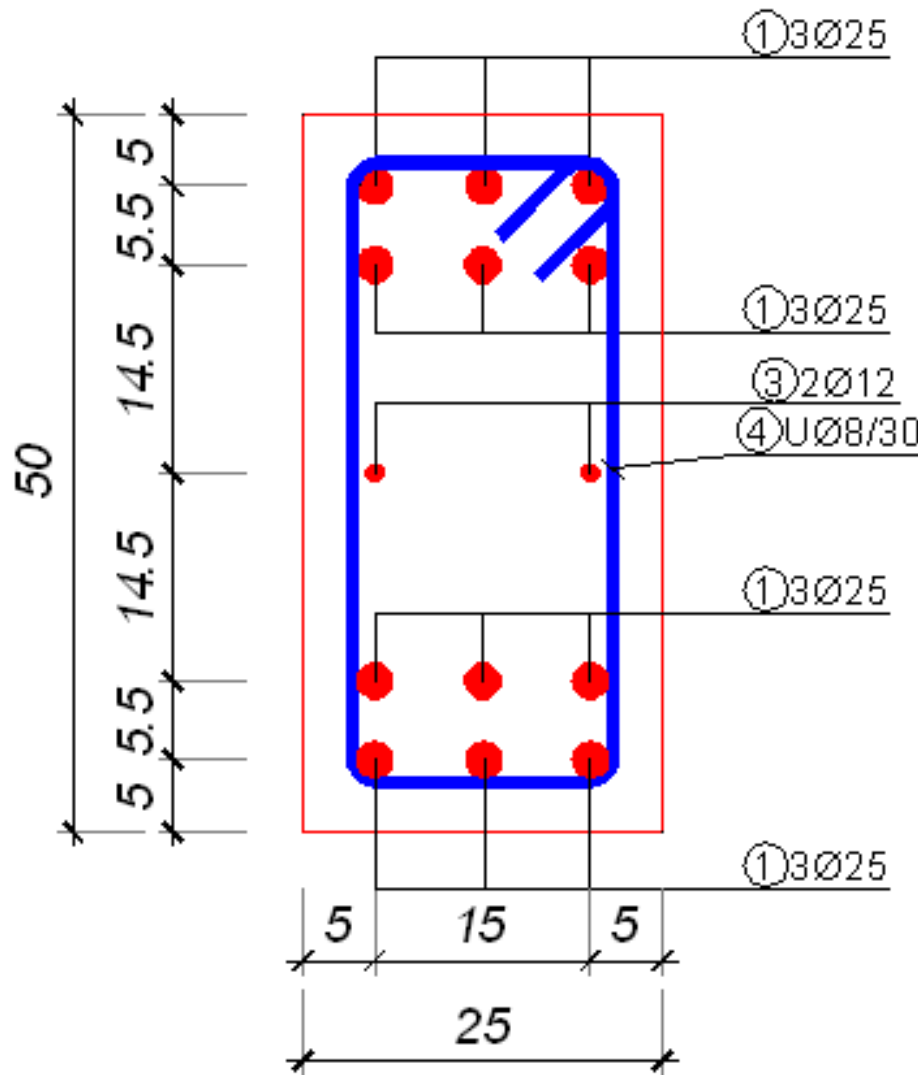
## Zadatak 15a – Moment nosivosti preseka bez $A_{s2}$

$\varepsilon_c$ (‰)	$\mu$	$\omega_1$ (%)	$\xi$	$\zeta$	$\varepsilon_{s1}$ (‰)	$K$
3.50	0.390	53.963	0.667	0.723	1.750	1.601

$$M_{Rds} = M_{Eds} = \left(\frac{d}{k}\right)^2 \cdot b \cdot f_{cd} = \left(\frac{42.25}{1.601}\right)^2 \cdot 25 \cdot 2.27 = 395.22 \text{ kNm}$$

$$M_{Rd} = M_{Rds} - N_{Ed} \cdot \left(\frac{h}{2} - d_1\right) = 395.22 - 325 \left(\frac{0.5}{2} - 0.0725\right) = 337.5 \text{ kNm}$$

## Zadatak 15b – Moment nosivosti preseka sa $A_{s2}$



C40/50

B500B

$N_{Ed} = 325$  kN

$d^I = 5.0$  cm

$d^{II} = 10.5$  cm

$d_1 = (3 \times 5 + 3 \times 10.5) / 6$

$d_1 = 7.75$  cm

$d = 50 - 7.75 = 42.25$  cm

$A_{s1} = 29.45$  cm<sup>2</sup> (6Ø25)

$A_{s2} = 29.45$  cm<sup>2</sup> (6Ø25)

# Zadatak 15b – Moment nosivosti preseka sa $A_{s2}$

## 1. ITERACIJA

1. Pretpostavka:  $\xi=0,259$

$\varepsilon_c$ (‰)	$\varepsilon_{s1}$ (‰)	$\xi$	$\zeta$	$\omega_1$ (%)	$k$	$\mu$
3.50	10.00	0.259	0.892	20.988	2.311	0.187

2. Sračunavanje unutrašnjih sila u preseku:

$$F_c = 0.810\xi dbf_{cd} = 0.81 \cdot 0.259 \cdot 42.25 \cdot 25 \cdot 2.27 = 503kN$$

$$F_{s1} = A_{s1} \cdot \sigma_{s1} = 29.45 \cdot 43.5 = 1281.1kN$$

3. Sračunavanje dilatacije u pritisnutoj armaturi:

$$\varepsilon_{s2} = \frac{\xi^{III} - \frac{d_2}{d}}{\xi^{III}} \varepsilon_{cu2} = \frac{0.259 - \frac{7.75}{42.25}}{0.259} \cdot 3.5 = 1.021\text{‰} < 2.175\text{‰}$$

4. Sila u pritisnutoj armaturi:  $F_{s2} = 29.45 \cdot 1.021 \cdot 200 / 10 = 601kN$



# Zadatak 15b – Moment nosivosti preseka sa $A_{s2}$

## 1. ITERACIJA

5. Suma sila:

$$503 + 601 - 1281.1 - 325 = -502 \text{ kN}$$

**Povećati silu u pritisnutom betonu, tj. spustiti neutralnu liniju, tj. povećati  $\xi$  !!!**

### ITERATIVNI POSTUPAK:

$\varepsilon_{c2}$ (‰)	$\varepsilon_{s1}$ (‰)	$\xi$ (-)	$\sigma_{s1}$ (kN/cm <sup>2</sup> )	$\varepsilon_{s2}$ (‰)	$\sigma_{s2}$ (kN/cm <sup>2</sup> )	$F_c$ (kN)	$F_{s1}$ (kN)	$F_{s2}$ (kN)	$N_{Ed}$ (kN)	$\Sigma N_u$ (kN)
3.5	10.00	0.259	43.50	1.021	204.2	503.0	1281.1	601	325	-501.6
3.5	5.00	0.412	43.50	1.942	388.3	800.2	1281.1	1144	325	337.8
3.5	7.00	0.333	43.50	1.572	314.4	646.7	1281.1	926	325	-33.4
3.5	6.80	0.340	43.50	1.612	322.3	660.3	1281.1	949	325	3.6





## Zadatak 15b – Moment nosivosti preseka sa $A_{s2}$

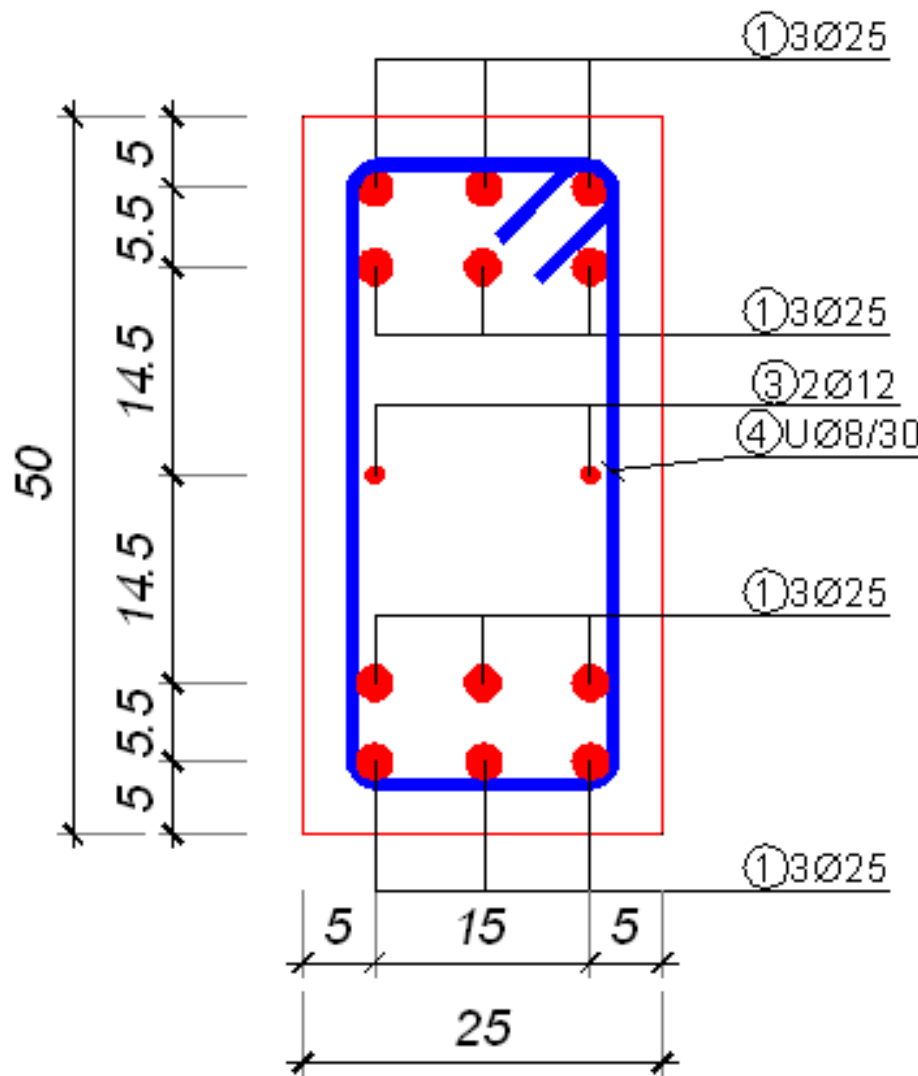
6. Sračunavanje sume momenata unutrašnjih sila u preseku oko težišta zategnute armature:

$$\begin{aligned} M_{Rds} &= F_c \cdot z + F_{s2} \cdot (d - d_2) = \\ &= 660.3 \cdot 0.859 \cdot 42.25 + 949 \cdot (42.25 - 7.75) = 567 \text{ kNm} \end{aligned}$$

