

# BETONSKE KONSTRUKCIJE

## Vežba br.2

***Jelena Dragaš dipl. građ. inž.                      Kabinet br. 3***

***Miodrag Stojanović dipl. građ. inž.            Kabinet br. 3***

Konsultacije:

Kabinet br. 3 – Pon. 14-16h, Uto. 14-16h, Čet. 15-16h

jelenad@imk.grf.bg.ac.rs

***Semestar: V***

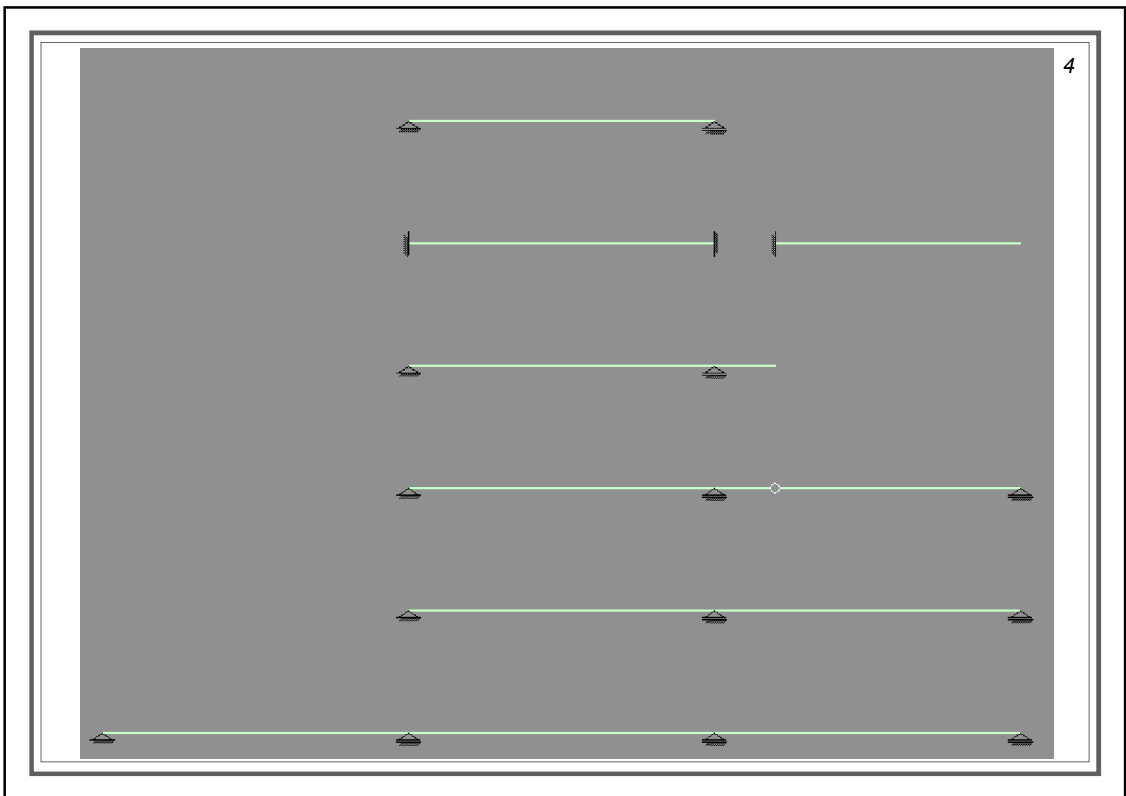
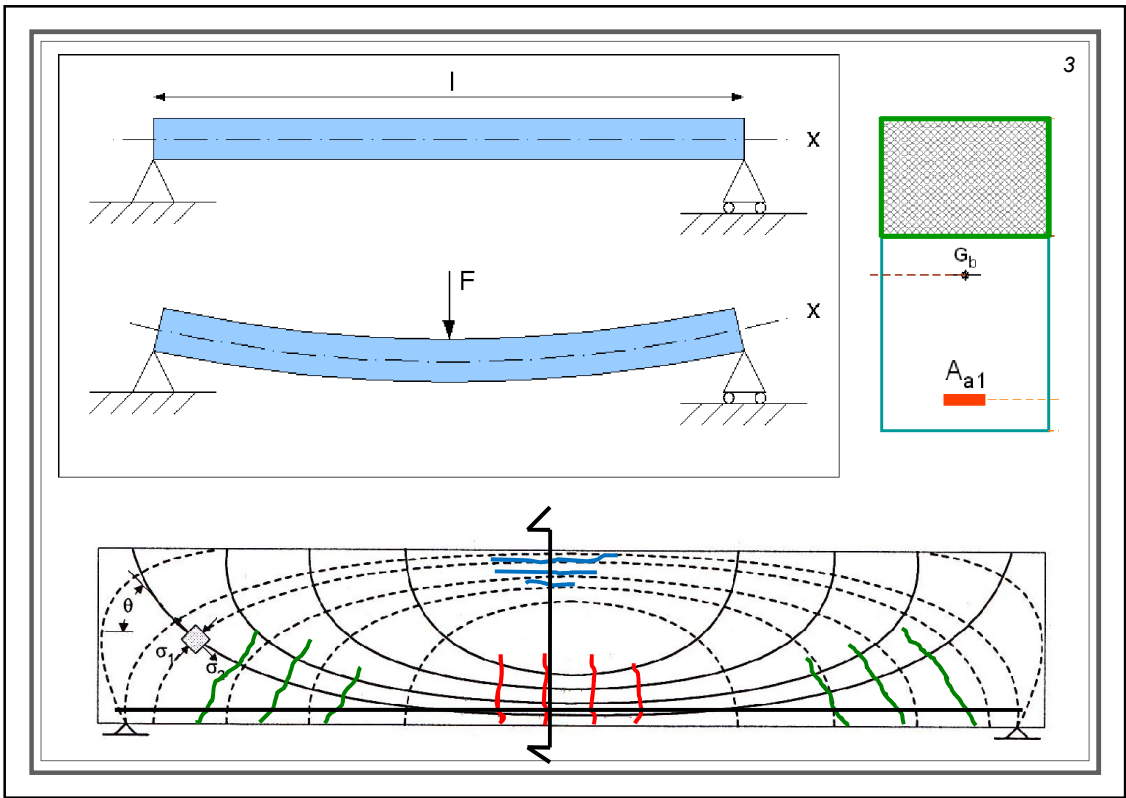
***ESPB: 6***

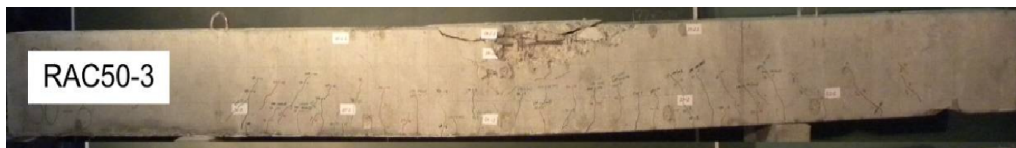
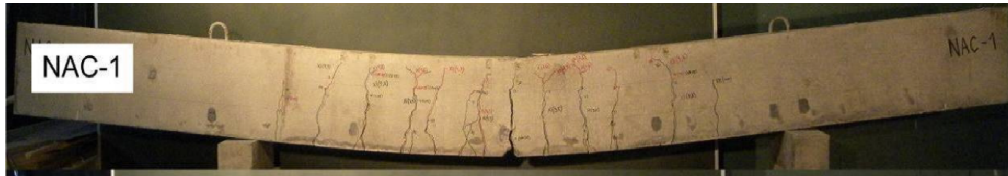
***1. Veliki ekscentricitet - Čisto savijanje***

