BETONSKE KONSTRUKCIJE

Vežba br.7

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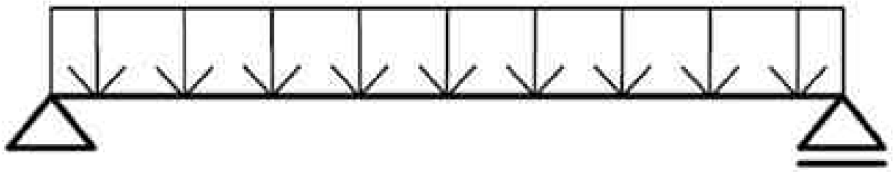
Semestar: V ***ESPB: 6***

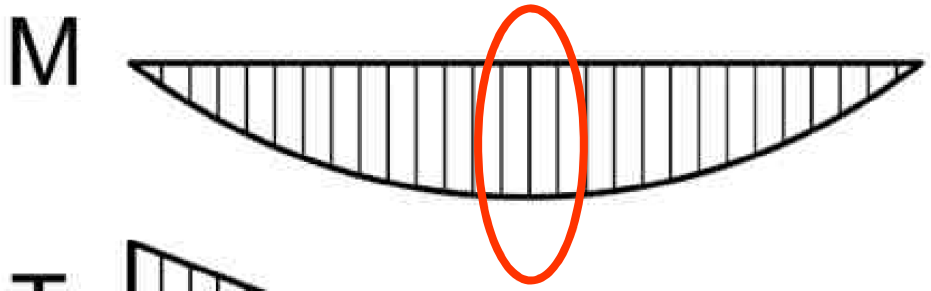
1. *Proračunski model i osnovni pojmovi*