7. Sračunavanje momenta nosivosti preseka za zadatu normalnu silu u preseku:

$$\begin{aligned} M_{Ed} &= M_{Rds} - N_{Ed} \cdot \left( \frac{h}{2} - d_1 \right) = \\ &= 567 - 325 \cdot \left( \frac{0.5}{2} - 0.075 \right) = 510.2 \text{ kNm} \end{aligned}$$

# Zadatak 15 - rezime



C405/50

B500B

$$N_{Ed} = 325 \text{ kN}$$

Zadatak 15a (bez  $A_{s2}$ ):

$$M_{Rd} = 337.5 \text{ kNm}$$

Zadatak 15b (sa  $A_{s2}$ ):

$$M_{Rd} = 510.2 \text{ kNm}$$

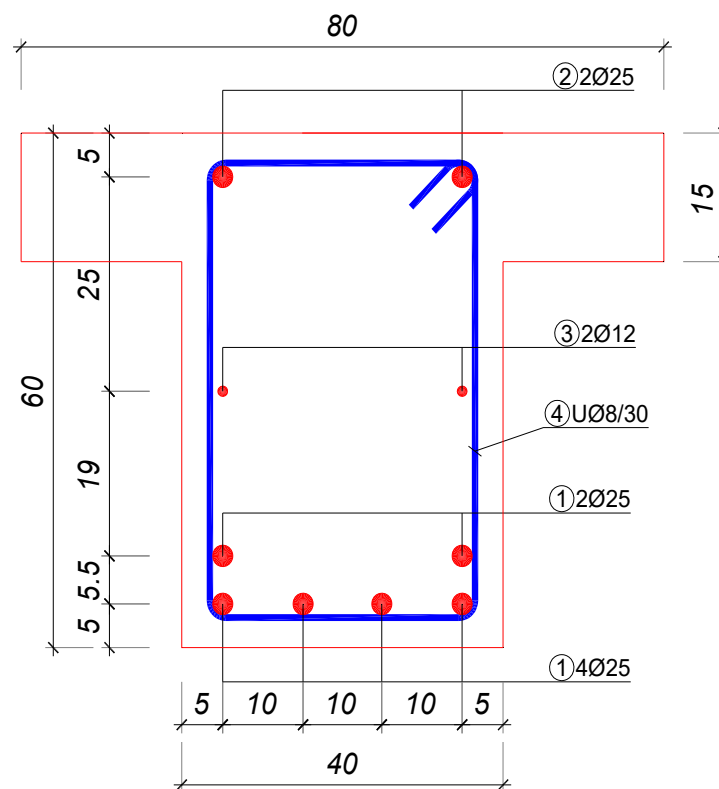


## Zadatak 16a – Moment nosivosti "T" preseka

Odrediti moment nosivosti preseka prikazanog na skici, koji je osim momenta savijanja opterećen i silom pritiska  $N_{Ed} = 1000 \text{ kN}$ . Proračun sprovesti uz zanemarenje nosivosti pritiskute armature.

C30/37

B500 B

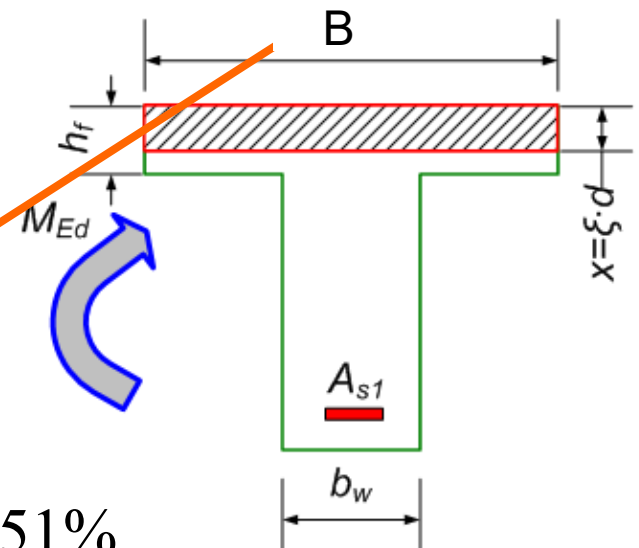


## Zadatak 16a – Moment nosivosti "T" preseka

C30/37  $f_{cd} = 0,85 \cdot 30 / 1,5 = 17 \text{ MPa} = 1,7 \text{ kN/cm}^2$

B500 B  $f_{yd} = 500 / 1,15 = 435 \text{ MPa} = 43,5 \text{ kN/cm}^2$

Pretpostavka: neutralna linija je u ploči, tj.  $x < h_f$



$$\omega_1 = \frac{A_{s1} \cdot f_{yd} + N_{Ed}}{B \cdot d \cdot f_{cd}} = \frac{29,46 \cdot 43,5 + 1000}{80 \cdot 53,17 \cdot 1,7} = 31,551\%$$

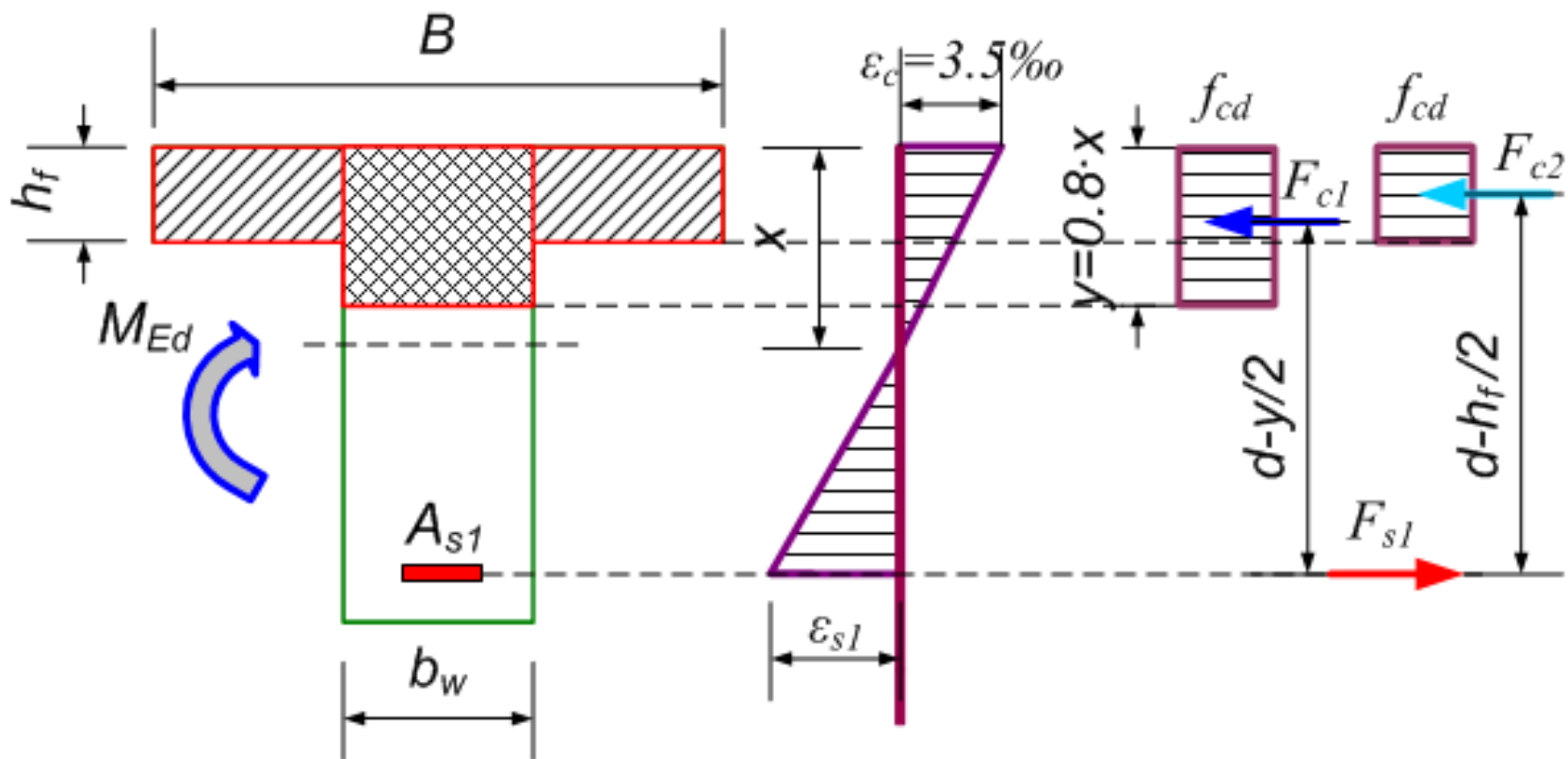
$\varepsilon_c$ (‰)	$\mu$	$\omega_1$ (%)	$\xi$	$\zeta$	$\varepsilon_{s1}$ (‰)	$K$
3.50	0.265	31.646	0.391	0.837	5.453	1.943

## Zadatak 16a – Moment nosivosti "T" preseka

Provera položaja neutralne linije:

$$x = \xi \cdot d = 0.391 \cdot 53.17 = 20.8 \text{ cm} > 15 \text{ cm} = h_f$$

Pretpostavka nije dobra, n-n linija je u rebru!



