

1. Odrediti napone u betonu i armaturi za presek prikazan na skici desno. Presek je opterećen momentima savijanja  $M_g = 29.6$  kNm i  $M_p = 26$  kNm. MB 30, GA 240/360.

$$a_1 = \frac{3 \times 4.5 + 2 \times 9.5}{5} = 6.5 \text{ cm} ; a_2 = 4.5 \text{ cm}$$

$$h = 40 - 6.5 = 33.5 \text{ cm} ; \alpha_2 = \frac{a_2}{h} = \frac{4.5}{33.5} = 0.134$$

$$A_{a1} = 15.71 \text{ cm}^2 \Rightarrow \mu_1 = \frac{A_{a1}}{b \times h} = \frac{15.71}{20 \times 33.5} = 2.34\%$$

$$A_{a2} = 2.26 \text{ cm}^2 \Rightarrow \mu_2 = \frac{A_{a2}}{b \times h} = \frac{2.26}{20 \times 33.5} = 0.34\%$$

$$\text{MB 30} \Rightarrow E_b = 31.5 \text{ GPa} \Rightarrow n = \frac{E_a}{E_b} = \frac{210}{31.5} = 6.67$$

$$s^2 + 2 \times n \times (\mu_1 + \mu_2) \times s - 2 \times n \times (\mu_1 + \mu_2 \times \alpha_2) = 0$$

$$s^2 + 2 \times 6.67 \times (2.34 + 0.34) \times 10^{-2} \times s - 2 \times 6.67 \times (2.34 + 0.34 \times 0.134) \times 10^{-2} = 0$$

$$s^2 + 0.358 \times s - 0.319 = 0 \Rightarrow s = \mathbf{0.414}$$

$$J_{Ib} = \frac{s^2}{2} \times \left(1 - \frac{s}{3}\right) = \frac{0.414^2}{2} \times \left(1 - \frac{0.414}{3}\right) = 0.074$$

$$M_a = M = M_g + M_p = 29.6 + 26.0 = 55.6 \text{ kNm}$$

$$\sigma_b = \frac{M_a}{b \times h^2} \times \frac{s}{J_{Ib} + n \times \mu_2 \times (s - \alpha_2) \times (1 - \alpha_2)}$$

$$\sigma_b = \frac{55.6 \times 10^2}{20 \times 33.5^2} \times \frac{0.414}{0.074 + 6.67 \times 0.34 \times 10^{-2} \times (0.414 - 0.134) \times (1 - 0.134)} = 1.29 \text{ kN/cm}^2$$

$$\sigma_{a1} = n \times \sigma_b \times \frac{1-s}{s} = 6.67 \times 1.29 \times \frac{1-0.414}{0.414} = 12.23 \text{ kN/cm}^2$$

$$\sigma_{a2} = n \times \sigma_b \times \frac{s-\alpha_2}{s} = 6.67 \times 1.29 \times \frac{0.414-0.134}{0.414} = 5.82 \text{ kN/cm}^2$$

$$\varepsilon_b = \frac{\sigma_b}{E_b} = \frac{1.29}{3.15 \times 10^3} = 0.411\text{‰}$$

$$\varepsilon_{a1} = \frac{\sigma_{a1}}{E_a} = \frac{12.23}{21 \times 10^3} = 0.583\text{‰} ; \varepsilon_{a2} = \frac{\sigma_{a2}}{E_a} = \frac{5.82}{21 \times 10^3} = 0.277\text{‰}$$



#### Napomene:

U praktičnim zadacima **nije obavezno** uvrstiti u proračun površinu pritisnute armature u preseku (ukoliko nije računski potrebna). Za slučaj  $A_{a2} = 0$ , sledi  $s = \mathbf{0.424}$ ,  $\sigma_b = \mathbf{13.6 \text{ MPa}}$ , odnosno  $\sigma_{a1} = \mathbf{123.0 \text{ MPa}}$ .

Potrebno je uočiti da su vrednosti sračunatih napona vrlo bliske vrednostima dopuštenih napona u betonu i armaturi. Naime, pored dimenzionisanja po teoriji granične nosivosti (Godišnji zadatak, list 2), isti primer je dimenzionisan i u Zadatku 1 - klasična teorija i u računskom smislu bio JEDNOSTRUKO ARMIRAN).