***2. Slobodno dimenzionisanje***

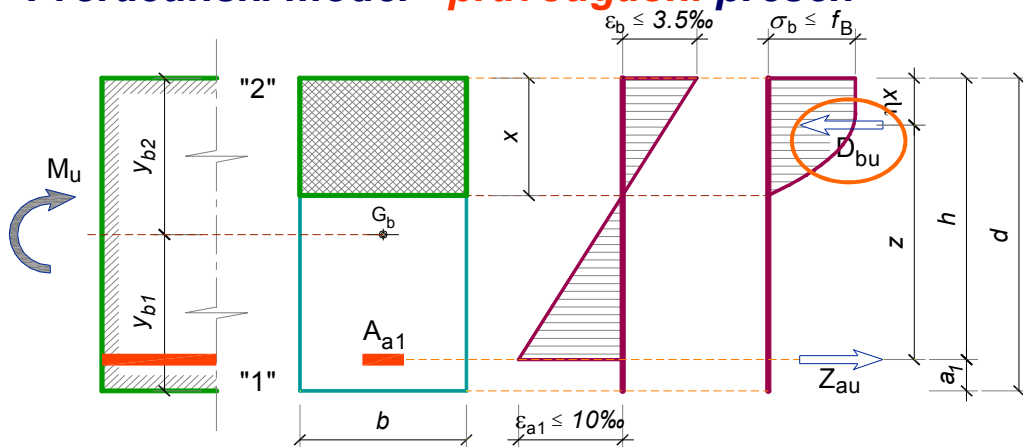
***3. Vezano dimenzionisanje***

***4. Dvostruko armirani preseci***





### Proračunski model - pravougaoni presek



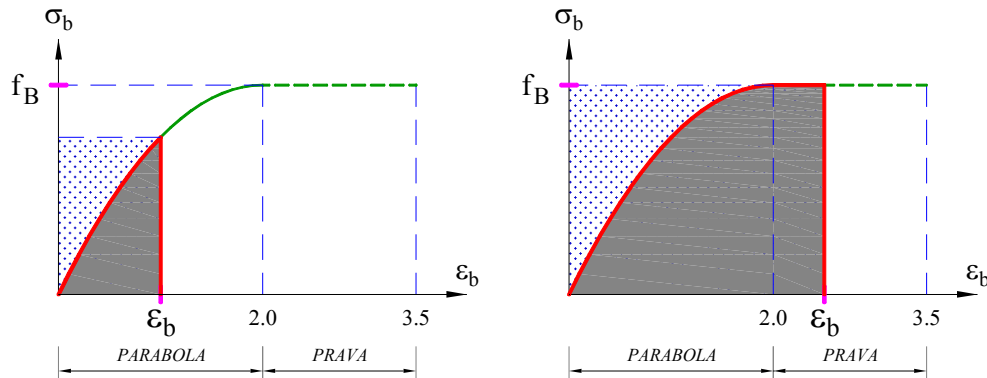
$$D_{bu} = \alpha_b \cdot b \cdot x \cdot f_B = \alpha_b \cdot s \cdot b \cdot h \cdot f_B$$

$$s = \frac{x}{h} = \frac{\epsilon_b}{\epsilon_b + \epsilon_{a1}} = \frac{1}{1 + \frac{\epsilon_{a1}}{\epsilon_b}}$$





## Koeficijent punoće naponskog dijagrama betona $\alpha_b$



$$\alpha_b = \frac{\epsilon_b}{12} \times (6 - \epsilon_b) \quad (0 \leq \epsilon_b \leq 2\text{‰})$$

$$\alpha_b = \frac{3\epsilon_b - 2}{3\epsilon_b} \quad (2\text{‰} \leq \epsilon_b \leq 3.5\text{‰})$$

| MB    | 15   | 20 | 30   | 40   | 50 | 60 |
|-------|------|----|------|------|----|----|
| $f_B$ | 10.5 | 14 | 20.5 | 25.5 | 30 | 33 |

## Položaj sile pritiska u betonu

$$\eta = \frac{8 - \epsilon_b}{4 \times (6 - \epsilon_b)} \quad (0 \leq \epsilon_b \leq 2\text{‰})$$

$$\eta = \frac{\epsilon_b \times (3\epsilon_b - 4) + 2}{2\epsilon_b \times (3\epsilon_b - 2)} \quad (2\text{‰} \leq \epsilon_b \leq 3.5\text{‰})$$

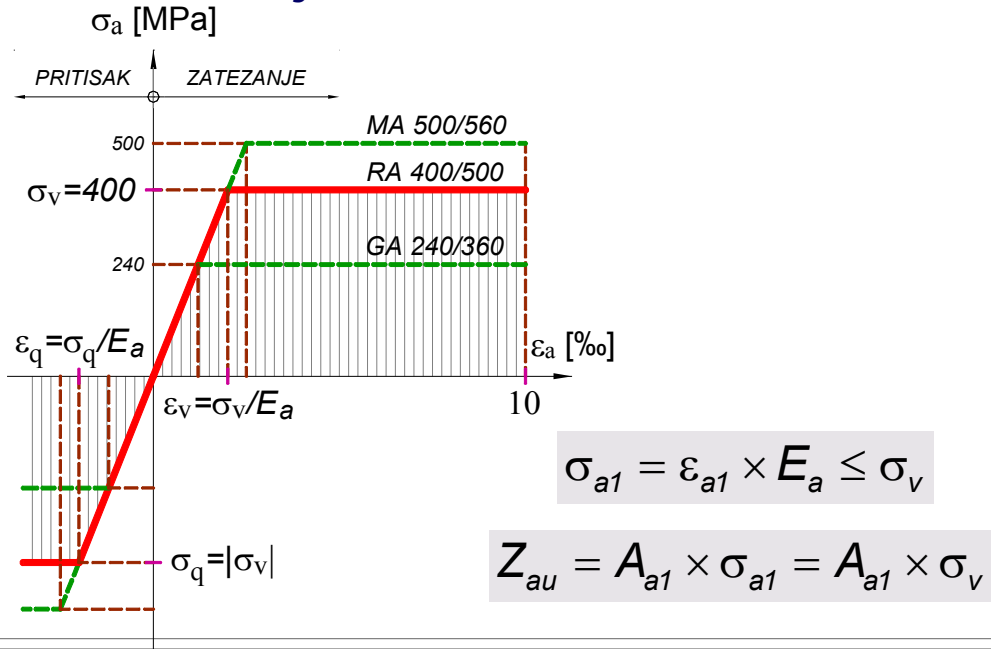
**Visina preseka :**  $d$

**Težište zategnute armature :**  $a_1$

**Statička visina :**  $h = d - a_1$

**Krak unutrašnjih sila**  $z = h - \eta \times x = h \times (1 - \eta \times s) = \zeta \times h$

## Sila zatezanja u armaturi



## Koeficijenti armiranja

### GEOMETRIJSKI KOEFICIJENT ARMIRANJA:

$$\mu_1 = \frac{A_{a1}}{b \times h} \geq \mu_{MIN.}$$

$$\mu_{MIN.} = \begin{cases} 0.25\% (GA) \\ 0.20\% (RA) \end{cases}$$

### MEHANIČKI KOEFICIJENT ARMIRANJA:

$$\bar{\mu}_1 = \mu_{1M} = \alpha \times s = \frac{A_{a1}}{b \times h} \times \frac{\sigma_v}{f_B} = \mu_1 \times \frac{\sigma_v}{f_B}$$

## Uslov ravnoteže **MOMENATA**

15

$$k = \frac{h}{\sqrt{\frac{M_u}{b \times f_B}}}$$

$$k = \sqrt{\frac{1}{\alpha_b \times s \times \zeta}}$$

## Uslov ravnoteže **NORMALNIH SILA**

$$A_{a1} = \bar{\mu}_1 \times b \times h \times \frac{f_B}{\sigma_v}$$

$$A_{a1} = \frac{M_u}{z \times \sigma_v} = \frac{M_u}{\zeta \times h \times \sigma_v}$$

## 2. Čisto savijanje – **SLOBODNO** dimenzionisanje

16

- **Poznato:**
  - statički uticaji za ( $M_i$ ) – sračunato
  - kvalitet materijala ( $f_B$ ,  $\sigma_v$ ) – usvojeno
- **Nepoznato:**
  - **dimenzije poprečnog preseka** ( $b$ ,  $d$ )
  - **površina armature** ( $A_a$ )
  - stanje dilatacija preseka ( $s$ )



## 2. Čisto savijanje –**SLOBODNO** dimenzionisanje <sup>17</sup>

1. Sračunavaju se granični računski statički uticaji

$$M_u = \sum_i \gamma_{u,i} \times M_i \quad (i = g, p, \Delta)$$

2. Usvajaju se  $\varepsilon_b$  i  $\varepsilon_a$ , pri čemu bar jedna mora dostići graničnu vrednost.

Za usvojene vrednosti dilatacija iz tabela se očitavaju koeficijenti  $k$  i  $\mu_{1M}$ , odnosno  $\zeta$ .

## 2. Čisto savijanje –**SLOBODNO** dimenzionisanje <sup>18</sup>

3. Sračunava se statička visina  $h$ :

$$\varepsilon_b / \varepsilon_a \xrightarrow{\text{TABL.}} k \Rightarrow h = k \sqrt{\frac{M_u}{b \times f_B}} \quad \text{Jedinice!}$$

i potrebna površina armature iz izraza:

$$\varepsilon_b / \varepsilon_a \xrightarrow{\text{TABL.}} \bar{\mu} \quad (\zeta)$$

$$A_a = \bar{\mu} \times \frac{b \times h}{100} \times \frac{f_B}{\sigma_v}$$

ili

$$A_a = \frac{M_u}{z \times \sigma_v} = \frac{M_u}{\zeta \times h \times \sigma_v}$$

## 2. Čisto savijanje –SLOBODNO dimenzionisanje

4. Usvaja se broj i prečnik šipki armature. Usvojena armatura se raspoređuje u preseku ( $a_0$ , čisto rastojanje između šipki)

5. Sračunava se položaj težišta  $a_1$  usvojene armature i usvaja visina preseka  $d$ :

$$d = h + a_1$$

6. Konačno se **konstruiše poprečni presek** i prikazuje u odgovarajućoj razmeri (1:10) sa svim potrebnim kotama i oznakama.