## Zadatak 16a – Moment nosivosti "T" preseka

$$F_{c1} = b_w \cdot y \cdot f_{cd} = 40 \cdot y \cdot 1.7 = 68y$$

$$F_{c2} = (B - b_w) \cdot h_f \cdot f_{cd} = (80 - 40) \cdot 15 \cdot 1.7 = 1020kN$$

$$F_{s1} = A_{s1} \cdot f_{yd} = 29.46 \cdot 43.5 = 1281.5kN$$

$$N_{Ed} = 1000kN$$

Uslov ravnoteže:  $\sum N = 0$

$$68y + 1020 - 1281.5 - 1000 = 0 \rightarrow y = 18.55cm$$

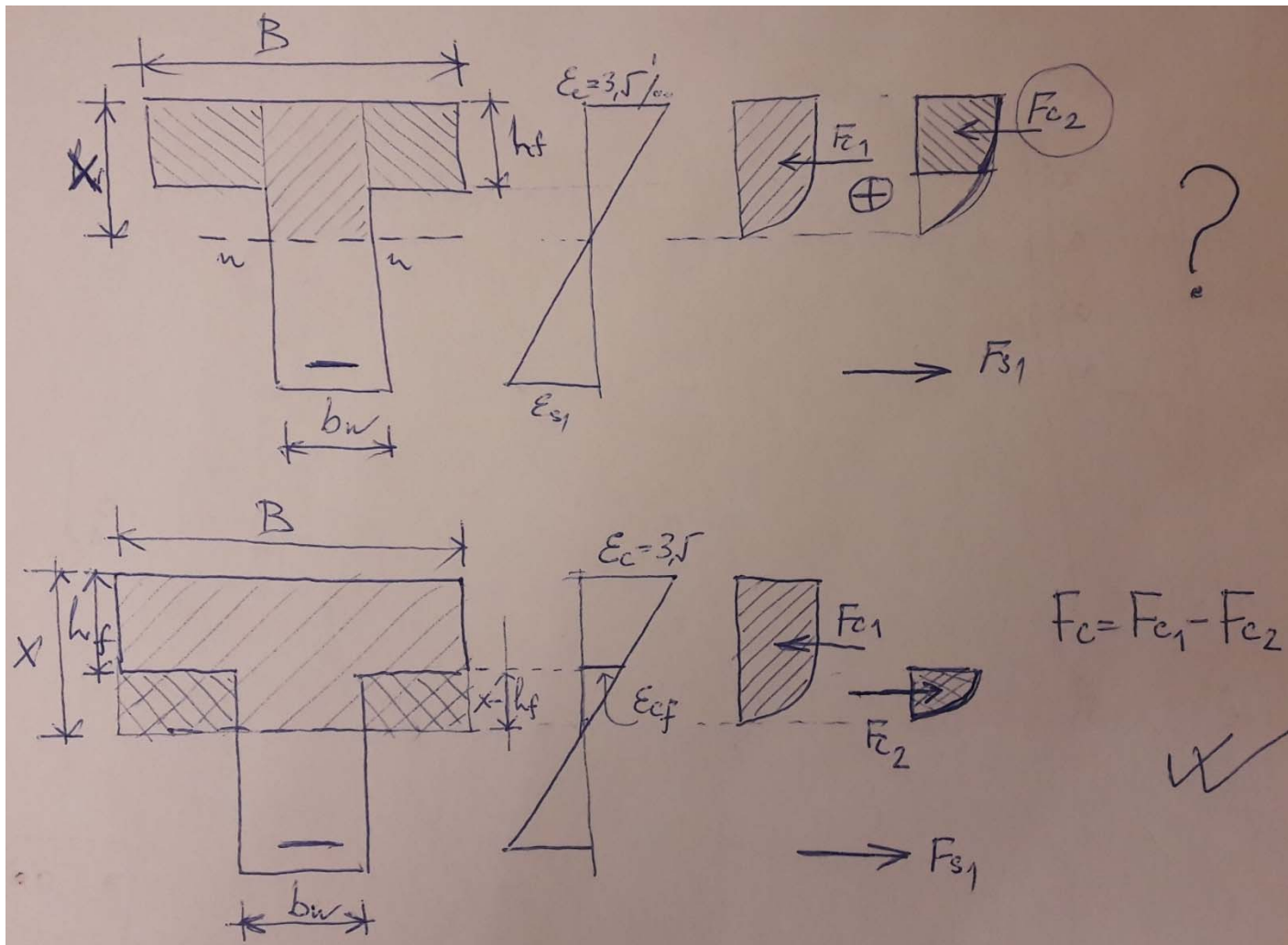
**Kontrolisati  $\epsilon_{s1}$ !!!**

Uslov ravnoteže:  $\sum M_s = 0$

$$M_{Rds} = f_{cd} \left[ b_w y \left( d - \frac{y}{2} \right) + (B - b_w) h_f \left( d - \frac{h_f}{2} \right) \right] = M_{Eds} = 1019.5kNm$$

$$M_{Ed} = M_{Rds} - N_{Ed} \cdot \left( \frac{h}{2} - d_1 \right) = 787.8kNm$$

# Zadatak 16b – Moment nosivosti "T" preseka (dijagram napona "parabola-prava")



## Zadatak 16b – Moment nosivosti "T" preseka (dijagram napona "parabola-prava")

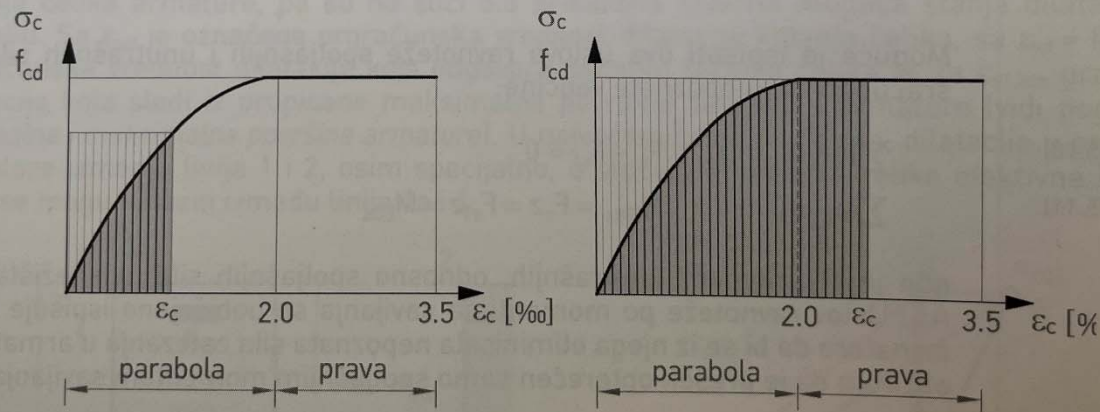
$$F_{c1} = \beta_{1,1} \cdot x \cdot f_{cd} \cdot B, \quad \beta_{1,1} = \beta_1 (3,5\%) = 0,81$$

$$F_{c2} = \beta_{1,2} \cdot (x - h_f) \cdot f_{cd} \cdot (B - b_f), \quad \beta_{1,2} = \beta_1 (\epsilon_{cf})$$

gde je  $\beta_1$  koeficijent punoće dijagrama napona pritiska u betonu a  $\xi = \frac{x}{d}$  koeficijent položaja neutralne linije. Koeficijent punoće naponskog dijagrama  $\beta_1$  zavisi od veličine maksimalne dilatacije betona u preseku  $\epsilon_c$  i iznosi:

$$\beta_1 = \frac{\epsilon_c (6 - \epsilon_c)}{12} \quad \text{za } 0 \leq \epsilon_c \leq 2\text{‰} \quad \beta_1 = \frac{3\epsilon_c - 2}{3\epsilon_c} \quad \text{za } 2\text{‰} \leq \epsilon_c \leq 3,5\text{‰}$$

U geometrijskom smislu, koeficijent punoće naponskog dijagrama  $\beta_1$  predstavlja odnos osenčene i šrafirane površine dijagrama napona pritiska u betonu, slika 5.5.