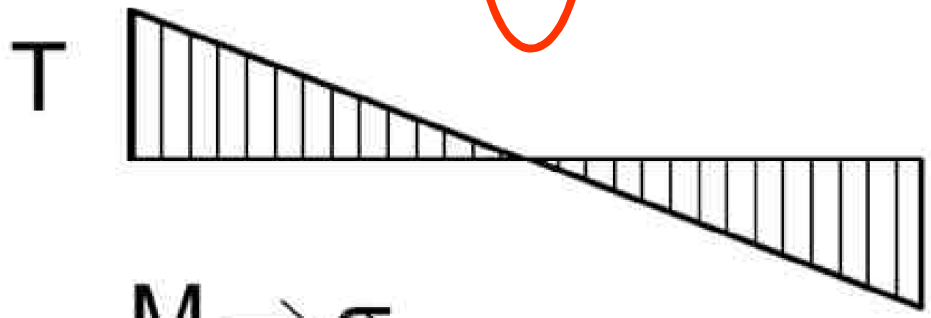
2. *Postupak proračuna*

3. *Primeri*

1. Proračunski model

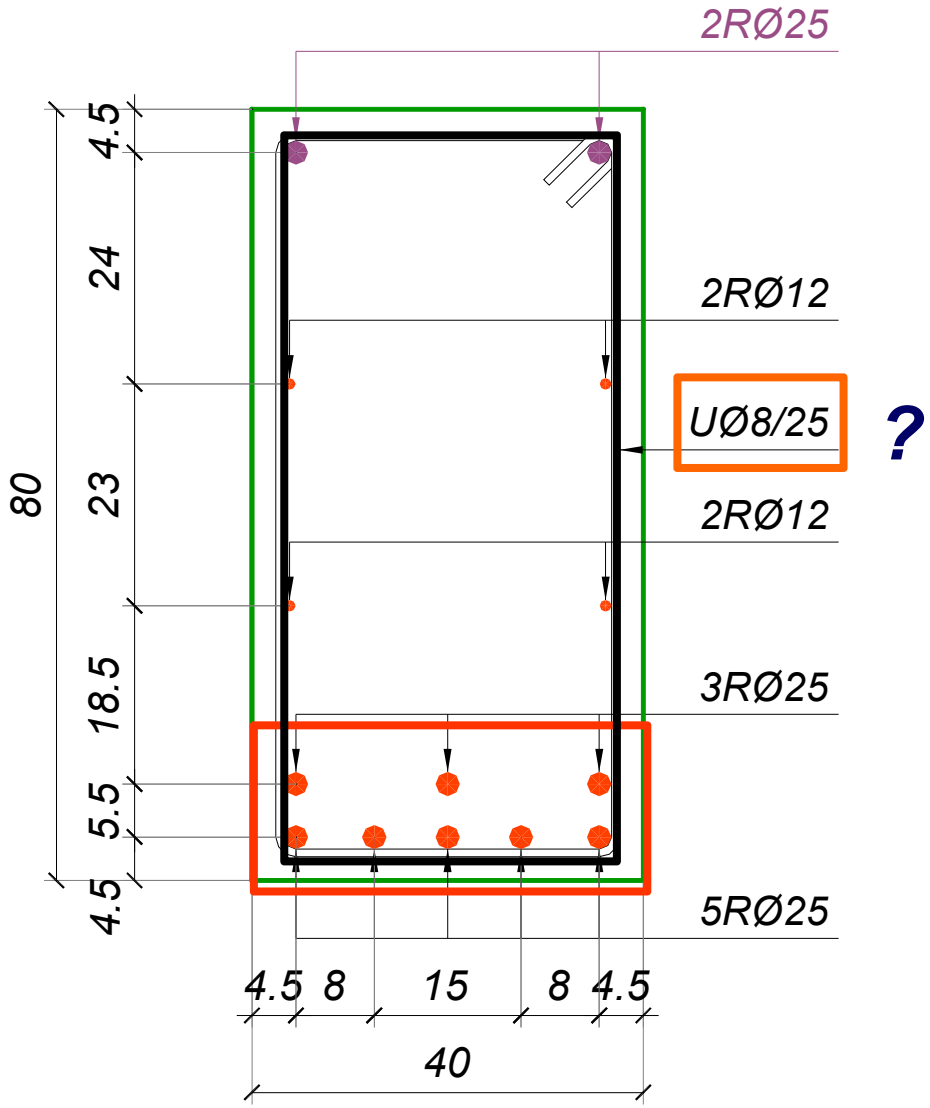


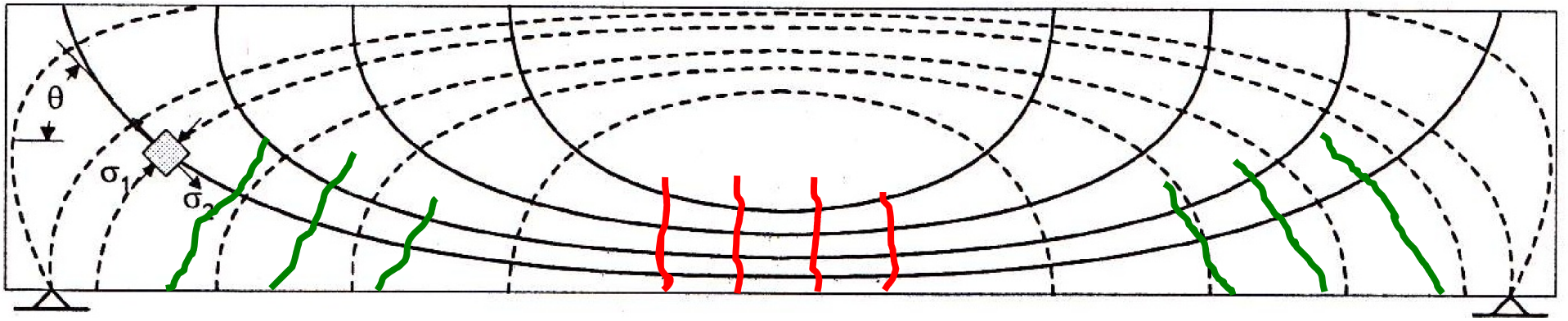




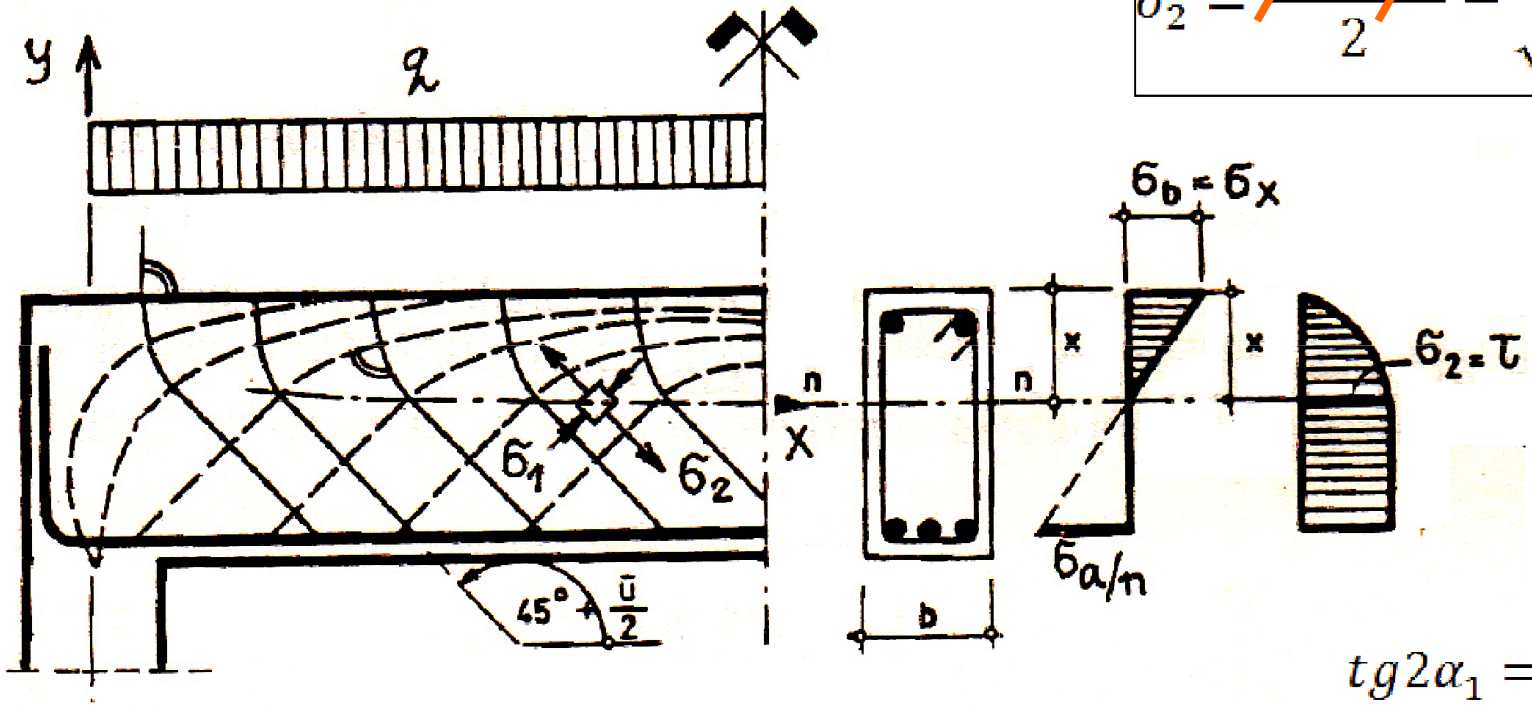
$M \rightarrow \sigma$

$T \rightarrow \tau$





$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau^2}$$

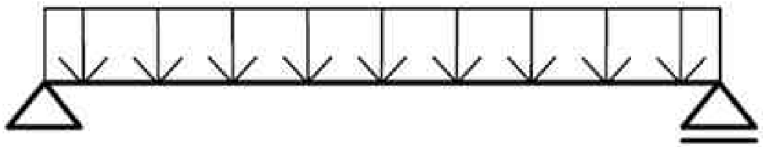


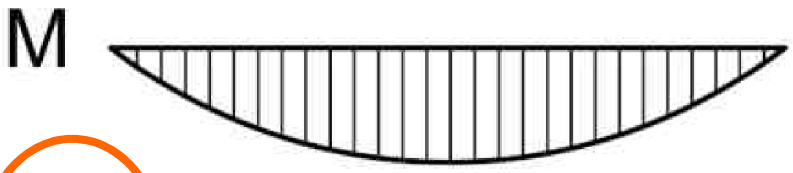
$$\sigma_{1,2} = \pm\tau$$

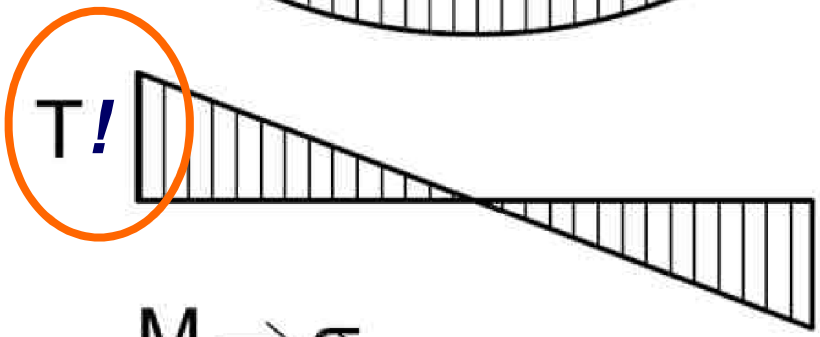
$$tg2\alpha_1 = \infty \quad \Rightarrow \quad \alpha_1 = 45^\circ$$

1. Proračunski model

$\sigma_{1,2} = \pm\tau$?







$M \rightarrow \sigma$

$T \rightarrow \tau$

- *Za presek sa prslinom :*

$$\tau = \frac{T}{bz} \qquad z = \frac{I_i}{S_i}$$

$$z = 0.9h \qquad \tau = \frac{T}{0.9bh}$$

PRORAČUN PRESEKA ZA GRANIČNE UTICAJE TRANSVERZALNIH SILA

- *PRORAČUN* ***PREMA*** *TEORIJI GRANIČNIH STANJA*

$$T_u = \sum_i \gamma_{u,i} \times T_i \qquad (i = g, p, \Delta)$$

$$za\ g\ i\ p:\ T_u = \gamma_g \times T_g + \gamma_p \times T_p = 1.6 \times T_g + 1.8 \times T_p$$

- *SAVIJANJE* ***AB PRESEKA POPREČNIM SILAMA***
  → ***KONTROLA*** *GLAVNIH NAPONA ZATEZANJA*

$$\tau_{max} = \frac{T \cdot S_i}{b_{min} \cdot I_i} = \frac{T}{b_{min} \cdot z} \quad ; \quad Z=0.9h$$

Proračun nominalnog napona smicanja

$$\tau_n = \frac{T_{mu}}{b \times z} \approx \frac{T_{mu}}{b \cdot 0.9 \cdot h}$$

(član 89 Pravilnika BAB 87)

MB	15	20	30	40	50	60	[MPa]
τ_r	0.6	0.8	1.1	1.3	1.5	1.6	[MPa]

$\tau_n \leq \tau_r$ - beton prihvata glavne napone zatezanja

$\tau_r \leq \tau_n \leq 5\tau_r$ - potrebno osiguranje armaturom

$\tau_n > 5\tau_r$ - nije dopušteno

Mogući slučajevi:

1. $\tau_n \leq \tau_r$

Nije potrebno osiguranje.
Glavne napone zatezanja prihvata sam beton (usvaja se konstruktivna poprečna armatura)

2. $\tau_r < \tau_n \leq 3\tau_r$

Potrebno je izvršiti osiguranje.
Deo glavnih napona zatezanja prihvata beton, a preostali deo napona se poverava armaturi

$$T_{bu} = \frac{1}{2} \times (3\tau_r - \tau_n) \times b \times z \qquad \tau_{bu} = \frac{T_{bu}}{0,9 \cdot b \cdot h} = \frac{1}{2} \cdot (3\tau_r - \tau_n)$$

$$T_{Ru} = T_{mu} - T_{bu} \qquad \tau_{ru} = \tau_n - \frac{1}{2} \cdot (3\tau_r - \tau_n) = \frac{3}{2} \cdot (\tau_n - \tau_r)$$

3. $3\tau_r < \tau_n \leq 5\tau_r$

Beton ne učestvuje u prijemu uticaja od transverzalnih sila.