### Primer 1 - SLOBODNO dimenzionisanje

Odrediti **visinu** i potrebnu **površinu armature** za presek pravougaonog oblika, opterećen momentima savijanja usled stalnog ( $M_g$ ) i povremenog ( $M_p$ ) opterećenja. Podaci za proračun:

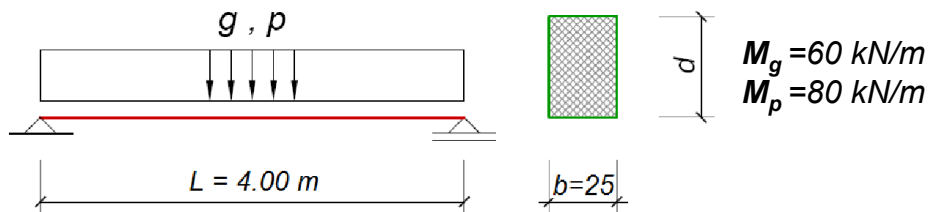
$$g = 30 \text{ kNm}$$

$$b = 25 \text{ cm}$$

MB 30

$$p = 40 \text{ kNm}$$

GA 240/360



$$\text{MB 30} \Rightarrow f_B = 20.5 \text{ MPa} = 2.05 \text{ kN/cm}^2$$

$$\text{GA 240/360} \Rightarrow \sigma_V = 240 \text{ MPa} = 24 \text{ kN/cm}^2$$

### Primer 1a - SLOBODNO dimenzionisanje

$$M_u = 1.6 \times 60 + 1.8 \times 80 = 240 \text{ kNm}$$

$$\text{usv. } \varepsilon_b / \varepsilon_{a1} = 3.5 / 10\text{‰}$$

$$k = 2.311 ; \mu_{1M} = 20.988\% ; \zeta = 0.892$$

| $\varepsilon_a$ | $\varepsilon_b$ | s     | $\alpha$ | $\eta$ | $\zeta$ | $\mu_{1M} \%$ | k     |
|-----------------|-----------------|-------|----------|--------|---------|---------------|-------|
| 10              | 3.5             | 0.259 | 0.810    | 0.416  | 0.892   | 20.988        | 2.311 |

$$h = k \sqrt{\frac{M_u}{b \times f_B}} = 2.311 \times \sqrt{\frac{240 \times 10^2}{25 \times 2.05}} = 50.0 \text{ cm}$$

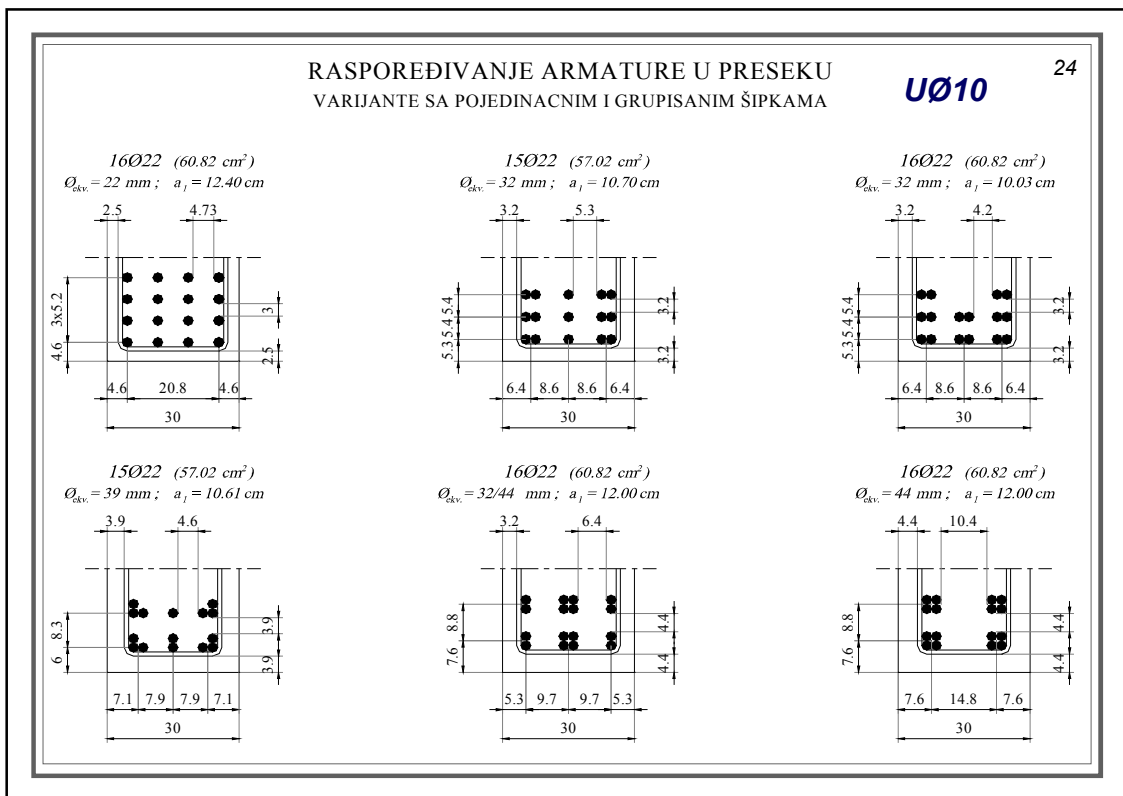
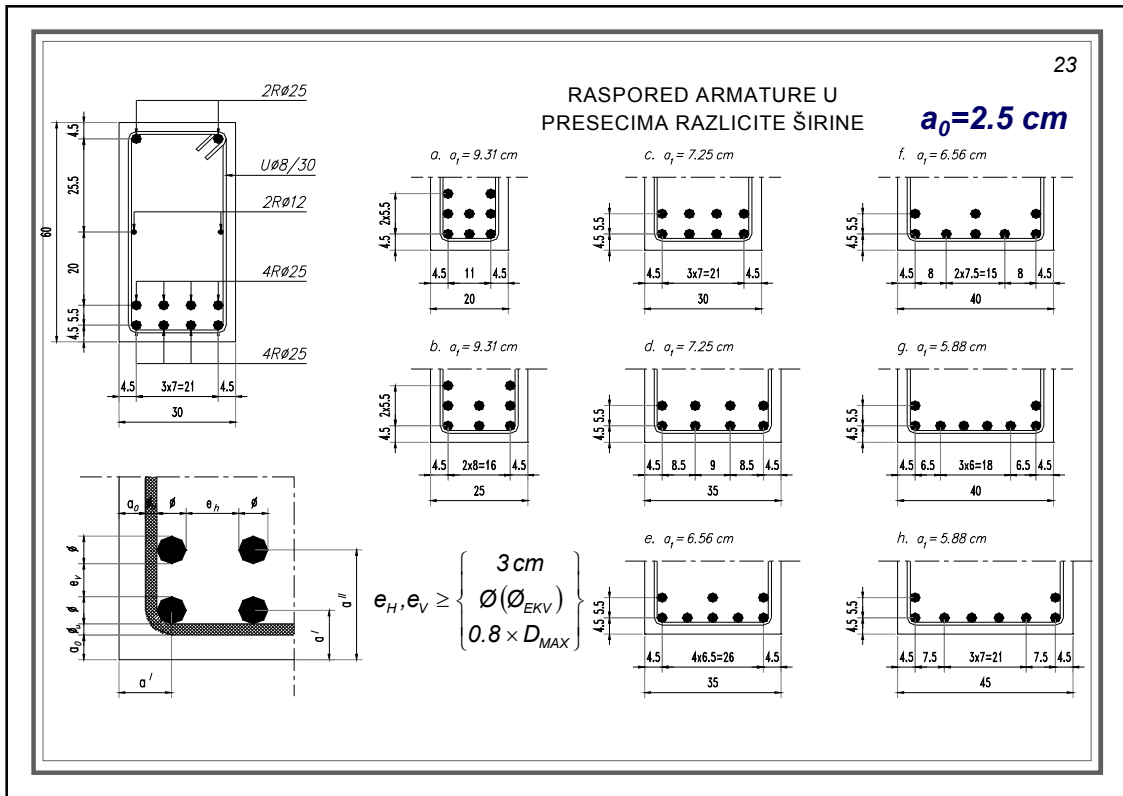
### Primer 1a - SLOBODNO dimenzionisanje

$$A_a = \mu \times \frac{b \times h}{100} \times \frac{f_B}{\sigma_v} = 20.988 \times \frac{25 \times 50.0}{100} \times \frac{2.05}{24} = 22.41 \text{ cm}^2$$

ili:

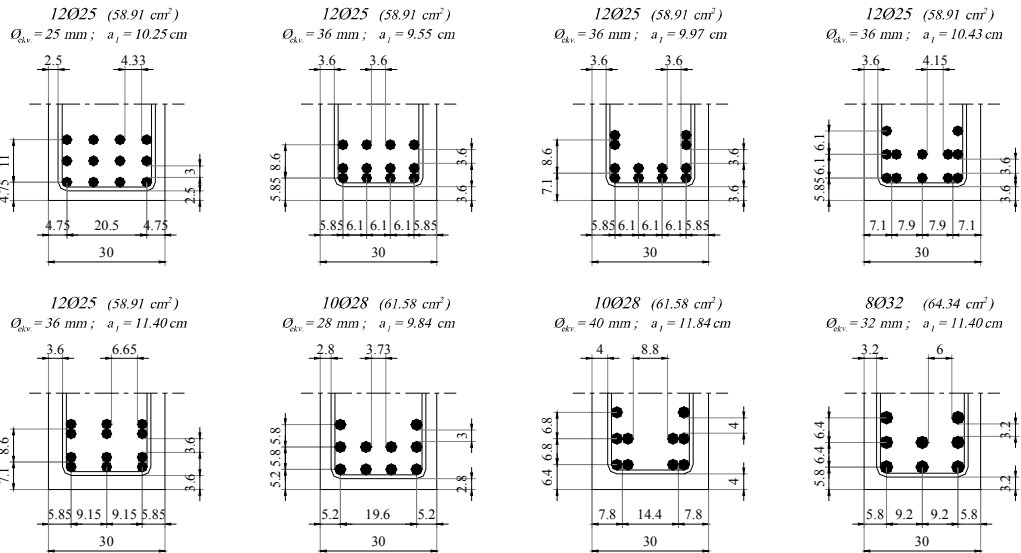
$$A_a = \frac{M_u}{z \times \sigma_v} = \frac{M_u}{\zeta \times h \times \sigma_v} = \frac{240 \times 10^2}{0.892 \times 50.0 \times 24} = 22.42 \text{ cm}^2$$

usvojeno: **6Ø22** (22.80 cm<sup>2</sup>)

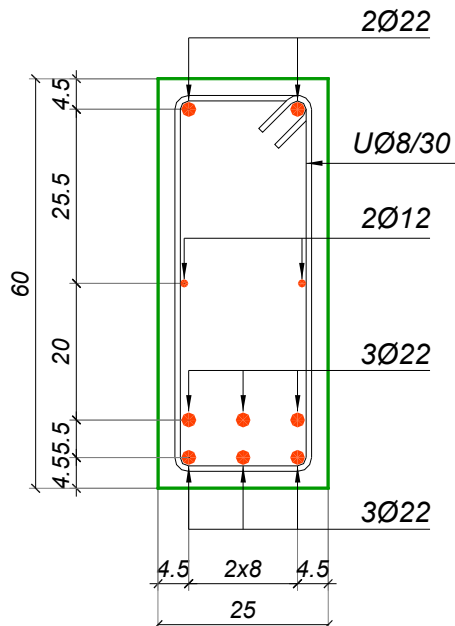


RASPOREĐIVANJE ARMATURE U PRESEKU  
VARIJANTE SA POJEDINACNIM I GRUPISANIM ŠIPKAMA

**UØ10**



**Primer 1a - SLOBODNO dimenzionisanje**



$$a' = 2.5 + 0.8 + 2.2/2 = 4.4 \text{ cm}$$

usv.  **$a' = 4.5 \text{ cm}$**

$$a'' = 4.5 + 3.0 + 2 \times 2.2/2 = 9.7 \text{ cm}$$

usv.  **$a'' = 10.0 \text{ cm}$**

$$a_1 = (3 \times 4.5 + 3 \times 10) / 6$$

**$a_1 = 7.25 \text{ cm}$**

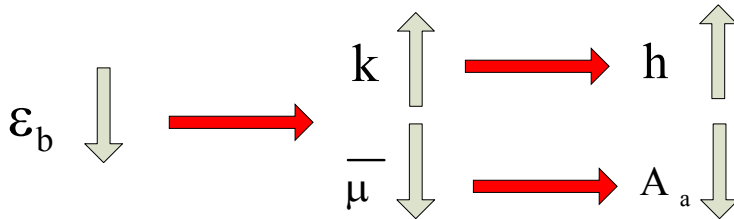
$$d = 50 + 7.25 = 57.25 \text{ cm}$$

**USV.  $d = 60 \text{ cm}$**

### Primer 1 - SLOBODNO dimenzionisanje

Tablice za dimenzionisanje

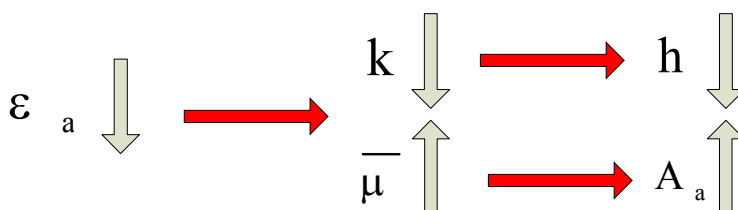
| $\varepsilon_a$ | $\varepsilon_b$ | s     | $\alpha$ | $\eta$ | $\zeta$ | $\mu_{1M} \%$ | k     |
|-----------------|-----------------|-------|----------|--------|---------|---------------|-------|
| 10              | 3.5             | 0.259 | 0.810    | 0.416  | 0.892   | 20.988        | 2.311 |
| 10              | 2.6             | 0.206 | 0.744    | 0.394  | 0.919   | 15.344        | 2.663 |
| 10              | 2.575           | 0.205 | 0.741    | 0.393  | 0.919   | 15.176        | 2.677 |
| 10              | 2.55            | 0.203 | 0.739    | 0.392  | 0.920   | 15.007        | 2.691 |



### Primer 1 - SLOBODNO dimenzionisanje

$\varepsilon_b = 3.5 \text{ ‰} : \alpha = 0.810 ; \eta = 0.416$

| $\varepsilon_a$ | $\varepsilon_b$ | s     | $\zeta$ | $\mu_{1M} \%$ | k     |
|-----------------|-----------------|-------|---------|---------------|-------|
| 10              | 3.5             | 0.259 | 0.892   | 20.988        | 2.311 |
| 9.95            | 3.5             | 0.260 | 0.892   | 21.066        | 2.307 |
| 9.90            | 3.5             | 0.261 | 0.891   | 21.144        | 2.303 |
| 9.85            | 3.5             | 0.262 | 0.891   | 21.223        | 2.300 |



### Primer 1b - SLOBODNO dimenzionisanje

Lom po armaturi:

$$M_u = 1.6 \times 60 + 1.8 \times 80 = 240 \text{ kNm}$$

$$\text{usv. } \varepsilon_b / \varepsilon_{a1} = 2.6 / 10\text{‰}$$

$$k = 2.311 ; \mu_{1M} = 20.988\% ; \zeta = 0.892$$

| $\varepsilon_a$ | $\varepsilon_b$ | s     | $\alpha$ | $\eta$ | $\zeta$ | $\mu_{1M} \%$ | k     |
|-----------------|-----------------|-------|----------|--------|---------|---------------|-------|
| 10              | 2.6             | 0.206 | 0.744    | 0.394  | 0.919   | 15.344        | 2.663 |

$$h = k \sqrt{\frac{M_u}{b \times f_B}} = 2.663 \times \sqrt{\frac{240 \times 10^2}{25 \times 2.05}} = 57.6 \text{ cm}$$

### Primer 1b - SLOBODNO dimenzionisanje

$$A_a = \mu \times \frac{b \times h}{100} \times \frac{f_B}{\sigma_v} = 15.344 \times \frac{25 \times 57.6}{100} \times \frac{2.05}{24} = 18.89 \text{ cm}^2$$

ili:

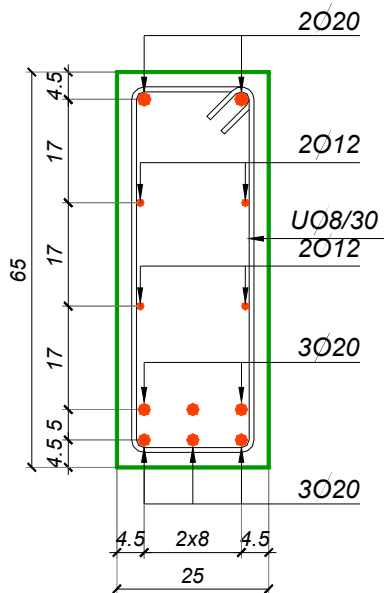
$$A_a = \frac{M_u}{z \times \sigma_v} = \frac{M_u}{\zeta \times h \times \sigma_v} = \frac{240 \times 10^2}{0.919 \times 57.6 \times 24} = 18.89 \text{ cm}^2$$

usvojeno: **5Ø22** (19.01 cm<sup>2</sup>)

ili:

usvojeno: **6Ø20** (18.85 cm<sup>2</sup>)