## Zadatak 16b – Moment nosivosti "T" preseka (dijagram napona "parabola-prava")

$$F_{c1} = \beta_{1,1} \cdot x \cdot f_{cd} \cdot B, \quad \beta_{1,1} = \beta_1(3,5\%) = 0,81$$

$$F_{c2} = \beta_{1,2} \cdot (x - h_f) \cdot f_{cd} \cdot (B - b_w), \quad \beta_{1,2} = \beta_1(\epsilon_{cf})$$

$$\beta_1 = \frac{\epsilon_c}{12} \cdot (6 - \epsilon_c), \quad \epsilon_c \leq 2\% \quad ; \quad \beta_1 = \frac{3\epsilon_c - 2}{3\epsilon_c}, \quad 2\% \leq \epsilon_c \leq 3,5\%$$

kontrolna deformacija u armaturi:

$$\frac{3,5}{20,8 \cdot x} = \frac{3,5 + \epsilon_{s1}}{53,17 \cdot d} \Rightarrow \epsilon_{s1} = 5,45\% > 2,5\%$$

$$\Rightarrow \sigma_{s1} = f_{yd}$$

1. iteracija

$$\frac{3,5}{20,8} = \frac{\epsilon_{cf}}{20,8 - 15} \Rightarrow \epsilon_{cf} = 0,976\% \Rightarrow \beta_1 = 0,317$$

$$\beta_1 = 0,409$$

$$F_{c1} = 0,81 \cdot 20,8 \cdot 1,7 \cdot 80 = 2291,3 \text{ kN}$$

$$F_{c2} = 0,317 \cdot (20,8 - 15) \cdot 1,7 \cdot (80 - 40) = 125 \text{ kN}$$

## Zadatak 16b – Moment nosivosti "T" preseka (dijagram napona "parabola-prava")

$$F_{s1} = 1281,5 \text{ kN} ; F_{c1} = 0,81 \cdot 20,8 \cdot 1,7 \cdot 80 = 2291,3 \text{ kN}$$

$$N_{ed} = 1000 \text{ kN} ; F_{c2} = 0,409 \cdot (20,8 - 15) \cdot 1,7 \cdot (80 - 40) = 161,3 \text{ kN}$$

$$\Sigma N = 0 : (2291,3 - 161,3) - 1281,5 - 1000 = -151,0 \text{ kN} \neq 0$$

$\Rightarrow$  сусветити неутралну линију

2. интеграција

$$x = 22 \text{ cm} \quad \frac{3,5}{22} = \frac{\epsilon_{cf}}{22 - 15} \Rightarrow \epsilon_{cf} = 1,114 \text{ ‰}$$

$$\beta_1 = 0,454$$

$$F_{c1} = 0,81 \cdot 22 \cdot 1,7 \cdot 80 = 2423,5 \text{ kN}$$

$$F_{c2} = 0,454 \cdot (22 - 15) \cdot 1,7 \cdot (80 - 40) = 216 \text{ kN}$$

$$\Sigma N = 0 : (2423,5 - 216) - 1281,5 - 1000 = -74,1 \text{ kN}$$



## Zadatak 16b – Moment nosivosti "T" preseka (dijagram napona "parabola-prava")

3. ušegranuša

$$x = 24 \quad \epsilon_{cf} = 1.3125 \quad \beta_1 = 0.513$$

$$F_{c1} = 2643,8 \text{ kN}$$

$$F_{c2} = 313,9 \text{ kN}$$

$$\Sigma N = 48,3 \text{ kN}$$

изменити се знак!

4. ušegranuša

$$x = 23 \quad \epsilon_{cf} = 1.217 \quad \beta_1 = 0.485$$

$$F_{c1} = 2533,7 \text{ kN}$$

$$F_{c2} = 263,8 \text{ kN}$$

$$\Sigma N = -11,64$$



## Zadatak 16b – Moment nosivosti "T" preseka (dijagram napona "parabola-prava")

$$b. \quad x = 23,3 \quad \epsilon_{ef} = 1,247 \quad \beta_1 = 0,494$$

$$F_{c1} = 2566,7 \text{ kN}$$

$$F_{c2} = 278,8 \text{ kN}$$

$$\Sigma N = \phi \cdot (2566,7 - 278,8) - 1281,5 - 1000 = 6,4 \text{ kN}$$

$$\beta_2 = \frac{\delta - \epsilon_c}{4 \cdot (6 - \epsilon_c)}, \quad 0 \leq \epsilon_c \leq 2\%$$

$$\beta_2 = \frac{\epsilon_c \cdot (3\epsilon_c - 4) + 2}{2\epsilon_c \cdot (3\epsilon_c - 2)}, \quad 2 \leq \epsilon_c \leq 3,5\%$$

$$\beta_{2,1} = \beta_2(3,5\%) = 0,416$$

$$\beta_{2,2} = \beta_2(1,247\%) = 0,355$$



Zadatak 16b – Moment nosivosti "T" preseka  
(dijagram napona "parabola-prava")

$$M_{Rds} = F_{c1} \cdot (d - \beta_{z1} \cdot x) - F_{s2} \cdot (d - h_f - \beta_{z2} \cdot (x - h_f))$$

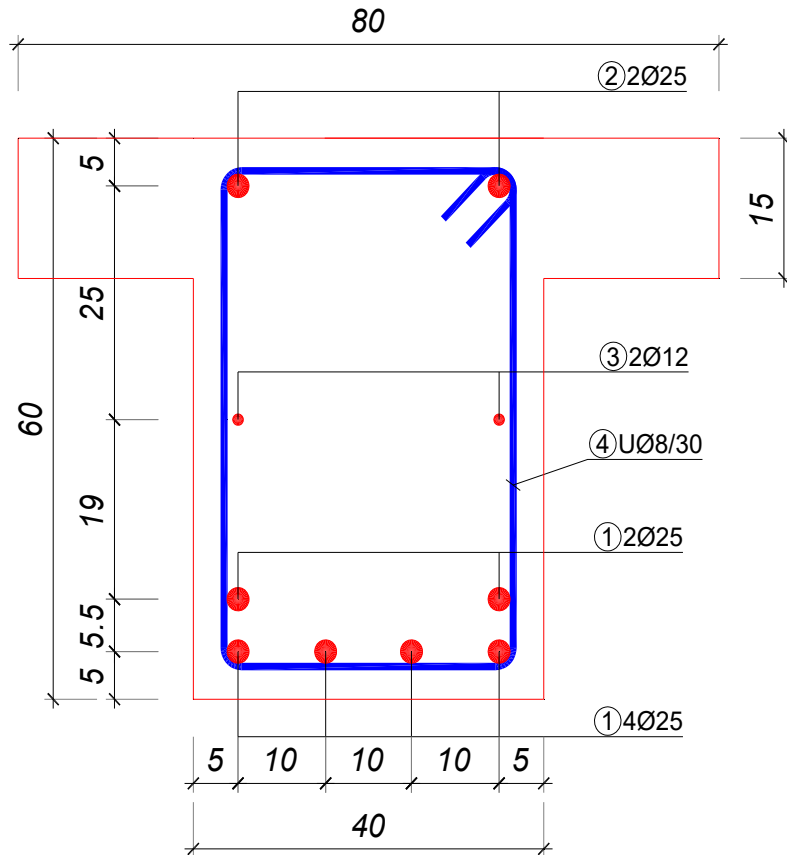
$$M_{Rds} = 2566,7 (53,17 - 0,416 \cdot 23,3) - 278,8 (53,17 - 15 - 0,355 \cdot (23,3 - 15))$$

$$M_{Rds} = 1017,7 \text{ kNm}$$

$$M_{ed} = M_{Rds} - N_{ed} \left( \frac{h}{2} - d_1 \right) = 1017,7 - 1000 \cdot \left( \frac{0,6}{2} - 0,0683 \right)$$

$$M_{ed} = 786 \text{ kNm}$$

# Zadatak 16 – rezime



C30/37

B500B

$N_{Ed} = 1000 \text{ kN}$

Zadatak 16a

(„blok dijagram“ napona u betonu):

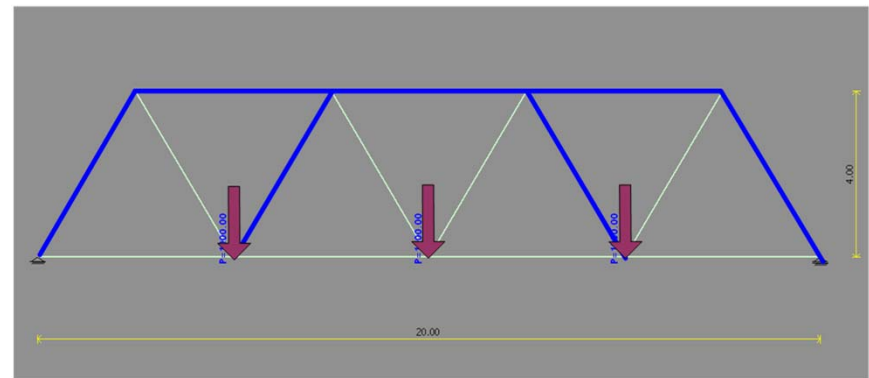
$M_{Rd} = 787.8 \text{ kNm}$

Zadatak 16b (dijagram parabola-prava u betonu):