5. Odrediti napone u betonu i armaturi za presek prikazan na skici desno. Presek je opterećen momentima savijanja  $M_g = 29.6 \text{ kNm}$  i  $M_p = 26 \text{ kNm}$ . Računska aktivna širina ploče  $B = 105 \text{ cm}$ . MB 30, GA 240/360.

$$a_1 = \frac{3 \times 4.5 + 2 \times 9.5}{5} = 6.5 \text{ cm}$$

$$h_1 = 40 - 6.5 = 33.5 \text{ cm}$$

$$a_2 = 4.5 \text{ cm} \Rightarrow \alpha_2 = \frac{a_2}{h} = \frac{4.5}{33.5} = 0.134$$

Iz praktičnih razloga se pretpostavlja da se neutralna linija nalazi u ploči, pa se  $s$  određuje za pravougaoni presek širine  $B = 105 \text{ cm}$ :

$$A_{a1} = 12.72 \text{ cm}^2 \Rightarrow \mu_1 = \frac{A_{a1}}{B \times h} = \frac{12.72}{105 \times 33.5} = 0.36\%$$

$$A_{a2} = 2.26 \text{ cm}^2 \Rightarrow \mu_2 = \frac{A_{a2}}{B \times h} = \frac{2.26}{105 \times 33.5} = 0.06\%$$

$$\text{MB 30} \Rightarrow E_b = 31.5 \text{ GPa} \Rightarrow n = \frac{E_a}{E_b} = \frac{210}{31.5} = 6.67$$

$$s^2 + 2 \times n \times (\mu_1 + \mu_2) \times s - 2 \times n \times (\mu_1 + \mu_2 \times \alpha_2) = 0$$

$$s^2 + 2 \times 6.67 \times (0.36 + 0.06) \times 10^{-2} \times s - 2 \times 6.67 \times (0.36 + 0.06 \times 0.134) \times 10^{-2} = 0$$

$$s^2 + 0.057 \times s - 0.049 = 0 \Rightarrow s = 0.196 < \delta = \frac{d_p}{h} = \frac{10}{33.5} = 0.299$$

Pretpostavka o položaju neutralne linije je tačna, pa dalje sledi:

$$J_{Iib} = \frac{s^2}{2} \times \left(1 - \frac{s}{3}\right) = \frac{0.196^2}{2} \times \left(1 - \frac{0.196}{3}\right) = 0.018$$

$$M_a = M = M_g + M_p = 29.6 + 26.0 = 55.6 \text{ kNm}$$

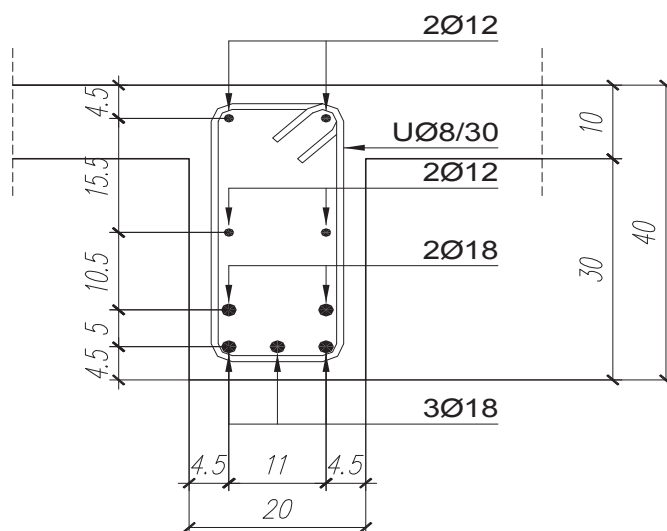
$$\sigma_b = \frac{M_a}{B \times h^2} \times \frac{s}{J_{Iib} + n \times \mu_2 \times (s - \alpha_2) \times (1 - \alpha_2)}$$

$$\sigma_b = \frac{55.6 \times 10^2}{105 \times 33.5^2} \times \frac{0.196}{0.018 + 6.67 \times 0.06 \times 10^{-2} \times (0.196 - 0.134) \times (1 - 0.134)} = 0.51 \text{ kN/cm}^2$$

$$\sigma_{a1} = n \times \sigma_b \times \frac{1-s}{s} = 6.67 \times 0.51 \times \frac{1-0.196}{0.196} = 13.97 \text{ kN/cm}^2$$

$$\sigma_{a2} = n \times \sigma_b \times \frac{s - \alpha_2}{s} = 6.67 \times 0.51 \times \frac{0.196 - 0.134}{0.196} = 1.06 \text{ kN/cm}^2$$

$$\varepsilon_b = \frac{\sigma_b}{E_b} = \frac{0.51}{3.15 \times 10^3} = 0.162\%$$



$$\varepsilon_{a1} = \frac{\sigma_{a1}}{E_a} = \frac{13.97}{21 \times 10^3} = 0.665\text{‰} \quad ; \quad \varepsilon_{a2} = \frac{\sigma_{a2}}{E_a} = \frac{1.06}{21 \times 10^3} = 0.051\text{‰}$$

Napomene:

Za slučaj  $A_{a2} = 0$ , sledi  $s = 0.197$ ,  $\sigma_b = 5.1 \text{ MPa}$ , odnosno  $\sigma_{a1} = 139.6 \text{ MPa}$ .