Celokupni glavni naponi zatezanja poveravaju se armaturi.

$$T_{bu} = 0$$

$$T_{Ru} = T_{mu}$$

4. $\tau_n > 5\tau_r$

Ne dozvoljava se.

Moraju se promeniti dimenzije preseka ili MB ili nešto u konstrukciji.

Određivanje redukovanog napona τ_{Ru}

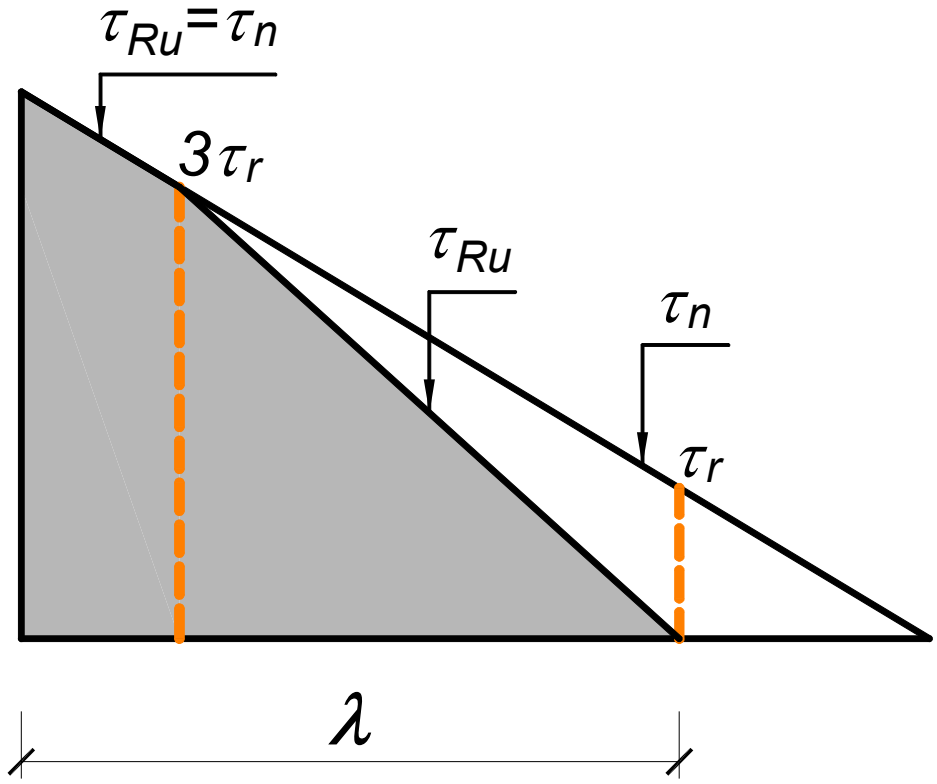
$\tau_r \le \tau_n \le 3\tau_r$

$$T_{bu} = \frac{1}{2} \times (3\tau_r - \tau_n) \times b \times z$$

$$T_{Ru} = T_{mu} - T_{bu}$$

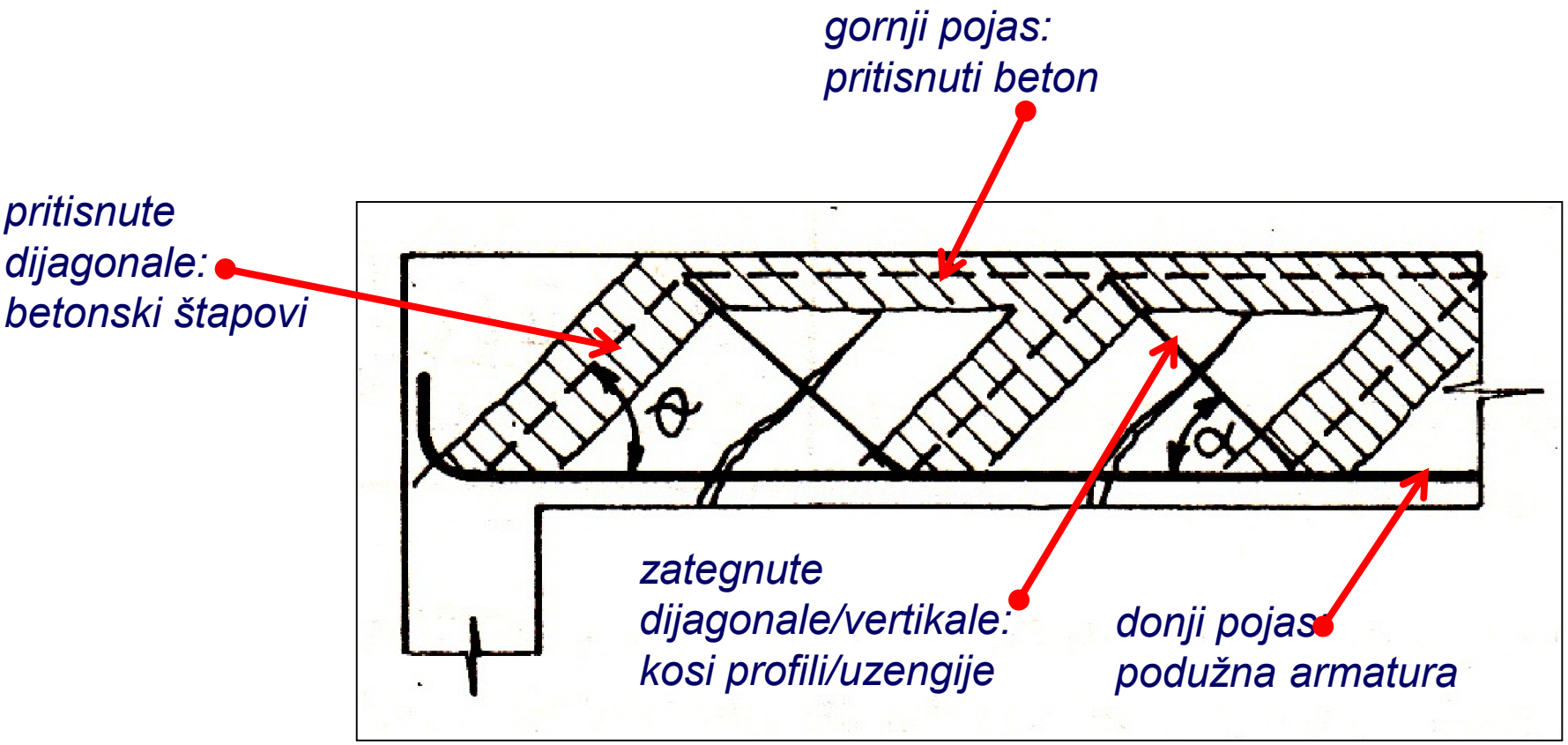
$3\tau_r \le \tau_n \le 5\tau_r$