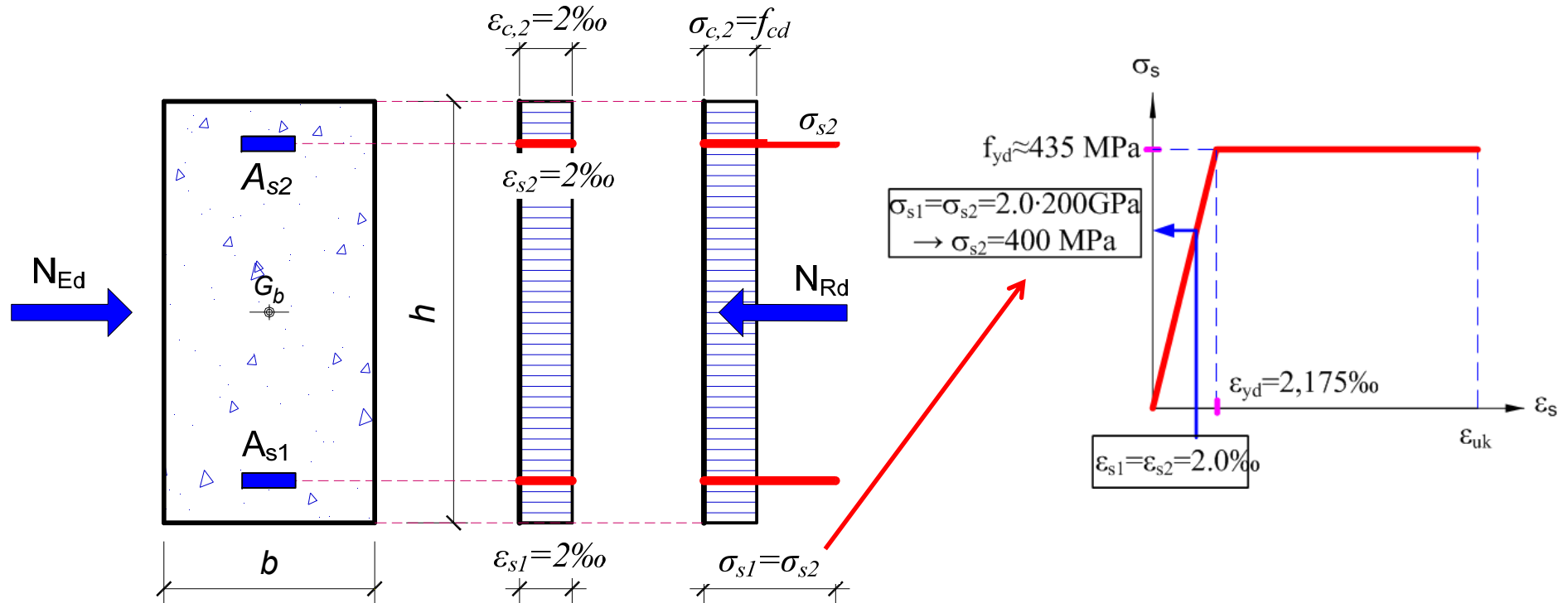
$M_{Rd} = 786 \text{ kNm}$



# Centrični pritisak



# Centrični pritisak



$$\Sigma N = 0: \quad N_{Rd} = N_{Ed} = A_c \cdot f_{cd} + A_s \cdot \sigma_s$$



# 3 moguća zadatka

$$\Sigma N = 0 : \quad N_{Rd} = N_{Ed} = A_c \cdot f_{cd} + A_s \cdot \sigma_s$$

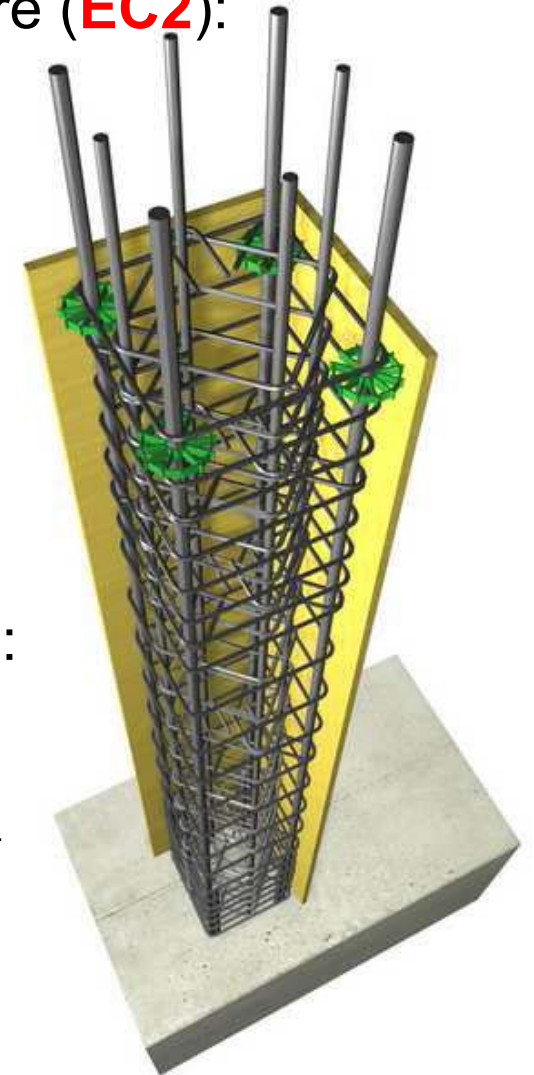
1. Dato: površina betona ( $A_c$ ) i  $N_{Ed}$ , traži se  $A_s$
2. Dato:  $A_c$  i  $A_s$ , traži se  $N_{Ed}$  ( $N_{Rd}$ )
3. Dato:  $N_{Ed}$ , traži se  $A_c$  i  $A_s$  (zadatak 19)

## Zadatak 19 – CENTRIČNI PRITISAK

Minimalna i maksimalna površina podužne armature (EC2):

$$A_{s,\min} = \max \left\{ \begin{array}{l} 0.15 \cdot \frac{N_{Ed}}{f_{yd}} \\ 0.003 \cdot A_c \\ 4\emptyset 12 \end{array} \right\}$$

$$A_{s,\max} = 0.04 \cdot A_c$$



Maksimalno rastojanje poprečne armature (EC2):

U sredini stuba:

Na krajevima (EC2/NA):

$$s_{cl,t\max} = \min \left\{ \begin{array}{l} 20\emptyset_{\min} \\ \min(b, h) \\ 40\text{cm} \end{array} \right\}$$

$$s_{cl,t\max} = \min \left\{ \begin{array}{l} 12\emptyset_{\min} \\ \min(b, h) \\ 30\text{cm} \end{array} \right\}$$

Razmak nepridržane od pridržane šipke  $\leq 15$  cm



## Zadatak 19 – CENTRIČNI PRITISAK

Odrediti potrebnu površinu armature i dimenzije poprečnog preseka, pravougaonog oblika, centrično pritisnutog elementa. Podaci za proračun:

$$N_{G,k} = 600 \text{ kN} \quad \text{C25/30}$$

XC3

$$N_{Q,k} = 800 \text{ kN} \quad \text{B500 B}$$

$$b = 35 \text{ cm}$$

$$\text{C25/30} \rightarrow f_{cd} = 0.85 \cdot 25 / 1.5 = 14.2 \text{ MPa} = 1.42 \text{ kN/cm}^2$$

$$\text{B500 B} \rightarrow f_{yd} = 500 / 1.15 = 435 \text{ MPa} = 43.5 \text{ kN/cm}^2$$



## Zadatak 19 – CENTRIČNI PRITISAK

Granična sila PRITISKA:

$$N_{Ed} = 1.35 \cdot 600 + 1.5 \cdot 800 = 2010 \text{ kN}$$

Uslov ravnoteže normalnih sila:

$$N_{Rd} = N_{Ed} = A_c \cdot f_{cd} + A_s \cdot \sigma_s$$

$$N_{Ed} = A_c \cdot f_{cd} \cdot \left( 1 + \frac{A_s}{A_c} \cdot \frac{\sigma_s}{f_{cd}} \right) = A_c \cdot f_{cd} \cdot \left( 1 + \frac{A_s}{A_c} \cdot \frac{\sigma_s}{f_{cd}} \cdot \frac{f_{yd}}{f_{yd}} \right) = A_c \cdot f_{cd} \cdot \left( 1 + \omega \cdot \frac{\sigma_s}{f_{yd}} \right)$$

Minimalni geometrijski koeficijent armiranja:

$$\rho_{l,\min} = 0.3\% \Rightarrow \omega = 0.3 \cdot 43.5 / 1.42 = 9.19\%$$

$$A_{c,\text{pot}} = \frac{N_{Ed}}{f_{cd} \cdot \left( 1 + \omega \frac{\sigma_s}{f_{yd}} \right)} = \frac{2010}{1.42 \cdot \left( 1 + 0.0919 \cdot \frac{40}{43.5} \right)} = 1305 \text{ cm}^2$$



## Zadatak 19 – CENTRIČNI PRITISAK

$$h_{\text{pot}} = A_{c,\text{pot}}/b = 1305/35 = 37.3 \text{ cm} \Rightarrow \text{usv. } h = 40 \text{ cm}$$

Sračunavanje potrebne površine armature:

$$A_s = \max \left\{ \begin{array}{l} 0.15 \cdot \frac{N_{Ed}}{f_{yd}} = 0.15 \cdot \frac{2010}{43.5} = 6.93 \text{ cm}^2 \\ 0.003 \cdot A_c = 0.003 \cdot 40 \cdot 35 = 4.2 \text{ cm}^2 \\ 4\emptyset 12 = 4 \cdot 1.13 = 4.48 \text{ cm}^2 \end{array} \right\} = 6.93 \text{ cm}^2$$

**8 Ø12 (8.96 cm<sup>2</sup>)**

Maksimalno rastojanje poprečne armature (**EC2**):

$$s_{cl,t\max} = \min \left\{ \begin{array}{l} 20\emptyset_{\min} \\ \min(b, h) \\ 40 \text{ cm} \end{array} \right\} = \min \left\{ \begin{array}{l} 20 \cdot 1.2 = 24 \text{ cm} \\ \min(b, h) = 35 \text{ cm} \\ 40 \text{ cm} \end{array} \right\} = 24 \text{ cm}$$



## Zadatak 19 – CENTRIČNI PRITISAK

Maksimalno rastojanje poprečne armature (**EN1992-1-1/NA**):