Jasno je da kod preseka sa ovako velikom pritisnutom površinom betona doprinos armature smeštene uz pritisnutu ivicu preseka treba zanemariti. Očekivano, sračunati napon u betonu je znatno ispod dopuštenog rubnog napona, dok je napon u zategnutoj armaturi ponovo vrlo blizak dopuštenom.

Ponovo se ukazuje da iz pretpostavljenog položaja neutralne linije sledi proračun za presek **pravougaonog oblika**, koga karakteriše SAMO JEDNA širina (u konkretnom slučaju,  $B=105 \text{ cm}$ ), bez obzira da li je u opštim izrazima za proračun napona kod pravougaonog preseka označena malim ili velikim slovom. Za slučaj da se neutralna linija nađe u rebru, proračun se sprovodi za odgovarajući presek oblika **T**.

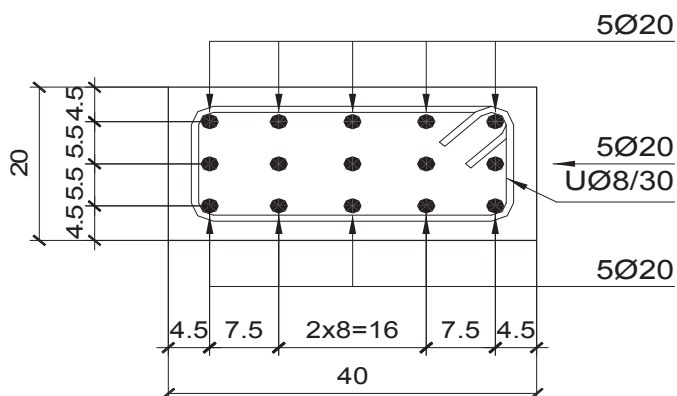
6. Odrediti napon u armaturi za presek prikazan na skici desno. Presek je opterećen silama zatezanja  $Z_g = 305 \text{ kN}$  i  $Z_p = 337 \text{ kN}$ . MB 25, GA 240/360.

$$A_a = 47.12 \text{ cm}^2 \quad (15\text{Ø}20)$$

$$Z = Z_g + Z_p = 305 + 337 = 642 \text{ kN}$$

$$\sigma_a = \frac{Z}{A_a} = \frac{642}{47.12} = 13.62 \text{ kN/cm}^2$$

$$\varepsilon_a = \frac{\sigma_a}{E_a} = \frac{13.62}{21 \times 10^3} = 0.649\text{‰}$$



7. Odrediti napone u betonu i armaturi za presek prikazan na skici desno. Presek je opterećen silama pritiska  $N_g = 630 \text{ kN}$  i  $N_p = 398 \text{ kN}$ . MB 25, GA 240/360.

$$\text{MB 25} \Rightarrow E_b = 30 \text{ GPa} \Rightarrow n = \frac{E_a}{E_b} = \frac{210}{30} = 7$$

$$A_a = 6.79 \text{ cm}^2 \quad (6\text{Ø}12)$$

$$A_b = \frac{D^2 \times \pi}{4} = \frac{40^2 \times \pi}{4} = 1257 \text{ cm}^2$$

Površina idealizovanog preseka:

$$A_i = A_b + n \times A_a = 1257 + 7 \times 6.79 = 1304 \text{ cm}^2$$

$$N = N_g + N_p = 630 + 398 = 1028 \text{ kN}$$

$$\sigma_b = \frac{N}{A_i} = \frac{1028}{1304} = 0.79 \text{ kN/cm}^2 \Rightarrow \varepsilon_b = \frac{\sigma_b}{E_b} = \frac{0.79}{3 \times 10^3} = 0.263\text{‰} = \varepsilon_a$$

$$\sigma_a = \varepsilon_a \times E_a = n \times \sigma_b = 0.263 \times 10^{-3} \times 21 \times 10^3 = 5.52 \text{ kN/cm}^2$$