### Primer 1b - SLOBODNO dimenzionisanje



$$a' = 2.5 + 0.8 + 2.2/2 = 4.4 \text{ cm}$$

usv.  $a' = 4.5 \text{ cm}$

$$a'' = 4.5 + 3.0 + 2 \times 2.0/2 = 9.5 \text{ cm}$$

usv.  $a'' = 9.5 \text{ cm}$

$$a_1 = (3 \times 4.5 + 3 \times 9.5)/6$$

$a_1 = 7.0 \text{ cm}$

$$d = 57.6 + 7.0 = 64.6 \text{ cm}$$

USV.  $d = 65 \text{ cm}$

### Primer 1c - SLOBODNO dimenzionisanje

Lom po betonu:

$$M_u = 1.6 \times 60 + 1.8 \times 80 = 240 \text{ kNm}$$

$$\text{usv. } \varepsilon_b/\varepsilon_{a1} = 3.5/5.0\%$$

$$k = 1.903 ; \mu_{1M} = 33.333\% ; \zeta = 0.829$$

| $\varepsilon_a$ | $\varepsilon_b$ | s     | $\alpha$ | $\eta$ | $\zeta$ | $\mu_{1M} \%$ | k     |
|-----------------|-----------------|-------|----------|--------|---------|---------------|-------|
| 5.0             | 3.5             | 0.412 | 0.810    | 0.416  | 0.829   | 33.333        | 1.903 |

$$h = k \sqrt{\frac{M_u}{b \times f_b}} = 1.903 \times \sqrt{\frac{240 \times 10^2}{25 \times 2.05}} = 41.2 \text{ cm}$$



### Primer 1c - SLOBODNO dimenzionisanje

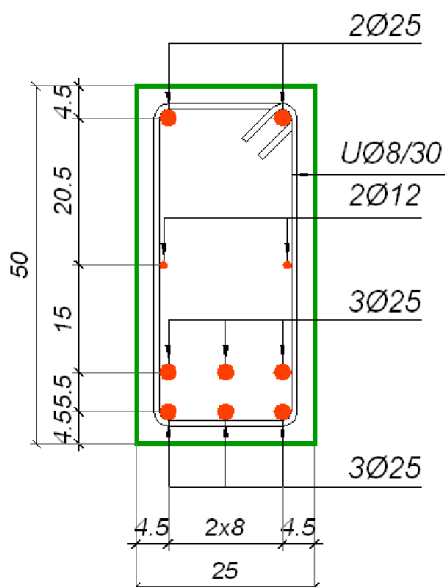
$$A_a = \mu \times \frac{b \times h}{100} \times \frac{f_b}{\sigma_v} = 33.333 \times \frac{25 \times 41.2}{100} \times \frac{2.05}{24} = 29.31 \text{ cm}^2$$

ili:

$$A_a = \frac{M_u}{z \times \sigma_v} = \frac{M_u}{\zeta \times h \times \sigma_v} = \frac{240 \times 10^2}{0.829 \times 41.2 \times 24} = 29.28 \text{ cm}^2$$

usvojeno: **6Ø25** (29.45 cm<sup>2</sup>)

### Primer 1c - SLOBODNO dimenzionisanje



$$a' = 2.5 + 0.8 + 2.5/2 = 4.5 \text{ cm}$$

usv. **a' = 4.5 cm**

$$a'' = 4.5 + 3.0 + 2 \times 2.5/2$$

usv. **a'' = 10 cm**

$$a_1 = (3 \times 4.5 + 3 \times 10)/6$$

**a<sub>1</sub> = 7.3 cm**

$$d = 41.2 + 7.3 = 48.5 \text{ cm}$$

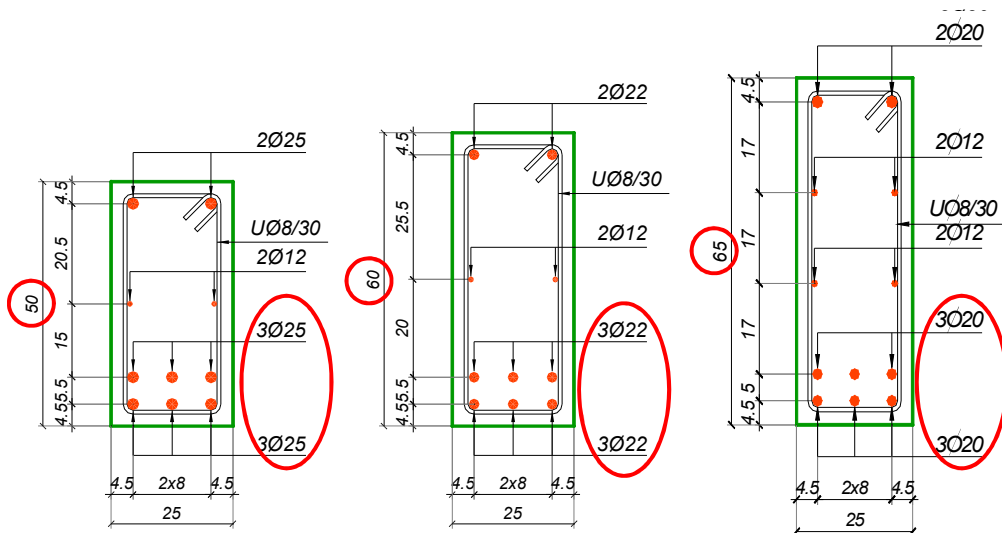
**USV. d = 50 cm**

## Primer 1 - SLOBODNO dimenzionisanje

LOM PO BETONU

SIMULTANI LOM

LOM PO ARMATURI



## Čisto savijanje - VEZANO dimenzionisanje

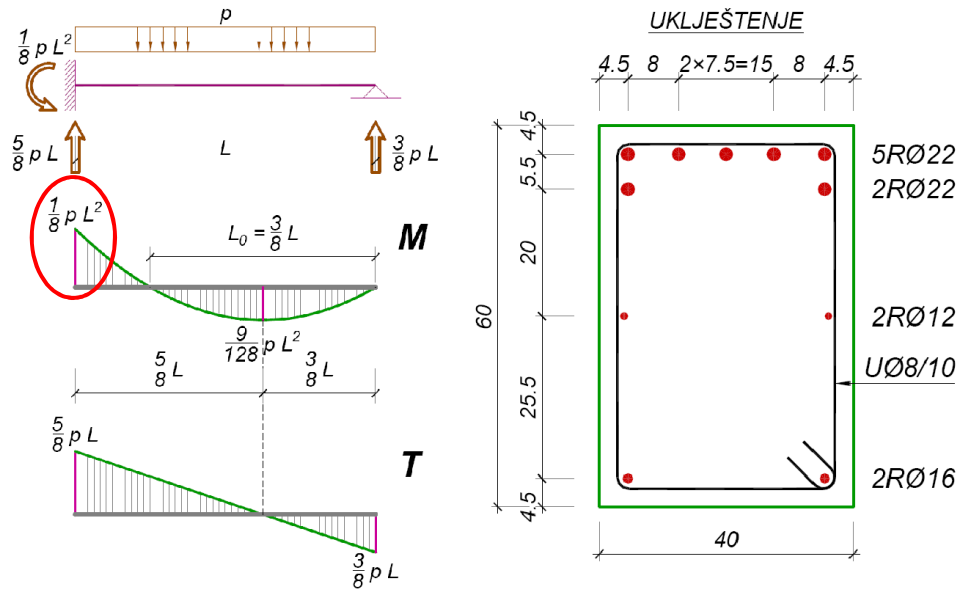
- **Poznato:**

- statički uticaji za ( $M_i$ ) – sračunato
- kvalitet materijala ( $f_B, \sigma_v$ ) – usvojeno
- **dimenzije poprečnog preseka** ( $b, d$ )

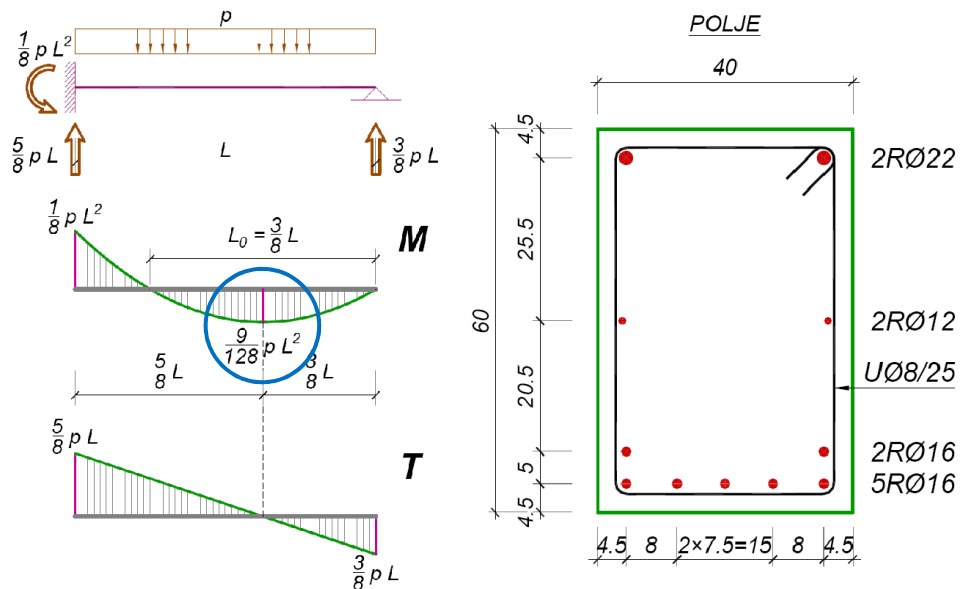
- **Nepoznato:**

- **površina armature** ( $A_a$ )
- stanje dilatacija preseka ( $s$ )