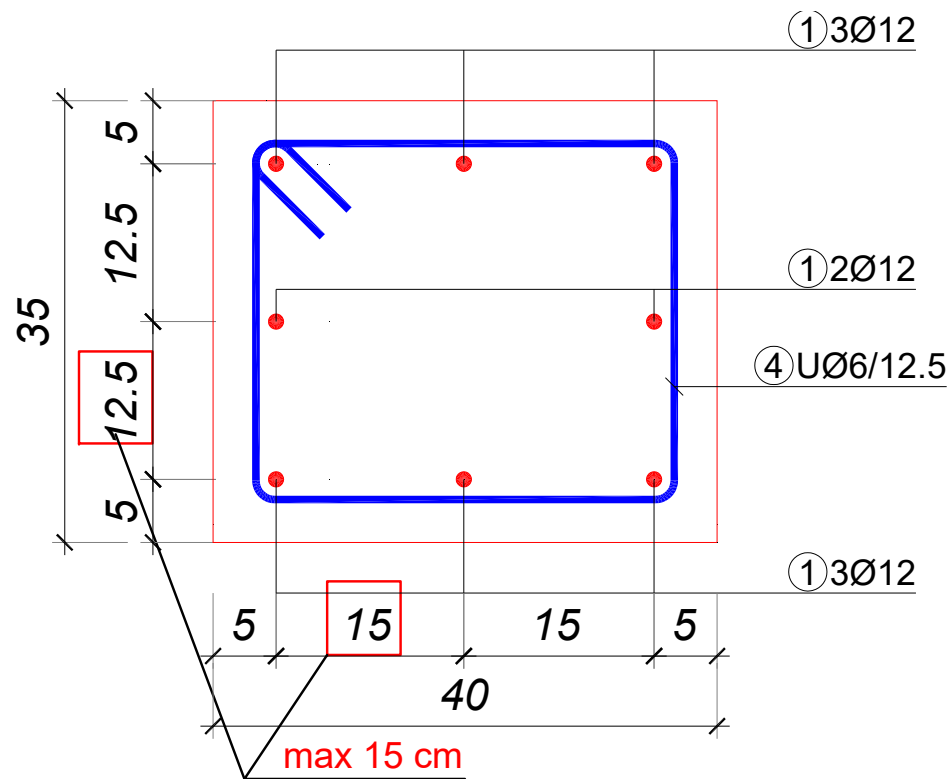
$$s_{cl,t\max} = \min \left\{ \begin{array}{l} 12\varnothing_{\min} \\ \min(b, h) \\ 30cm \end{array} \right\} = \min \left\{ \begin{array}{l} 12 \cdot 1.2 = 14.4cm \\ \min(b, h) = 35cm \\ 30cm \end{array} \right\} = 14.4cm$$



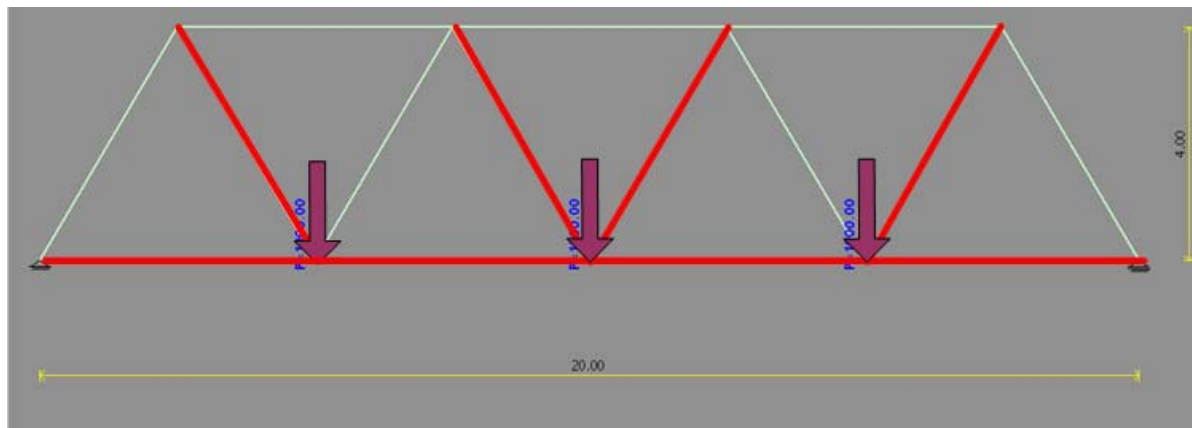
# Zadatak 19 – CENTRIČNI PRITISAK

Usvojena podužna armatura: 8 Ø12 (8.96 cm<sup>2</sup>)

Usvojena poprečna armatura: Ø6/12.5



# Centrično zatezanje





## Zadatak 17 – **CENTRIČNO ZATEZANJE**

Odrediti **potrebnu površinu armature i oblikovati poprečni presek**, pravougaonog oblika, centrično zategnutog elementa. Podaci za proračun:

$$N_{G,k} = -400 \text{ kN}$$


C25/30

XD1

$$N_{Q,k} = -500 \text{ kN}$$

B500 B

**Treba ?**

C25/30   $f_{cd} = 0.85 \cdot 25 / 1.5 = 14.2 \text{ MPa} = 1.42 \text{ kN/cm}^2$

B500 B   $f_{yd} = 500 / 1.15 = 435 \text{ MPa} = 43.5 \text{ kN/cm}^2$



## Zadatak 17 – **CENTRIČNO ZATEZANJE**

Granična sila **ZATEZANJA**:

$$N_{Ed} = 1.35 \cdot (-400) + 1.5 \cdot (-500) = -1290 \text{ kN}$$

Proračun površine armature:

$$A_{s1} = \frac{N_{Ed}}{f_{yd}} = \frac{1290}{43.5} = 29.7 \text{ cm}^2$$

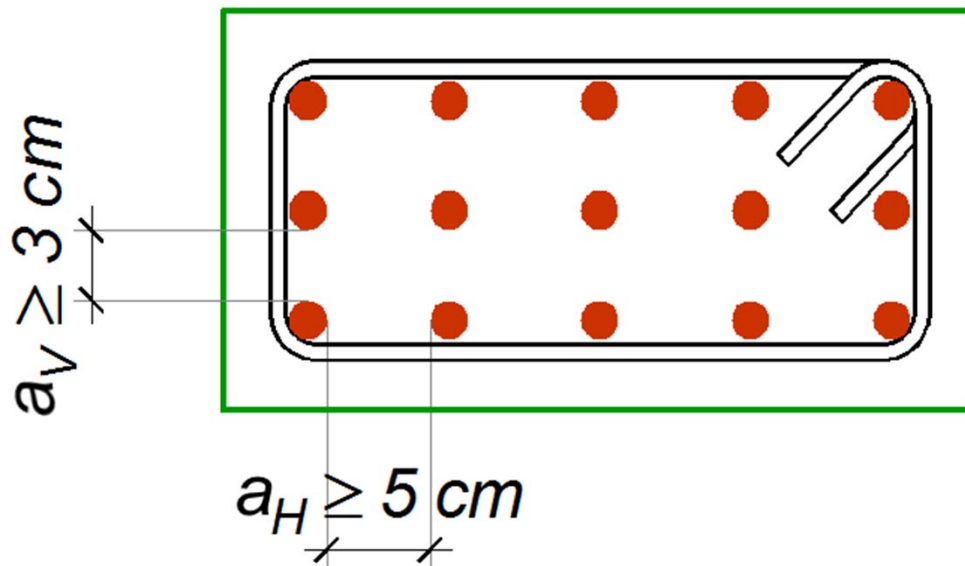
Usvojeno: **15 Ø16** (30.15 cm<sup>2</sup>)



# Zadatak 17 – CENTRIČNO ZATEZANJE

$$b \geq 2c_{\text{nom}} + 2\varnothing_s + m \times \varnothing + (m-1) \times a_H$$

$$h \geq 2c_{\text{nom}} + 2\varnothing_s + n \times \varnothing + (n-1) \times a_V$$



XD1  $\Rightarrow c_{\text{nom}} = 35 + 10 = 45 \text{ mm}$

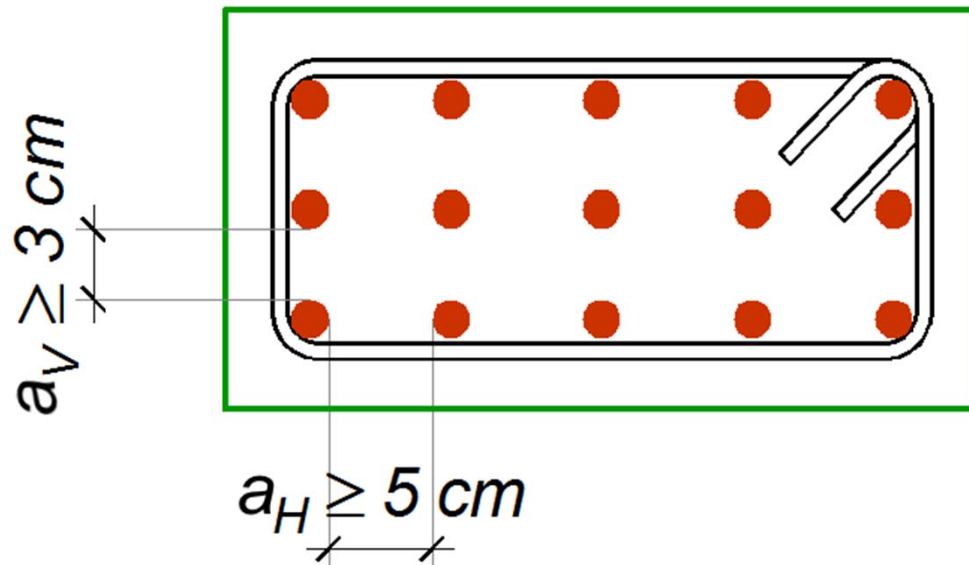
Pretp.  $\Rightarrow \varnothing_s = 8 \text{ mm}$