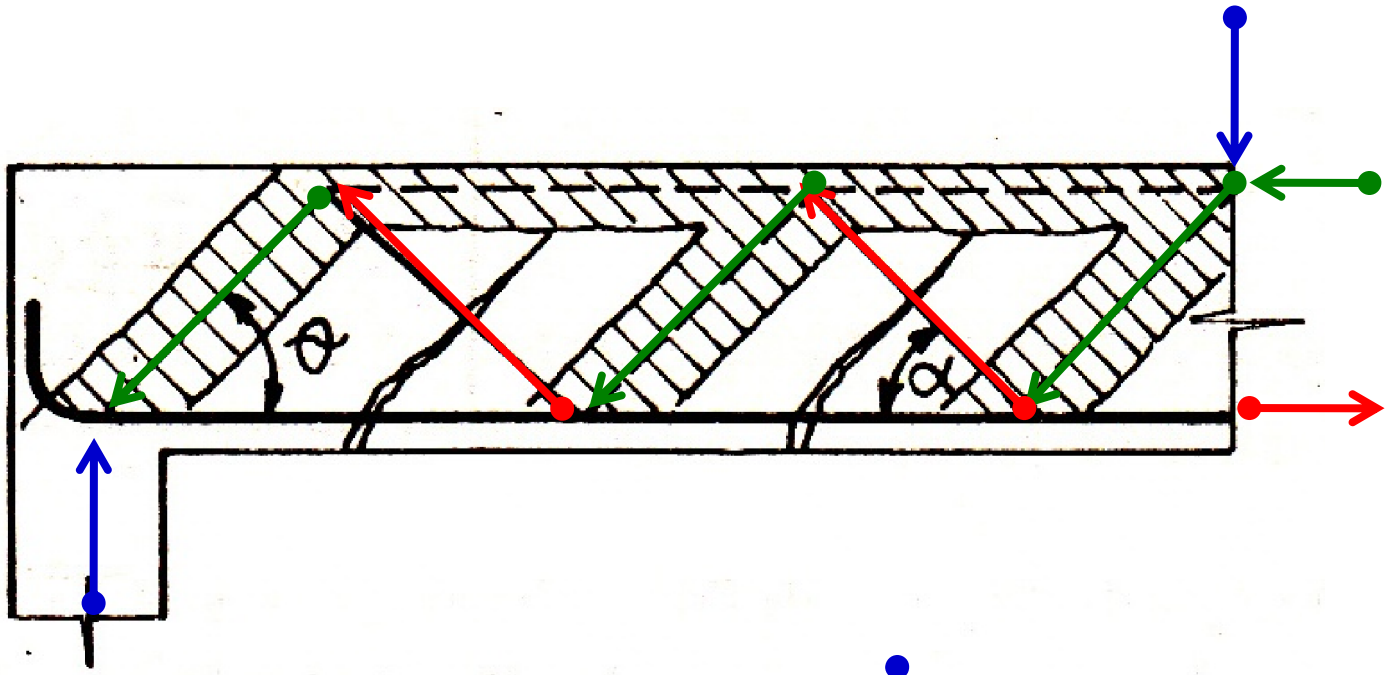
$$T_{bu} \equiv 0 \quad \Rightarrow \quad T_{Ru} = T_{mu}$$



$$\tau_{Ru} = \frac{T_{Ru}}{b \times z}$$

- ***Model rešetke:***

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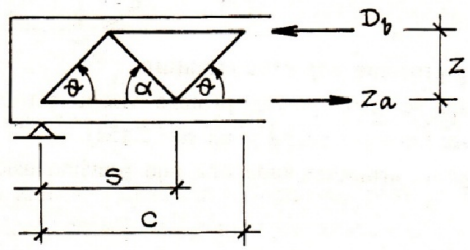
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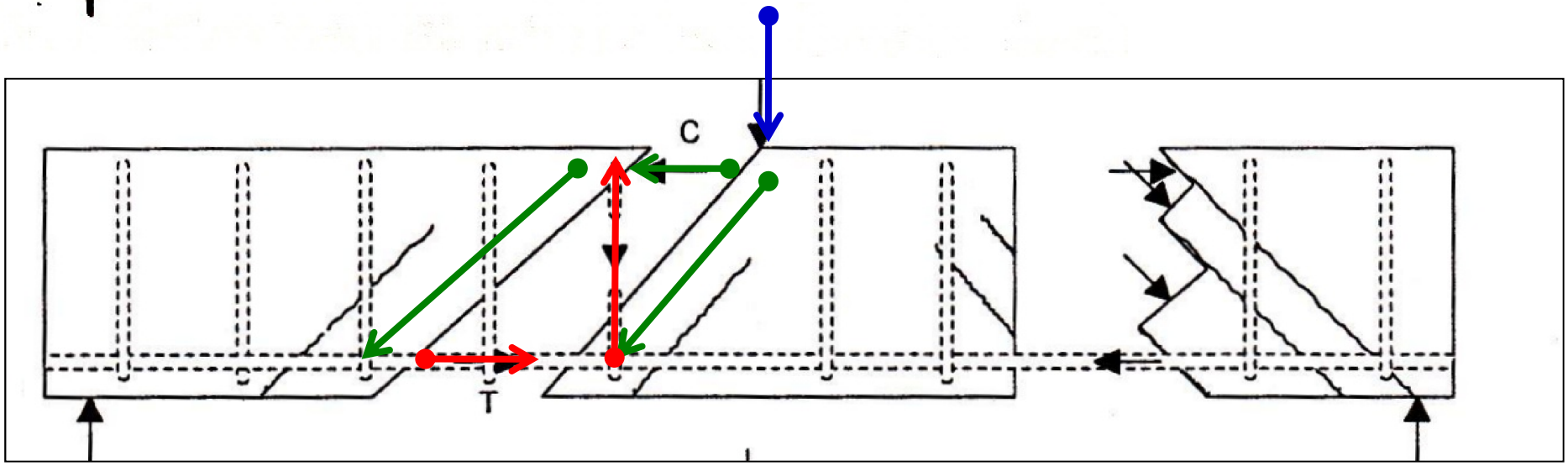
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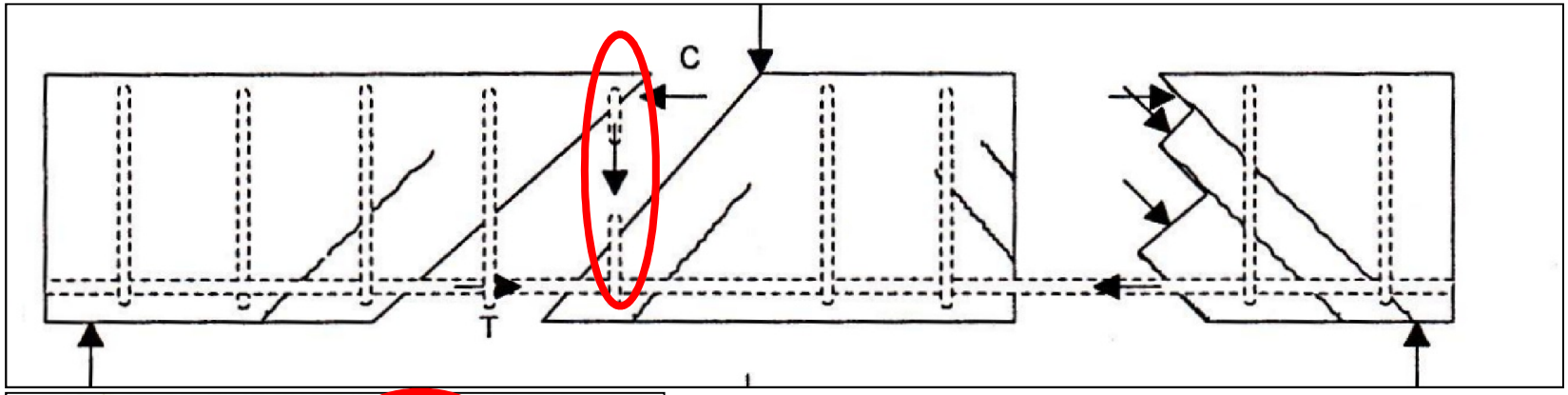
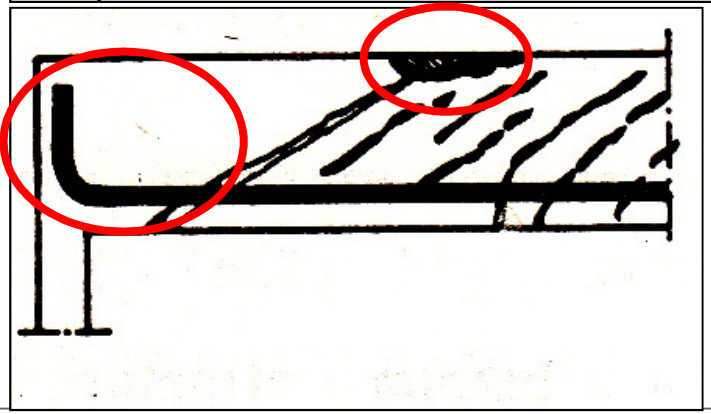
Lom nastaje iz tri razloga:

1. Nedostatak ili mali procenat poprečne armature
2. Lom betona kada se kosa prslina proteže visoko po preseku
3. Proklizavanje zategnute armature kada nije pravilno usidrena nad osloncima





- ***Ukupna redukovana merodavna sila smicanja na dužini osiguranja (horizontalna sila veze):***

$$H_{vu} = \int_{x=a}^{x=b} \tau_{Ru} b\,dx = \int_{x=a}^{x=b} \overline{T}_{Ru}\,dx = \int_{x=a}^{x=b} \frac{T_{Ru}}{z}\,dx$$

- ***Ukupna potrebna površina preseka poprečne armature:***

$$A_{ak} = \frac{1}{\sigma_v (\mathrm{ctg}\theta + \mathrm{ctg}\alpha)\sin\alpha} \int_{x=a}^{x=b} \frac{T_{Ru}}{z}\,dx$$

$$A_{ak} = \frac{H_{vu}}{\sigma_v (\cos\alpha + \sin\alpha \cdot ctg\theta)}$$

Prihvatanje glavnih napona zatezanja može se izvršiti:

1. Vertikalnim uzengijama
2. Kosim uzengijama
3. Kombinacijom vertikalnih uzengija i koso povijenom armaturom

Za kosa gvožđa: $\theta = 45^o$; $\alpha = 45^o$

$$A_{ak} = \frac{H_{vu}}{\sigma_v \cdot (\cos\alpha + \sin\alpha \cdot ctg\theta)}$$

$$A_{ak} = \frac{H_{vu,k}}{\sigma_v \sqrt{2}}$$

Određivanje podužne armature

(član 93 PBAB 87):

$$\Delta A_a = \frac{T_{mu}}{2\sigma_v} \times \left(ctg\theta - ctg\alpha\right)$$

U oblastima momentnih špiceva** (npr. srednji oslonci kontinualnih nosača) **nema potrebe za armaturom ΔA_a usled T sila (str. 268. BAB 87, Tom 1 - Priručnik)

Pomeranje linije zatežućih sila:

$$v = \frac{z}{2} \times \left(ctg\theta - ctg\alpha\right) \geq \begin{cases} 0.75 \times h \ \ (U\emptyset) \\ 0.50 \times h \ \ (U\emptyset + A_{ak}) \end{cases}$$

Za uzengije:

$$a_u^{(1)} = \frac{b \times \tau_{Ru}}{m \times \sigma_v} \times \frac{1}{\left(\cos\alpha + \sin\alpha \times \mathrm{ctg}\theta\right)} \times e_u$$ ***(Ø8, 10, 12)***

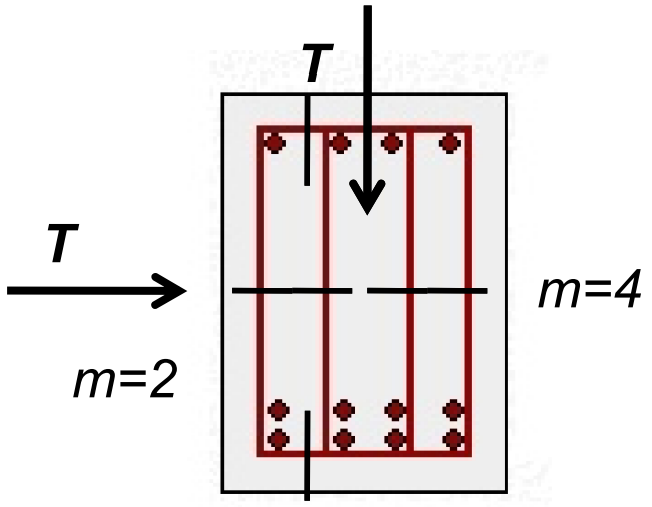
$$e_u = \frac{m \times a_u^{(1)}}{b \times \tau_{Ru}} \times \left(\cos\alpha + \sin\alpha \times \mathrm{ctg}\theta\right) \times \sigma_v$$ ***(10, 12.5, 15, 20, 25 cm)***

Vertikalne uzengije: $\theta = 45^o$; $\alpha = 90^o$

$$e_u \le \frac{m \times a_u^{(1)}}{b \times \tau_{Ru}} \times \sigma_v$$

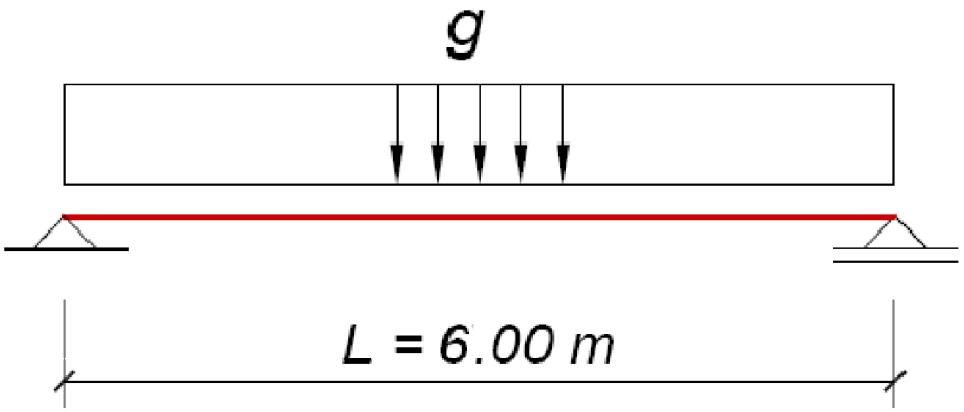
$$\max e_u \le \min \begin{cases} h/2 \\ b \\ 25\,\mathrm{cm} \end{cases}$$

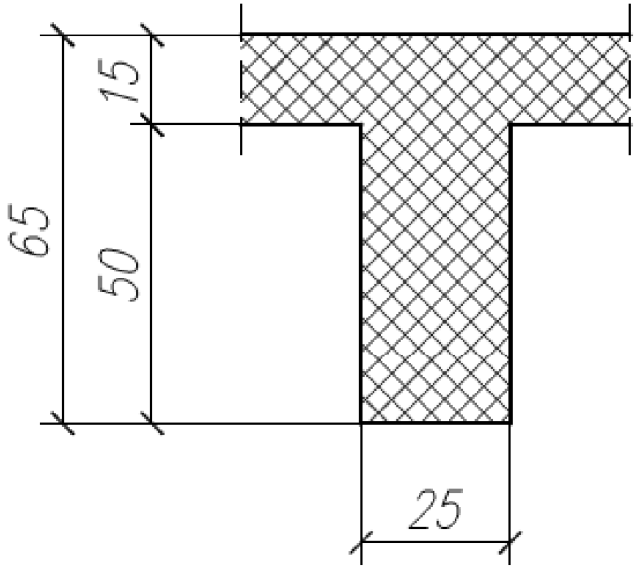
$$\min\mu_{uz} = 0.2\% \quad \rightarrow \quad e_u \le \frac{m \times a_u^{(1)}}{\min\mu \times b}$$



Primer 1

Dimenzionisati nosač sistema proste grede, raspona L = 6.0 m, opterećen jednako raspodeljenim stalnim opterećenjem g = 40 kN/m po čitavom rasponu. U opterećenje je uračunata i sopstvena težina nosača. Poprečni presek je pravougaoni, dimenzija b/d = 25/60 cm. Kvalitet materijala: MB 30, RA 400/500.