## Čisto savijanje - VEZANO dimenzionisanje



## Čisto savijanje - VEZANO dimenzionisanje



## Čisto savijanje - **VEZANO** dimenzionisanje

1. Sračunavaju se granični računski statički uticaji

$$M_u = \sum_i \gamma_{u,i} \times M_i \quad (i = g, p, \Delta)$$

2. Pretpostavlja se položaj težišta zategnute armature  $a_1$  i na osnovu toga sračunava statička visina

$$h = d - a_1$$

Veličina  $a_1$  se pretpostavlja u granicama  $(0.05-0.15) \times d$

## 2. Čisto savijanje - **VEZANO** dimenzionisanje

3. Sračunava se koeficijent  $k$ :

$$M_u \Rightarrow k = \frac{h}{\sqrt{\frac{M_u}{b \times f_B}}} \xrightarrow{\text{TABLICE}} \bar{\mu} (\zeta)$$

i pročitaju dilatacije  $\varepsilon_b, \varepsilon_{a1}$ .

Ako je  $\varepsilon_{a1} \geq 3\%$ , sračunava se potrebna površina armature iz izraza:

## Čisto savijanje - **VEZANO** dimenzionisanje

41

$$A_a = \mu \times \frac{b \times h}{100} \times \frac{f_B}{\sigma_v} \quad \text{ili:}$$

$$A_a = \frac{M_u}{z \times \sigma_v} = \frac{M_u}{\zeta \times h \times \sigma_v}$$

Ako je  $\varepsilon_{a1} < 3\text{‰}$ , presek se **DVOSTRUKO** armira

## Čisto savijanje - **VEZANO** dimenzionisanje

42

### 4. Usvaja se broj i prečnik šipki armature.

Usvojena armatura se raspoređuje u preseku ( $a_0$ , čisto rastojanje između šipki)

### 5. Sračunava se položaj težišta $a_1$ usvojene armature i statička visina $h$ i upoređuje sa pretpostavljenom.

➤ U slučaju znatnijih odstupanja, proračun se ponavlja sa korigovanom vrednošću  $a_1$ .

### 6. Konačno se **konstruiše poprečni presek** i prikazuje u odgovarajućoj razmeri (1:10) sa svim potrebnim kotama i oznakama.

## Primer 2 - VEZANO dimenzionisanje

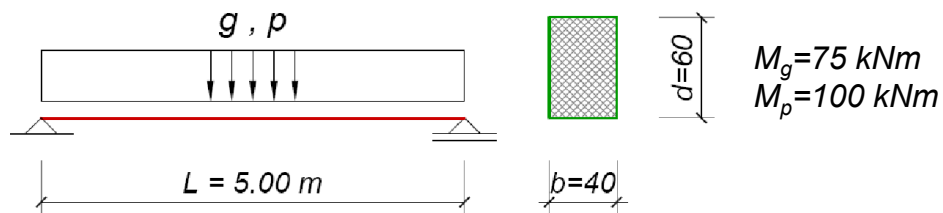
Odrediti **potrebnu površinu armature** za presek poznatih dimenzija, pravougaonog oblika, opterećen graničnim momentom savijanja  $M_u$ . Podaci za proračun:

$$g = 24 \text{ kN/m}$$

$$b = 40 \text{ cm} \quad \text{MB 30}$$

$$p = 32 \text{ kN/m}$$

$$d = 60 \text{ cm} \quad \text{RA 400/500}$$



$$\text{MB 30} \quad \Rightarrow \quad f_B = 20.5 \text{ MPa} = 2.05 \text{ kN/cm}^2$$

$$\text{RA 400/500} \quad \Rightarrow \quad \sigma_V = 400 \text{ MPa} = 40 \text{ kN/cm}^2$$

## Primer 2 - VEZANO dimenzionisanje

$$M_u = 1.6 \times 75 + 1.8 \times 100 = 300 \text{ kNm}$$

$$\text{pretp. } a_1 = 7 \text{ cm}$$

$$h = d - a_1 = 60 - 7 = 53 \text{ cm}$$

$$k = \frac{h}{\sqrt{\frac{M_u}{b \times f_B}}} = \frac{53}{\sqrt{\frac{300 \times 10^2}{40 \times 2.05}}} = 2.771$$

| $\epsilon_a$ | $\epsilon_b$ | s            | $\alpha$     | $\eta$       | $\zeta$      | $\mu_{IM} \%$ | k            |
|--------------|--------------|--------------|--------------|--------------|--------------|---------------|--------------|
| 10           | 2.450        | 0.197        | 0.728        | 0.389        | 0.923        | 14.324        | 2.750        |
| <b>10</b>    | <b>2.425</b> | <b>0.195</b> | <b>0.725</b> | <b>0.389</b> | <b>0.924</b> | <b>14.152</b> | <b>2.765</b> |
| 10           | 2.400        | 0.194        | 0.722        | 0.388        | 0.925        | 13.978        | 2.781        |

## Primer 2 - VEZANO dimenzionisanje

45

| $\varepsilon_a$ | $\varepsilon_b$ | s     | $\alpha$ | $\eta$ | $\zeta$ | $\mu_{IM} \%$ | k     |
|-----------------|-----------------|-------|----------|--------|---------|---------------|-------|
| 10              | 2.425           | 0.195 | 0.725    | 0.389  | 0.924   | 14.152        | 2.765 |

$$A_a = \mu \times \frac{b \times h}{100} \times \frac{f_B}{\sigma_v}$$

$$A_a = 14.152 \times \frac{40 \times 53}{100} \times \frac{2.05}{40} = 15.38 \text{ cm}^2$$

ili:

## Primer 2 - VEZANO dimenzionisanje

46

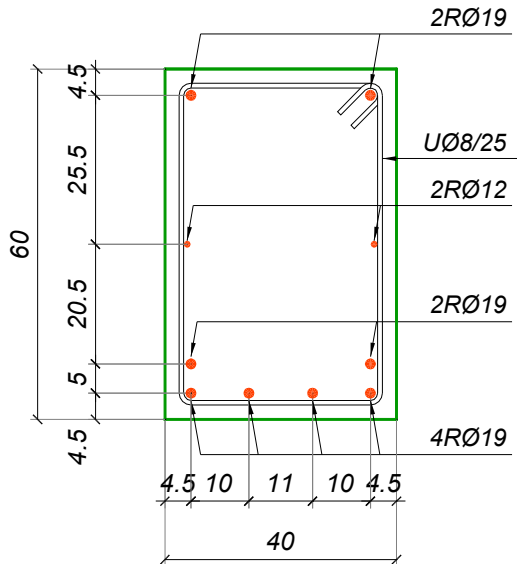
| $\varepsilon_a$ | $\varepsilon_b$ | s     | $\alpha$ | $\eta$ | $\zeta$ | $\mu_{IM} \%$ | k     |
|-----------------|-----------------|-------|----------|--------|---------|---------------|-------|
| 10              | 2.425           | 0.195 | 0.725    | 0.389  | 0.924   | 14.152        | 2.765 |

$$A_a = \frac{M_u}{z \times \sigma_v} = \frac{M_u}{\zeta \times h \times \sigma_v}$$

$$A_a = \frac{300 \times 10^2}{0.924 \times 53 \times 40} = 15.31 \text{ cm}^2$$

usvojeno: **6RØ19** (17.01 cm<sup>2</sup>)