Oznaka klase izloženosti	Minimalni zaštitni slojevi iz uslova trajnosti, $c_{\text{min,dur}}$ , za klasu konstrukcija S4									
	10	15	20	25	30	35	40	45	50	55
X0										
XC1										
XC2										
XC3										
XC4										
XD1										
XD2										
XD3										
XS1										
XS2										
XS3										



## Zadatak 17 – CENTRIČNO ZATEZANJE

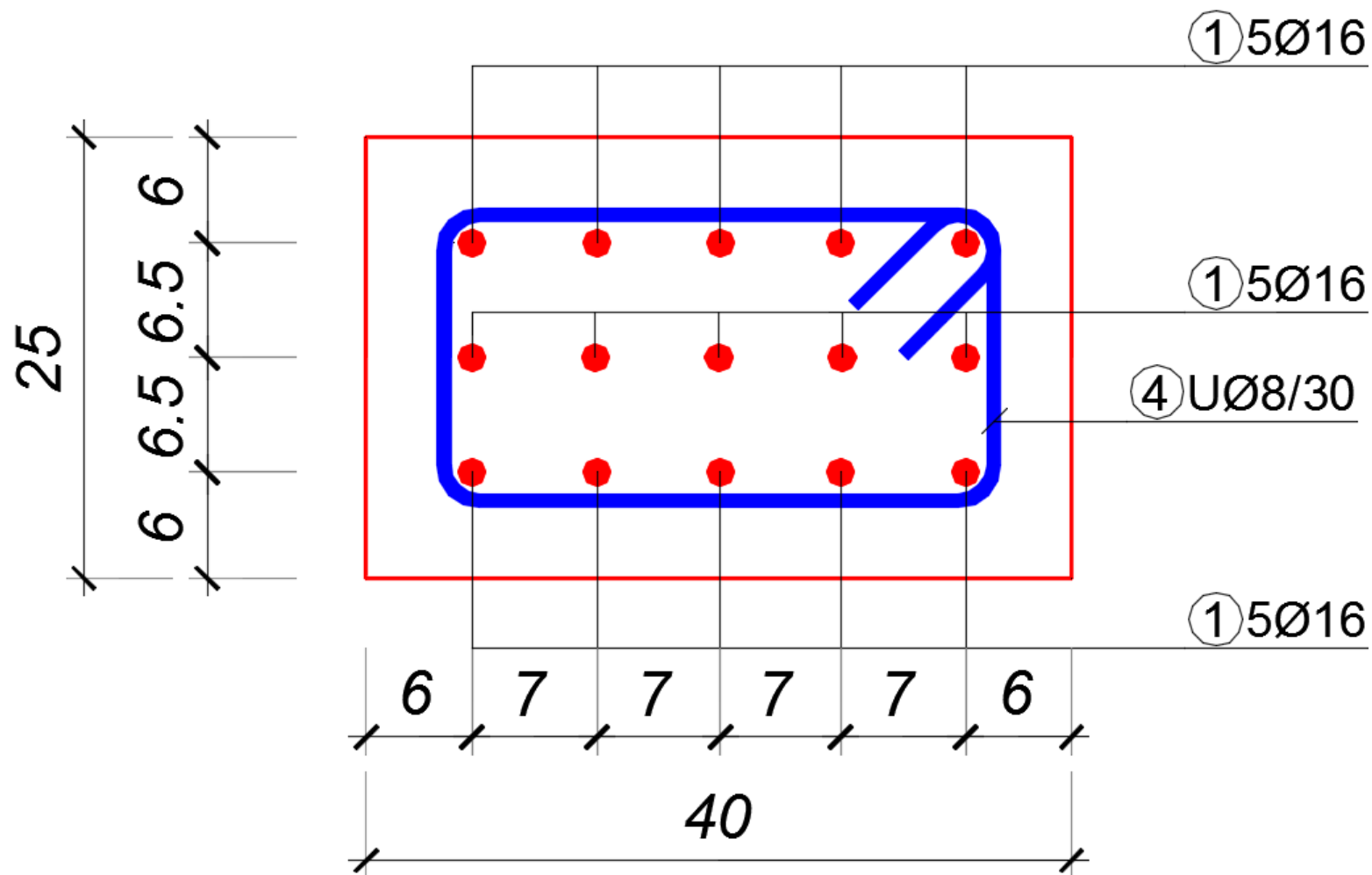
$$b \geq 2c_{\text{nom}} + 2\varnothing_s + m \times \varnothing + (m-1) \times a_H$$
$$h \geq 2c_{\text{nom}} + 2\varnothing_s + n \times \varnothing + (n-1) \times a_V$$



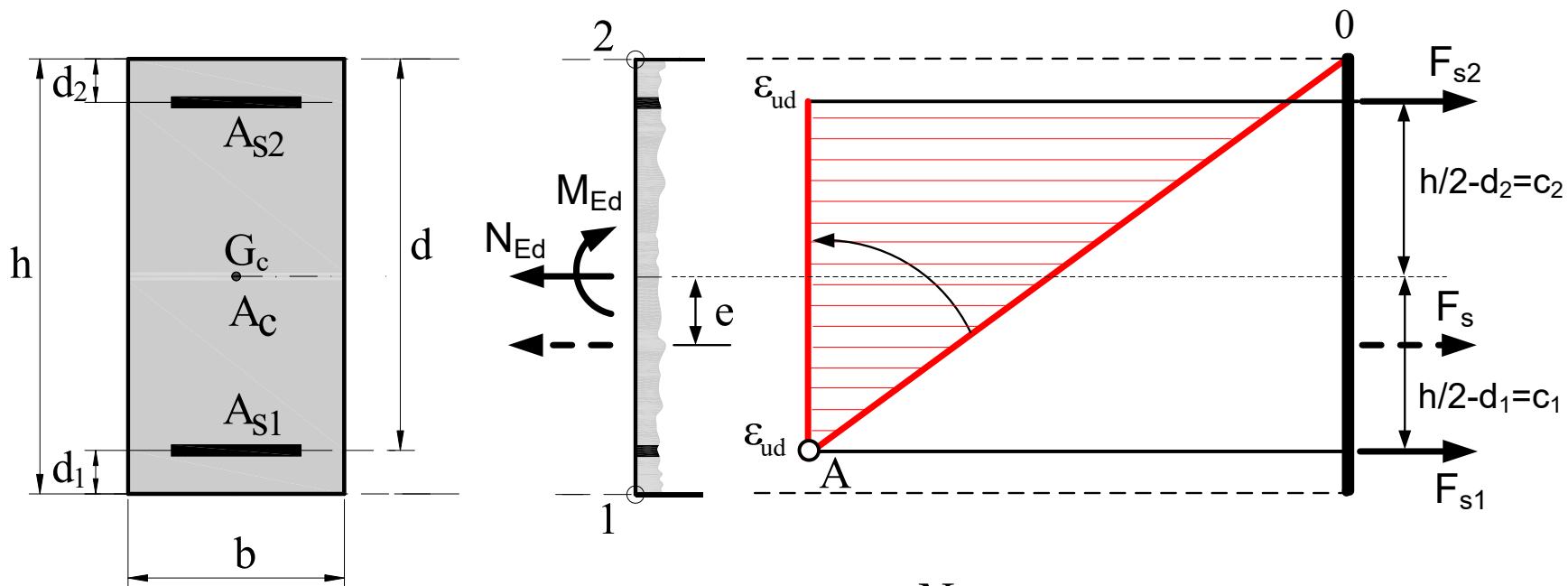
$$b \geq 2 \times 4.5 + 2 \times 0.8 + 5 \times 1.6 + (5-1) \times 5.0 = 38.6 \text{ cm} \Rightarrow b = 40 \text{ cm}$$

$$d \geq 2 \times 4.5 + 2 \times 0.8 + 3 \times 1.6 + (3-1) \times 3.0 = 21.4 \text{ cm} \Rightarrow d = 25 \text{ cm}$$