Dimenzionisanje prema momentima savijanja:

$M_g = 40 \cdot 6.0^2 / 8 = 180 kNm$ $\qquad$ $M_u = 1.6 \cdot 180 = 288 kNm$

pretp. $a_1 = 7cm$ $\qquad$ $h = 60 - 7 = 53cm$

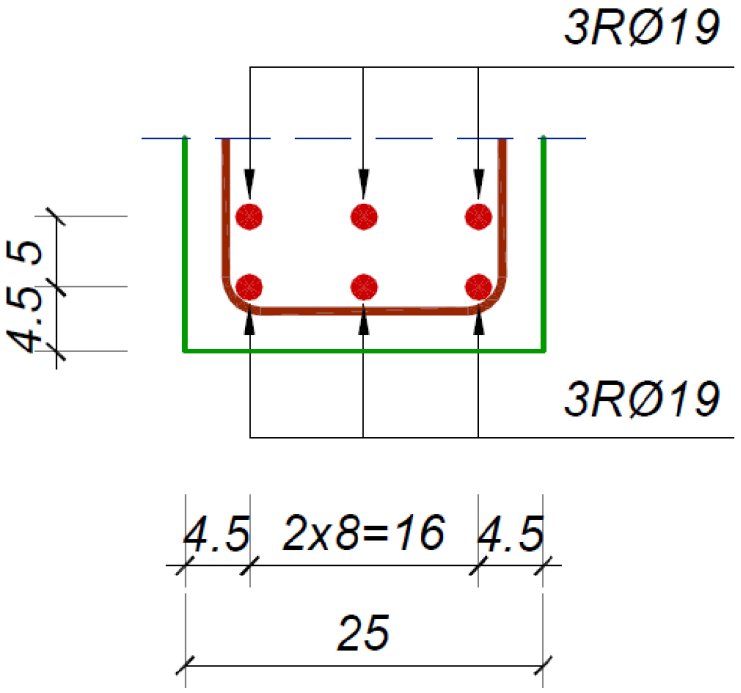
$$k = \frac{53}{\sqrt{\frac{288 \times 10^2}{25 \times 2.05}}} = 2.236 \Rightarrow \begin{matrix} \varepsilon_b / \varepsilon_a = 3.5 / 9.018‰ \\ \mu = 22.633\% \end{matrix}$$

$$A_a = 22.633 \times \frac{25 \times 53}{100} \times \frac{2.05}{40} = 15.37\ cm^2$$

usvojeno: **6RØ19** ($17.01\ cm^2$)

$$a_1 = \frac{3 \times (4.5 + 9.5)}{6} = 7\ cm$$

$h_{stv.} = 60 - 7 = 53\ cm = h_{pretp.}$



Određivanje nominalnog napona smicanja:

$$T_g = 40 \cdot 6.0 / 2 = 120 kN \qquad T_u = 1.6 \cdot 120 = 192 kN$$

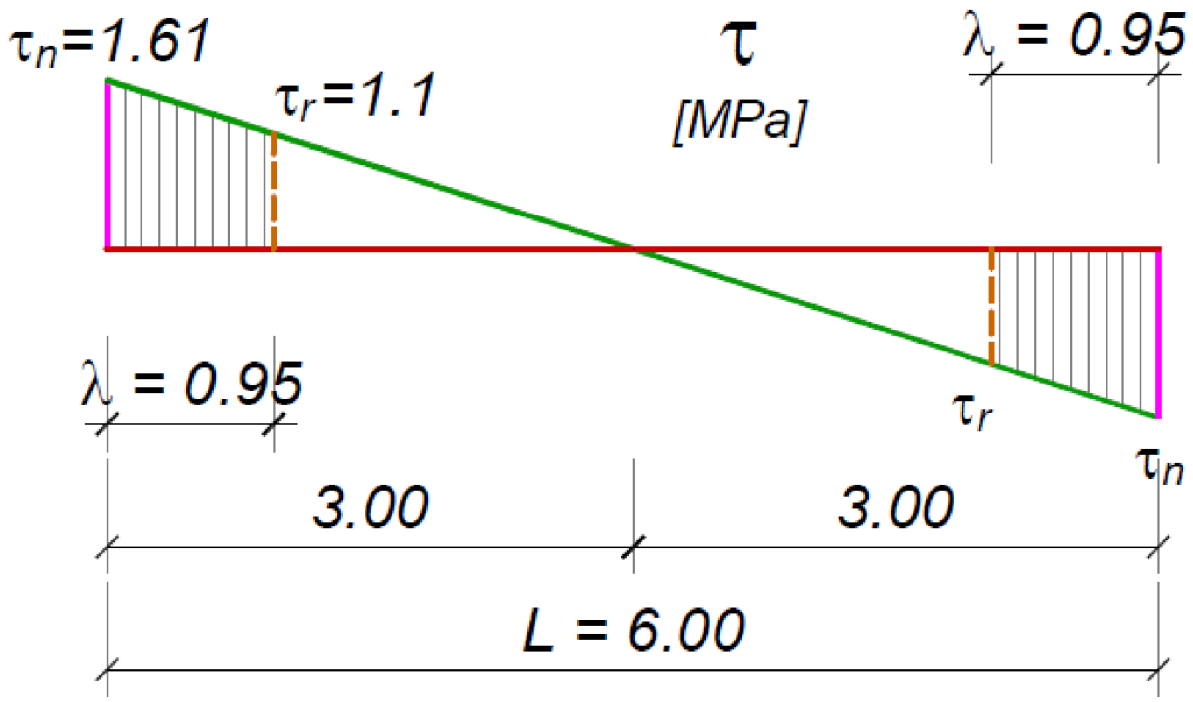
$$\tau_n = \frac{T_{mu}}{b \times z} \approx \frac{T_{mu}}{b \times 0.9 \times h} = \frac{192}{25 \times 0.9 \times 53} = 0.161\ kN/cm^2 = 1.61\ MPa$$

MB 30 $\Rightarrow$ $\tau_r = 1.1$ MPa (član 89. Pravilnika BAB 87)

$\tau_n = 1.61\ MPa < 5\tau_r = 5.5\ MPa$

=> nije potrebno korigovati dimenzije preseka ili marku betona

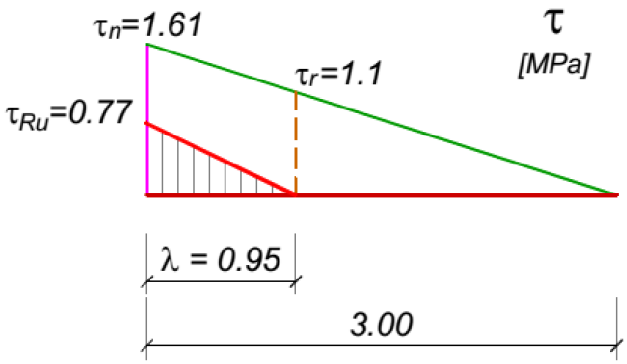
Određivanje dužine osiguranja:



$$\lambda = \frac{L}{2} \times \left(1 - \frac{\tau_r}{\tau_n}\right) = \frac{6.0}{2} \times \left(1 - \frac{1.1}{1.61}\right) = 0.95\ m$$

Određivanje potrebne armature:

Osiguranje poprečnom i podužnom armaturom



$\tau_n = 1.61\ MPa < 3\tau_r = 3.3\ MPa$

=> redukcija napona smicanja

$$\tau_{Ru} = \frac{3}{2} \times (\tau_n - \tau_r) = \frac{3}{2} \times (0.161 - 0.11) = 0.077\ kN/cm^2$$

Osiguranje dvosečnim vertikalnim uzengijama:

$m = 2 \quad ; \quad \theta = 45° \quad ; \quad \alpha = 90°$

$$e_u \le \frac{m \times a_u^{(1)}}{b \times \tau_{Ru}} \times \sigma_v \times (\cos\alpha + \sin\alpha \times \cot\theta)$$

$$e_u \le \frac{2 \times a_u^{(1)}}{25 \times 0.077} \times 40 \times (\cos 90° + \sin 90° \times \cot 45°) = 41.82 \times a_u^{(1)}$$

Osiguranje dvosečnim vertikalnim uzengijama:

pretp. RØ10 ($a_u^{(1)} = 0.785\ cm^2$) $e_u \leq 41.82 \times 0.785 = 32.9\ cm$

Maksimalno rastojanje na dužini osiguranja:

$$e_{u,max} = min.\left\{\begin{matrix} b = 25\,cm \\ h/2 = 53/2 = 26.5\,cm \\ 25\,cm \end{matrix}\right\} = 25\,cm$$

pretp. RØ8 ($a_u^{(1)} = 0.503\ cm^2$)