## Primer 2 - VEZANO dimenzionisanje



$$a' = a_0 + \varnothing_u + \varnothing/2$$

$$a' = 2.5 + 0.8 + 1.9/2 = 4.25 \text{ cm}$$

$$\text{usv. } a' = 4.5 \text{ cm}$$

$$a'' = a' + e_v + 2 \times \varnothing/2$$

$$a'' = 4.5 + 3.0 + 2 \times 1.9/2$$

$$\text{usv. } a'' = 9.5 \text{ cm}$$

$$a_1 = (4 \times 4.5 + 2 \times 9.5)/6$$

$$a_1 = 6.17 \text{ cm}$$

$$h = 60 - 6.17 = 53.83 \text{ cm} > 53 \text{ cm}$$

## Primer 2 - VEZANO dimenzionisanje

| $\varepsilon_a$ | $\varepsilon_b$ | s     | $\alpha$ | $\eta$ | $\zeta$ | $\mu_{IM} \%$ | k     |
|-----------------|-----------------|-------|----------|--------|---------|---------------|-------|
| 10              | 2.425           | 0.195 | 0.725    | 0.389  | 0.924   | 14.152        | 2.765 |

$$D_{bu} = \alpha_b \times s \times b \times h \times f_B$$

$$D_{bu} = 0.725 \times 0.195 \times 40 \times 53 \times 2.05 = 612.3 \text{ kN}$$

$$Z_{au} = A_{a1} \times \sigma_{a1} = A_{a1} \times \sigma_v$$

$$Z_{au} = 15.31 \times 40 = 612.3 \text{ kN} = D_{bu}$$

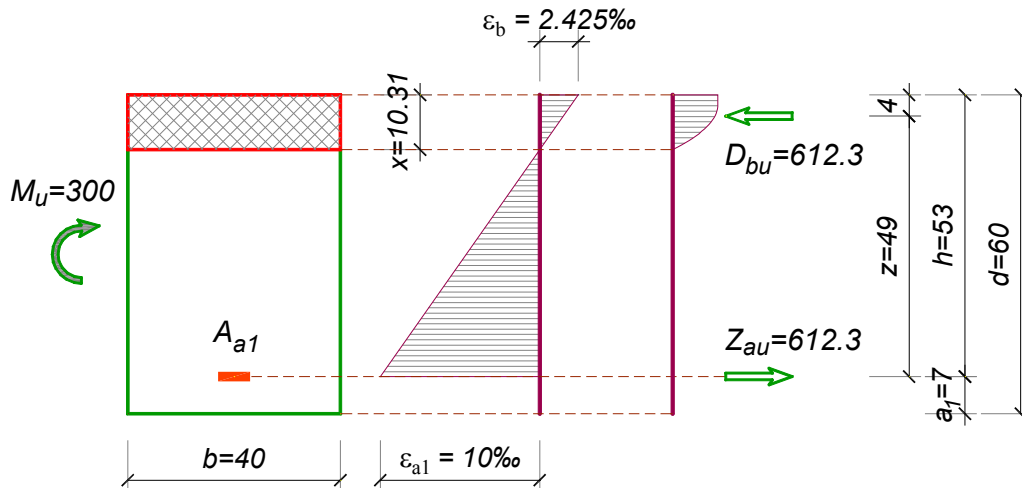
$$x = s \times h = 0.195 \times 53 = 10.31 \text{ cm}$$

$$z = \zeta \times h = 0.924 \times 53 = 49.0 \text{ cm}$$



## Primer 2 - VEZANO dimenzionisanje

49



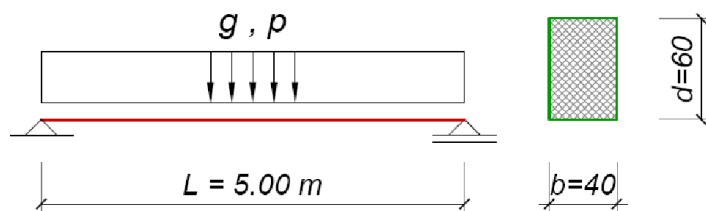
## Primer 3 - VEZANO dimenzionisanje

50

Odrediti potrebnu površinu armature za presek poznatih dimenzija, pravougaonog oblika, opterećen graničnim momentom savijanja  $M_u$ . Podaci za proračun:

$$M_u = 600 \text{ kNm} \quad b = 40 \text{ cm} \quad \text{MB 30}$$

$$d = 60 \text{ cm} \quad \text{RA 400/500}$$



$$\text{MB 30} \quad \Rightarrow \quad f_B = 20.5 \text{ MPa} = 2.05 \text{ kN/cm}^2$$

$$\text{RA 400/500} \quad \Rightarrow \quad \sigma_V = 400 \text{ MPa} = 40 \text{ kN/cm}^2$$

### Primer 3 - VEZANO dimenzionisanje

pretp.  $a_1 = 7 \text{ cm}$

$h = d - a_1 = 60 - 7 = 53 \text{ cm}$

$$k = \frac{h}{\sqrt{\frac{M_u}{b \times f_B}}} = \frac{53}{\sqrt{\frac{600 \times 10^2}{40 \times 2.05}}} = 1.959$$

| $\varepsilon_a$ | s            | $\zeta$      | $\mu_{1M} \%$ | k            |
|-----------------|--------------|--------------|---------------|--------------|
| 5.7             | 0.380        | 0.842        | 30.797        | 1.964        |
| <b>5.65</b>     | <b>0.383</b> | <b>0.841</b> | <b>30.965</b> | <b>1.960</b> |
| 5.6             | 0.385        | 0.840        | 31.136        | 1.955        |

### Primer 3 - VEZANO dimenzionisanje

| $\varepsilon_a$ | $\varepsilon_b$ | s            | $\alpha_b$   | $\eta$       | $\zeta$      | $\mu_{1M} \%$ | k            |
|-----------------|-----------------|--------------|--------------|--------------|--------------|---------------|--------------|
| <b>5.65</b>     | <b>3.5</b>      | <b>0.383</b> | <b>0.810</b> | <b>0.416</b> | <b>0.841</b> | <b>30.965</b> | <b>1.960</b> |

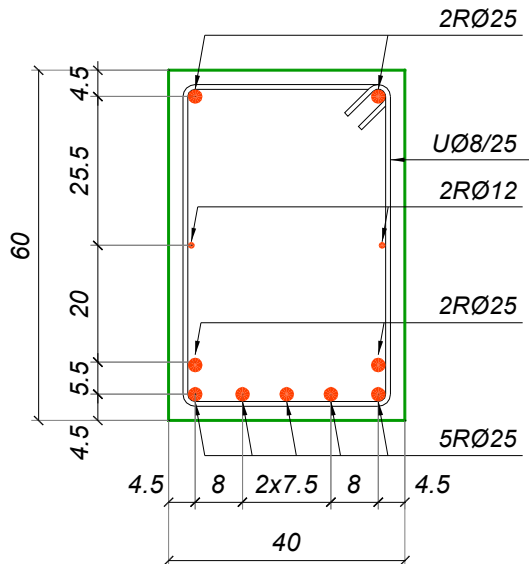
$$A_a = 30.965 \times \frac{40 \times 53}{100} \times \frac{2.05}{40} = 33.64 \text{ cm}^2$$

ili:

$$A_a = \frac{600 \times 10^2}{0.841 \times 53 \times 40} = 33.66 \text{ cm}^2$$

usvojeno: **7RØ25** (34.36 cm<sup>2</sup>)

### Primer 3 - VEZANO dimenzionisanje



$$a' = a_0 + \varnothing_u + \varnothing/2$$

$$a' = 2.5 + 0.8 + 2.5/2 = 4.55 \text{ cm}$$

usv.  $a' = 4.5 \text{ cm}$

$$a'' = a' + e_v + 2 \times \varnothing/2$$

$$a'' = 4.5 + 3.0 + 2 \times 2.5/2$$

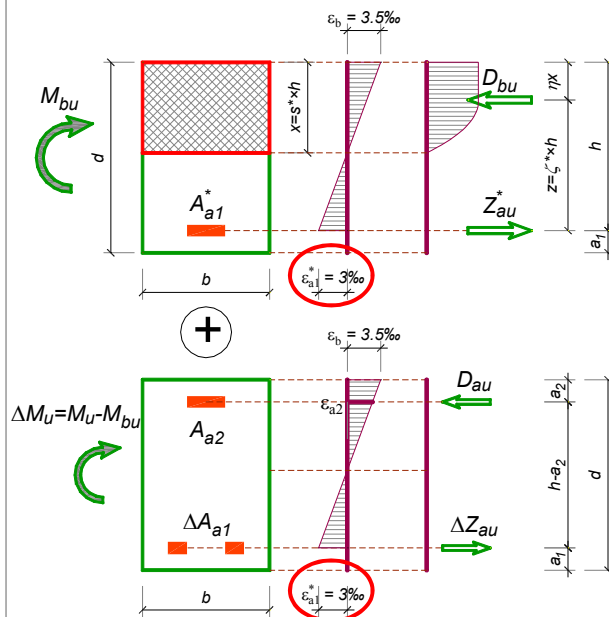
usv.  $a'' = 10 \text{ cm}$

$$a_1 = (5 \times 4.5 + 2 \times 10) / 7$$

$a_1 = 6.07 \text{ cm}$

$$h = 60 - 6.07 = 53.93 \text{ cm} > 53 \text{ cm}$$

### 4. Dvostruko armirani presecki



$$M_{bu} = \left( \frac{h}{k^*} \right)^2 \times b \times f_B$$

$$\Delta M_u = M_u - M_{bu}$$

## 4. Dvostruko armirani preseći

1. Sračunavaju se statički uticaji:

$$M_u = \sum \gamma_{ui} \times M_i$$

2. Pretpostavlja se  $a_1$  i sračunava statička visina

$$h = d - a_1$$

3. Sračunava se koeficijent  $k$ :

$$M_u \Rightarrow k = \frac{h}{\sqrt{\frac{M_u}{b \times f_B}}} \xrightarrow{\text{TABLICE}} \bar{\mu}(\zeta)$$

i pročitaju dilatacije  $\varepsilon_b$ ,  $\varepsilon_{a1}$ .