# Zadatak 17 – CENTRIČNO ZATEZANJE



# ULS – MALI EKSCENTRICITET, SILA ZATEZANJA

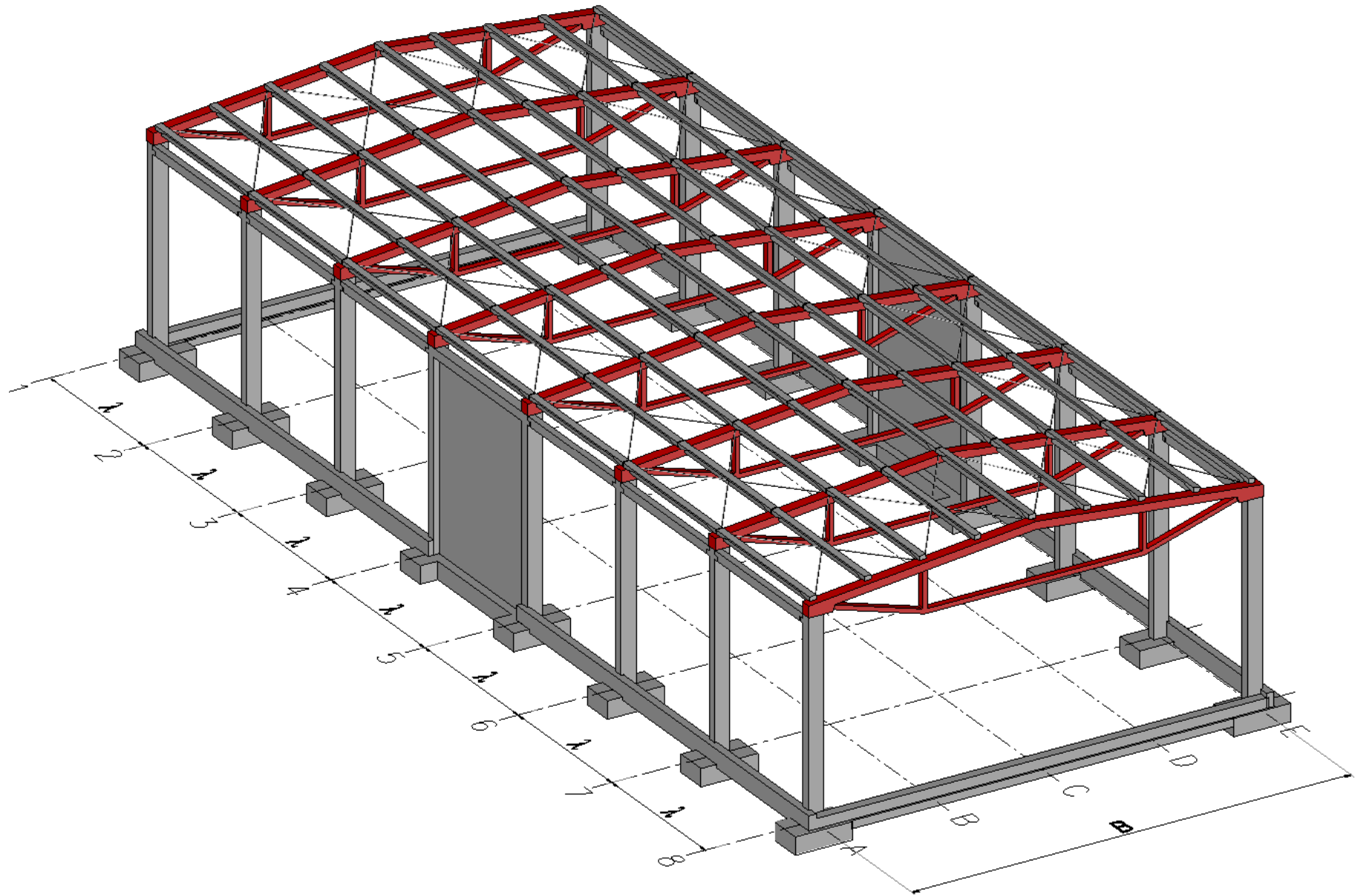


$$A_s = A_{s1} + A_{s2} = \frac{N_{Ed}}{f_{yd}}$$

$$A_{s1} = \frac{N_{Ed}}{f_{yd}} \frac{c_2 + e}{c_1 + c_2}$$

$$A_{s2} = \frac{N_{Ed}}{f_{yd}} \frac{c_1 - e}{c_1 + c_2}$$

# ULS – MALI EKSCENTRICITET, SILA ZATEZANJA



## Zadatak 18 – **EKSCENTRIČNO ZATEZANJE**

Dimenzionisati pravougaoni poprečni presek zadatih dimenzija, opterećen silom zatezanja i momentom savijanja u fazi malog ekscentriciteta.

Podaci za proračun:

$$N_{G,k} = -400 \text{ kN}$$

$$M_{G,k} = 10 \text{ kNm}$$

$$b = 40 \text{ cm}$$

$$N_{Q,k} = -500 \text{ kN}$$

$$h = 25 \text{ cm}$$

B500 B

$$f_{yd} = 500/1.15 = 435 \text{ MPa} = 43.5 \text{ kN/cm}^2$$





## Zadatak 18 – EKSCENTRIČNO ZATEZANJE

$$N_{Ed} = 1.35 \cdot (-400) + 1.5 \cdot (-500) = -1290 \text{ kN}$$

$$M_{Ed} = 1.35 \cdot 10 = 13.5 \text{ kNm}$$

$$e = \frac{M_{Ed}}{N_{Ed}} = \frac{13.5 \times 10^2}{1290} = 1.05 \text{ cm}$$

pretp.  $d_1 = d_2 = 6 \text{ cm}$

$$c_1 = c_2 = h/2 - d_1 = 25/2 - 6 = 6.5 \text{ cm}$$

$$A_s = A_{s1} + A_{s2} = \frac{N_{Ed}}{f_{yd}} = \frac{1290}{43.5} = 29.7 \text{ cm}^2$$



## Zadatak 18 – EKSCENTRIČNO ZATEZANJE

$$A_{s1} = \frac{N_{Ed}}{f_{yd}} \frac{c_2 + e}{c_1 + c_2} = 29.7 \cdot \frac{6.5 + 1.05}{6.5 + 6.5} = 17.2 \text{ cm}^2$$

$$A_{s2} = \frac{N_{Ed}}{f_{yd}} \frac{c_1 - e}{c_1 + c_2} = 29.7 \cdot \frac{6.5 - 1.05}{6.5 + 6.5} = 12.5 \text{ cm}^2$$

≈ 9Ø16

$d_1 = 8 \text{ cm}$

$c_1 = 25/2 - 8$   
 $= 4.5 \text{ cm}$

≈ 6Ø16

$d_2 = 6 \text{ cm}$

$c_2 = 25/2 - 6 =$   
 $6.5 \text{ cm}$



## Zadatak 18 – EKSCENTRIČNO ZATEZANJE

$$A_{s1} = \frac{N_{Ed}}{f_{yd}} \frac{c_2 + e}{c_1 + c_2} = 29.7 \cdot \frac{6.5 + 1.05}{4.5 + 6.5} = 20.4 \text{ cm}^2$$

$$A_{s2} = \frac{N_{Ed}}{f_{yd}} \frac{c_1 - e}{c_1 + c_2} = 29.7 \cdot \frac{4.5 - 1.05}{4.5 + 6.5} = 9.3 \text{ cm}^2$$

≈ 10Ø16

$$d_1 = 8.25 \text{ cm}$$

$$c_1 = 25/2 - 8.25 = 4.25 \text{ cm}$$

≈ 5Ø16

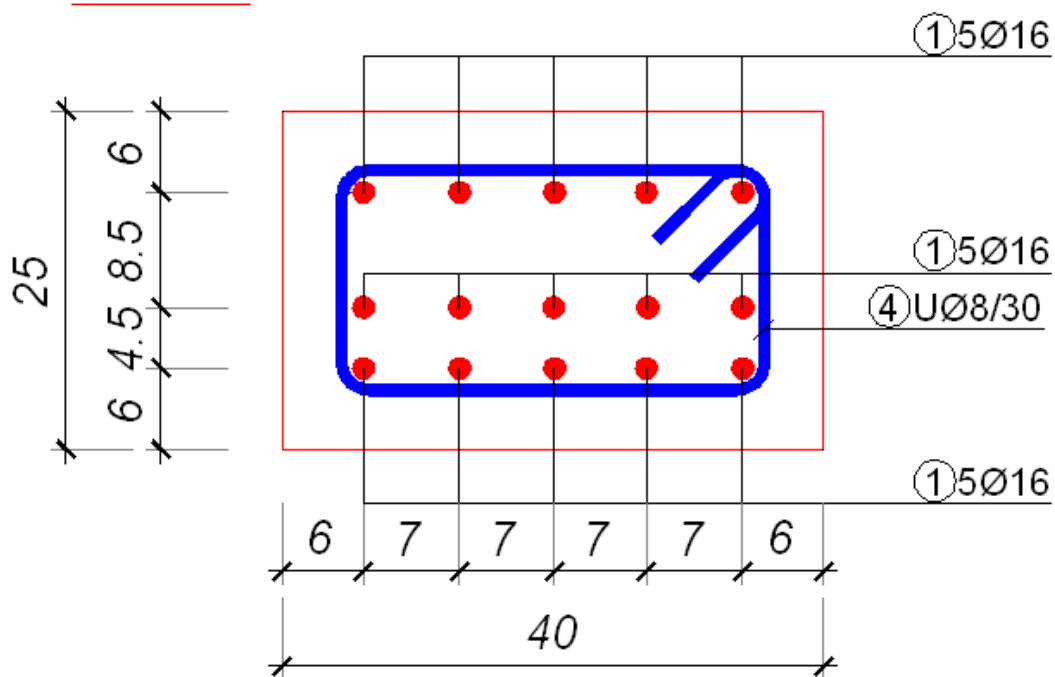
$$d_2 = 6 \text{ cm}$$

$$c_2 = 25/2 - 6 = 6.5 \text{ cm}$$



# Zadatak 18 – EKSCENTRIČNO ZATEZANJE

XD1



$$d^I = c_{\text{nom}} + \varnothing_s + \varnothing/2$$

$$a^I = 4.5 + 0.8 + 1.6/2 = 6.1 \text{ cm}$$

$$\text{usv. } d^I = 6 \text{ cm}$$

$$d^{II} = d^I + a_v + 2 \times \varnothing/2$$

$$d^{II} = 6 + 3.0 + 2 \times 1.6/2 = 10.6 \text{ cm}$$

$$\text{usv. } d^{II} = 10.5 \text{ cm}$$

$$a_1 = (5 \times 6 + 5 \times 10.5) / 10$$

$$d_1 = 8.25 \text{ cm}$$