$e_u \leq 41.82 \times 0.503 = 21.0\ cm < 25\ cm = e_{u,max.}$

Osiguranje dvosečnim vertikalnim uzengijama:

$$\mu_{uz} = \frac{m \times a_u^{(1)}}{b \times e_u} \geq 0.2\%$$

$$UR\text{Ø}8 \Rightarrow \; e_u \leq \frac{m \times a_u^{(1)}}{b \times \mu_{uz,min.}} = \frac{2 \times 0.503}{25 \times 0.2 \times 10^{-2}} = 20.1\ cm > e_{u,rač.} = 21.0\ cm$$

usvojeno: **URØ8/20** ($m=2$) na dužini $\lambda = 1$ m

Dodatna zategnuta armatura:

$$\Delta A_a = \frac{T_{mu}}{2\sigma_v}\left(\cot\theta - \cot\alpha\right)$$

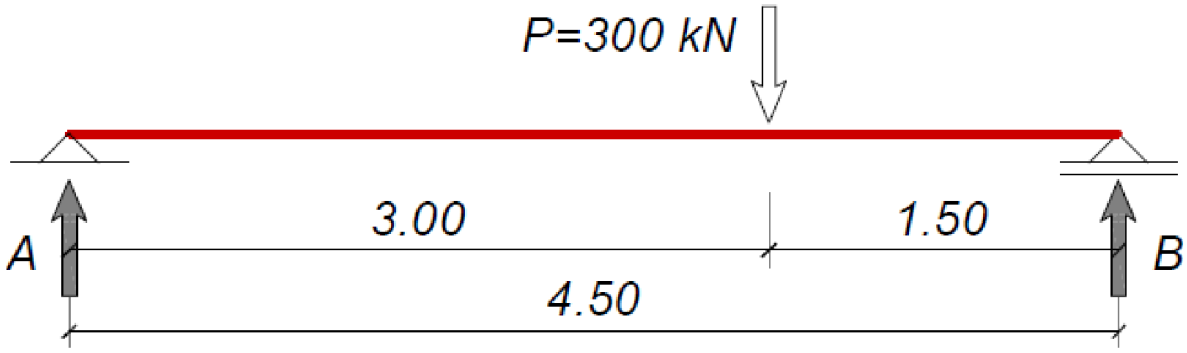
$$\Delta A_a = \frac{192}{2 \times 40}\left(\cot 45° - \cot 90°\right) = 2.40\ cm^2$$

usvojeno: **2RØ19** ($5.68\ cm^2$)

Primer 2

$b/d = 30/65$ cm

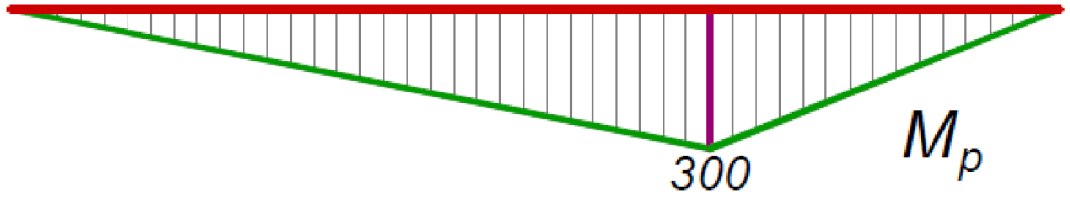
MB 25, RA 400/500

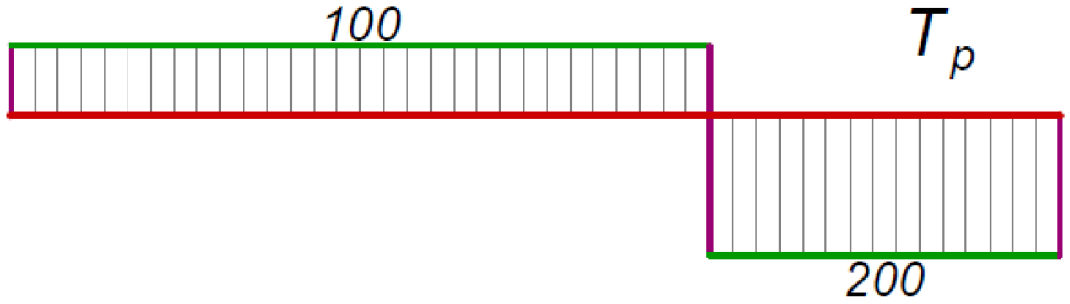


$A_p = 300 \times 1.5/4.5 = 100$ kN

$B_p = 300 - 100 = 200$ kN

$M_{p,max} = 200 \times 1.5 = 300$ kNm





Dimenzionisanje prema momentima savijanja

$$M_u = 1.8 \times 300 = 540\ kNm$$

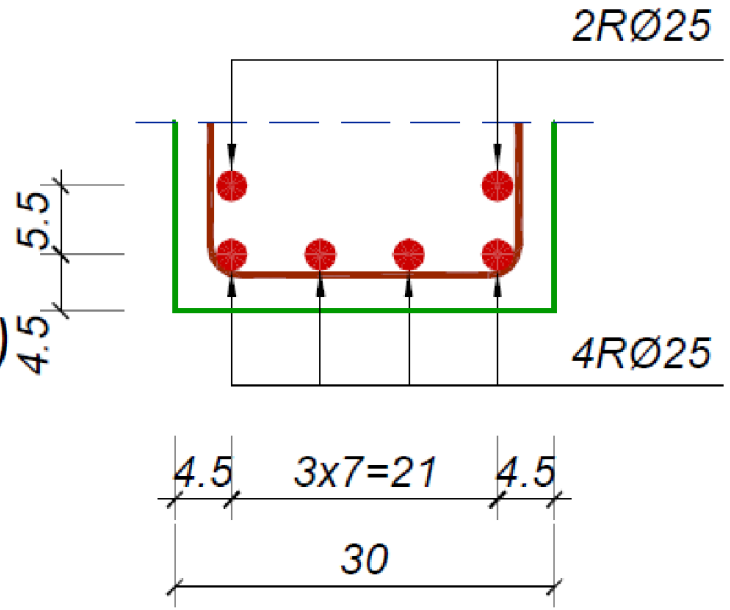
$$a_1 = 7\ cm \Rightarrow \quad h = 65 - 7 = 58\ cm$$

$$MB\ 25 \quad \Rightarrow \quad f_B = 17.25\ MPa$$

$$k = \frac{58}{\sqrt{\frac{540 \times 10^2}{30 \times 1.725}}} = 1.796 \Rightarrow \begin{matrix} \varepsilon_b / \varepsilon_a = 3.5 / 3.82‰ \\ \bar{\mu} = 38.709\% \end{matrix}$$

$$A_a = 38.709 \times \frac{30 \times 58}{100} \times \frac{1.725}{40} = 29.05\ cm^2$$

usvojeno: **6RØ25** ($29.45\ cm^2$)



Osiguranje na delu uz oslonac B

usvojeno: $z \approx 0.9h = 0.9 \times 58 = 52.2\ cm = const.$

$$T_{u,max.} = T_u^B = 1.8 \times 200 = 360\ kN$$

$$\tau_n = \frac{T_u^B}{b \times z} = \frac{360}{30 \times 52} = 0.230\,\frac{kN}{cm^2} = 2.30\,MPa \quad \begin{cases} > \tau_r = 0.95\,MPa \\ < 3\tau_r = 2.85\,MPa \end{cases}$$

$$\tau_{Ru}^B = \frac{3}{2} \times (0.230 - 0.095) = 0.202\,\frac{kN}{cm^2} = 2.02\,MPa$$

$$e_u \le \frac{2 \times a_u^{(1)}}{30 \times 0.202} \times 40 \times (\cos 90° + \sin 90° \times \cot 45°) = 13.18 \times a_u^{(1)}$$

pretp. RØ10 ($a_u^{(1)} = 0.785\ cm^2$) $\Rightarrow$ $e_u \le 13.18 \times 0.785 = 10.35\ cm$