Ako je  $\varepsilon_{a1} < 3\text{‰}$ , presek se **DVOSTRUKO ARMIRA**.

## 4. Dvostruko armirani preseći

3a. Određuje se **MOMENT NOSIVOSTI JEDNOSTRUKO ARMIRANOG PRESEKA**, sa procentom armiranja  $\mu_{1M}^*$  i koeficijentom  $k^*$  koji odgovaraju dilataciji armature koja se želi zadržati (po pravilu  $\varepsilon_{a1}^* = 3\text{‰}$ )

$$M_{bu} = \left( \frac{h}{k^*} \right)^2 \times b \times f_B$$

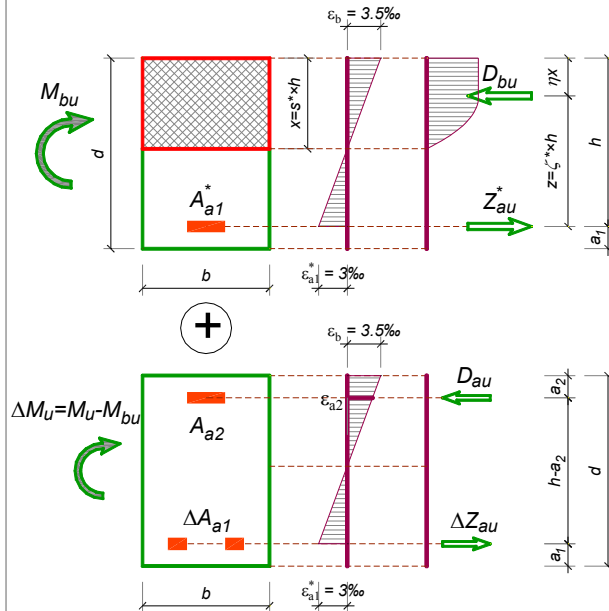
Preostali deo spoljašnjeg momenta savijanja:

$$\Delta M_u = M_u - M_{bu}$$

se prihvata **dodatnom zategnutom i pritisnutom armaturom**.

Pretpostavlja se položaj težišta pritisnute armature  $a_2$  i određuju se površine zategnute i pritisnute armature u preseku, iz izraza:

## 4. Dvostruko armirani presecci



$$A_{a2} = \frac{\Delta M_U}{(h - a_2) \times \sigma_V}$$

$$A_{a1} = \mu^* \times \frac{b \times h}{100} \times \frac{f_B}{\sigma_V} + A_{a2}$$

ili:

$$A_{a1} = \frac{M_{bu}}{\zeta^* \times h \times \sigma_V} + A_{a2}$$

## 4. Dvostruko armirani presecci

**4. Usvaja se broj i prečnik šipki armature. Usvojena armatura se raspoređuje u preseku ( $a_0$ , čisto rastojanje između šipki)**

**5. Sračunava se položaj težišta  $a_1$  usvojene armature i statička visina  $h$  i upoređuje sa pretpostavljenom.**

- U slučaju znatnijih odstupanja, proračun se ponavlja sa korigovanom vrednošću  $a_1$ .

**6. Konačno se konstruiše poprečni presek i prikazuje u odgovarajućoj razmeri (1:10) sa svim potrebnim kotama i oznakama.**

## Primer 4 – dvojno armiranje

Odrediti **potrebnu površinu armature** za presek poznatih dimenzija, pravougaonog oblika, opterećen graničnim momentom savijanja  $M_u$ .  
Podaci za proračun:

$$M_u = 1000 \text{ kNm} \quad b = 40 \text{ cm} \quad \text{MB 30} \\ d = 60 \text{ cm} \quad \text{RA 400/500}$$

$$\text{MB 30} \quad \Rightarrow \quad f_B = 20.5 \text{ MPa} = 2.05 \text{ kN/cm}^2$$

$$\text{RA 400/500} \quad \Rightarrow \quad \sigma_V = 400 \text{ MPa} = 40 \text{ kN/cm}^2$$

## Primer 4 – dvojno armiranje

$$\text{pretp. } a_1 = 8 \text{ cm}$$

$$h = d - a_1 = 60 - 8 = 52 \text{ cm}$$

$$k = \frac{52}{\sqrt{\frac{1000 \times 10^2}{40 \times 2.05}}} = 1.489 \Rightarrow \varepsilon_b / \varepsilon_{a1} = 3.5 / 0.5\text{‰}$$

Kako je  $\varepsilon_{a1} < 3\text{‰}$ , presek se **DVOSTRUKO ARMIRA**

## Primer 4 – dvojno armiranje

usvojeno  $\varepsilon_{a1}^* = 3\text{‰}$   $\Rightarrow$   $k^* = 1.719$ ,  $\mu_{1M}^* = 43.590\%$

| $\varepsilon_a$ | $\varepsilon_b$ | s     | $\alpha_b$ | $\eta$ | $\zeta$ | $\mu_{1M} \%$ | k     |
|-----------------|-----------------|-------|------------|--------|---------|---------------|-------|
| 3               | 3.5             | 0.538 | 0.810      | 0.416  | 0.776   | 43.590        | 1.719 |

$$M_{bu} = \left( \frac{52}{1.719} \right)^2 \times 40 \times 2.05 = 75000 \text{ kNcm} = 750 \text{ kNm}$$

$$\Delta M_u = 1000 - 750 = 250 \text{ kNm}$$

## Primer 4 – dvojno armiranje

$$a_2 = 5 \text{ cm} \Rightarrow A_{a2} = \frac{250 \times 10^2}{(52 - 5) \times 40} = 13.30 \text{ cm}^2$$

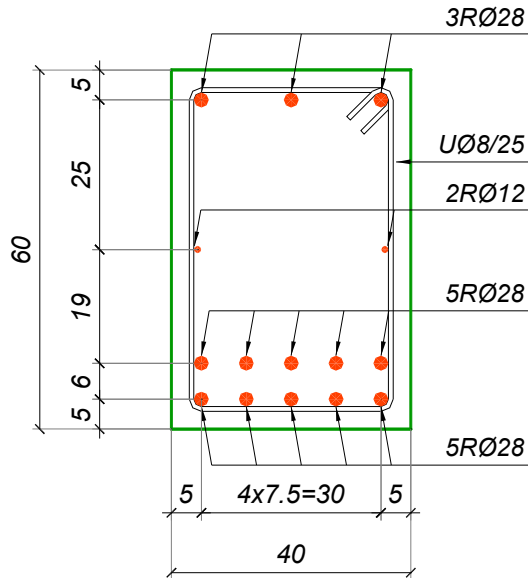
usvojeno: **3RØ28** (18.47 cm<sup>2</sup>)

| $\varepsilon_a$ | $\varepsilon_b$ | s     | $\alpha_b$ | $\eta$ | $\zeta$ | $\mu_{1M} \%$ | k     |
|-----------------|-----------------|-------|------------|--------|---------|---------------|-------|
| 3               | 3.5             | 0.538 | 0.810      | 0.416  | 0.776   | 43.590        | 1.719 |

$$A_{a1} = 43.590 \times \frac{40 \times 52}{100} \times \frac{2.05}{40} + 13.30 = 59.76 \text{ cm}^2$$

usvojeno: **10RØ28** (61.58 cm<sup>2</sup>)

## Primer 4 – dvojno armiranje



$$a' = a_0 + \varnothing_u + \varnothing/2$$

$$a' = 2.8 + 0.8 + 2.8/2 = 5.0 \text{ cm}$$

$$\text{usv. } a' = 5.0 \text{ cm}$$

$$\text{pretp } a_2 = a' = 5.0 \text{ cm}$$

$$a'' = a' + e_v + 2 \times \varnothing/2$$

$$a'' = 5.0 + 3.0 + 2 \times 2.8/2 = 10.8 \text{ cm}$$

$$\text{usv. } a'' = 11.0 \text{ cm}$$

$$a_1 = (5 \times 5.0 + 5 \times 11.0) / 10$$

$$a_1 = 8.0 \text{ cm}$$

$$h = 60 - 8.0 = 52.0 \text{ cm} = h_{\text{pretp.}}$$