usvojeno: **URØ10/10** (m=2) na dužini $\lambda_b = 1.5\ m$

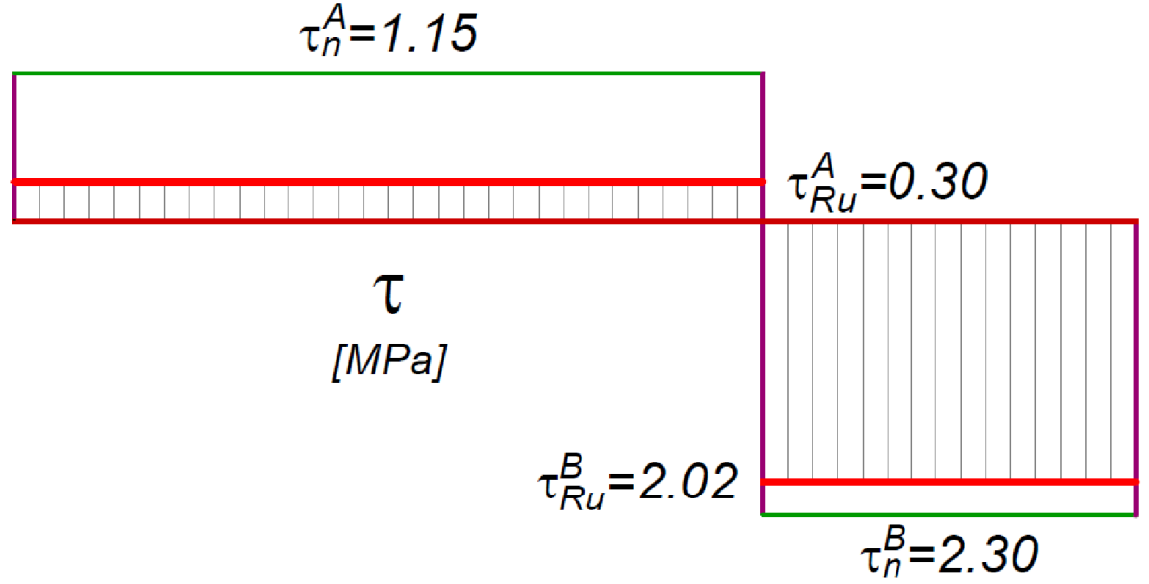
$$\Delta A_a = \frac{360}{2 \times 40}(\cot 45° - \cot 90°) = 4.50\ cm^2$$

usvojeno: **2RØ25** ($9.82\ cm^2$)

Osiguranje na delu uz oslonac A

$$T_u^A = 1.8 \times 100 = 180\ kN$$

$$\tau_n^A = \frac{180}{30 \times 52.2} = 0.115 \frac{kN}{cm^2} \quad \begin{cases} > \tau_r \\ < 3\tau_r \end{cases}$$



$$\tau_{Ru}^A = \frac{3}{2} \times (0.115 - 0.095) = 0.030 \frac{kN}{cm^2}$$

$$e_u \le \frac{2 \times a_u^{(1)}}{30 \times 0.030} \times 40 \times \left(\cos 90° + \sin 90° \times \cot 45°\right) = 89.15 \times a_u^{(1)}$$

pretp. RØ10 ($a_u^{(1)} = 0.785\ cm^2$) $\Rightarrow$ $e_u \le 89.15 \times 0.785 = 70.0\ cm$

$$\mu_{uz} = \frac{m \times a_u^{(1)}}{b \times e_u} \ge 0.2\%$$

$$e_u \le \frac{2 \times a_u^{(1)}}{30 \times 0.2 \times 10^{-2}} = 33.3 \times a_u^{(1)} < 89.15 \times a_u^{(1)}$$

URØ10 ($a_u^{(1)} = 0.785\ cm^2$) $\Rightarrow$ $e_u \le 33.3 \times 0.785 = 26.2\ cm > 25\ cm = e_{u,max.}$

URØ8 ($a_u^{(1)} = 0.503\ cm^2$) $\Rightarrow$ $e_u \le 33.3 \times 0.503 = 16.8\ cm$

usvojeno: **URØ8/15** ($m=2$) na dužini $\lambda_b = 3.0\ m$

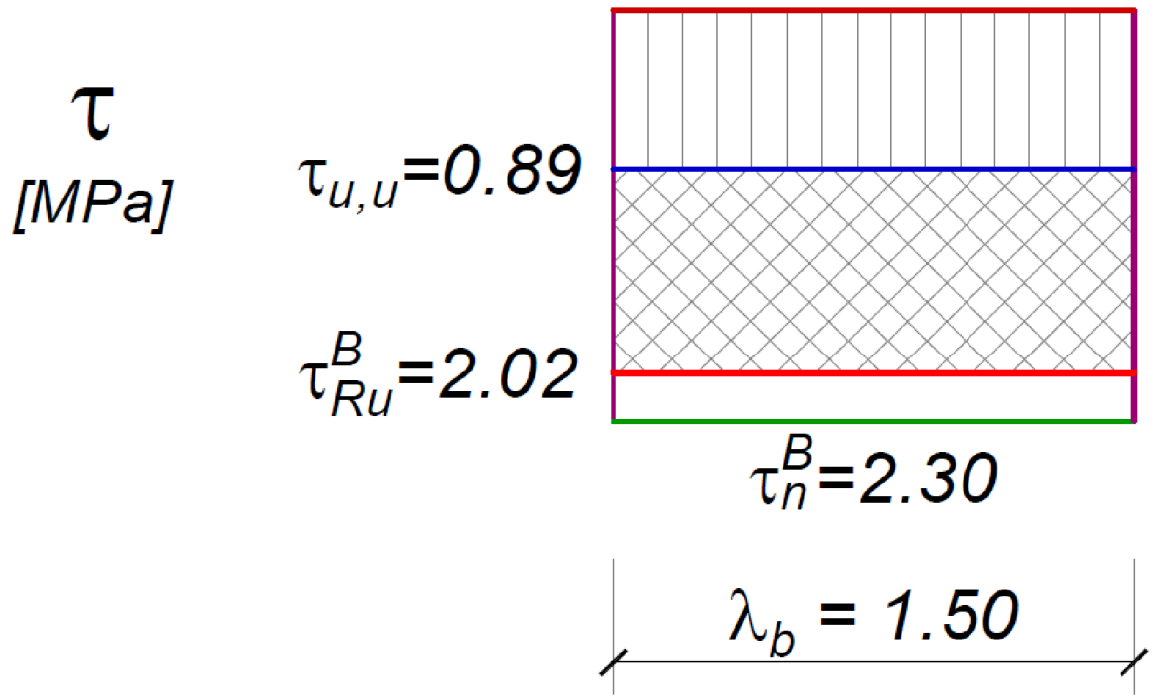
$$\Delta A_a = \frac{180}{2 \times 40}\left(\cot 45° - \cot 90°\right) = 2.25\ cm^2$$

usvojeno: **2RØ25** ($9.82\ cm^2$)[1]

Osiguranje na delu uz oslonac B - varijanta

$$\tau_{u,u} = \frac{2 \times 0.503}{30 \times 15} \times 40 \times (\cos 90° + \sin 90° \times \cot 45°)$$

$$\tau_{u,u} = 0.089 \frac{kN}{cm^2} = 0.89\,MPa$$



$$H_{vu,k} = (0.202 - 0.089) \times 150 \times 30 = 508.4\ kN$$

$$\alpha_k = 45° \quad \Rightarrow \quad A_{a,k} = \frac{H_{vu,k}}{\sigma_v \times (\cos\alpha_k + \sin\alpha_k \times \cot\theta)} = \frac{508.4}{40 \times (0.707 + 0.707 \times 1)} = 8.99\ cm^2$$

usvojeno: **2RØ25** *(9.82 cm²)*

$$T_{bu} = \frac{1}{2} \times (3 \times 0.095 - 0.230) \times 30 \times 52.2 = 43.2\ kN$$

$$T_{u,u} = \tau_{u,u} \times b \times z = 0.089 \times 30 \times 52.2 = 139.9\ kN$$

$$\Delta A_a = \frac{43.2 + 139.9}{2 \times 40} (\cot 45° - \cot 90°) = 2.29\ cm^2$$

usvojeno: **2RØ25** *(9.82 cm²)*