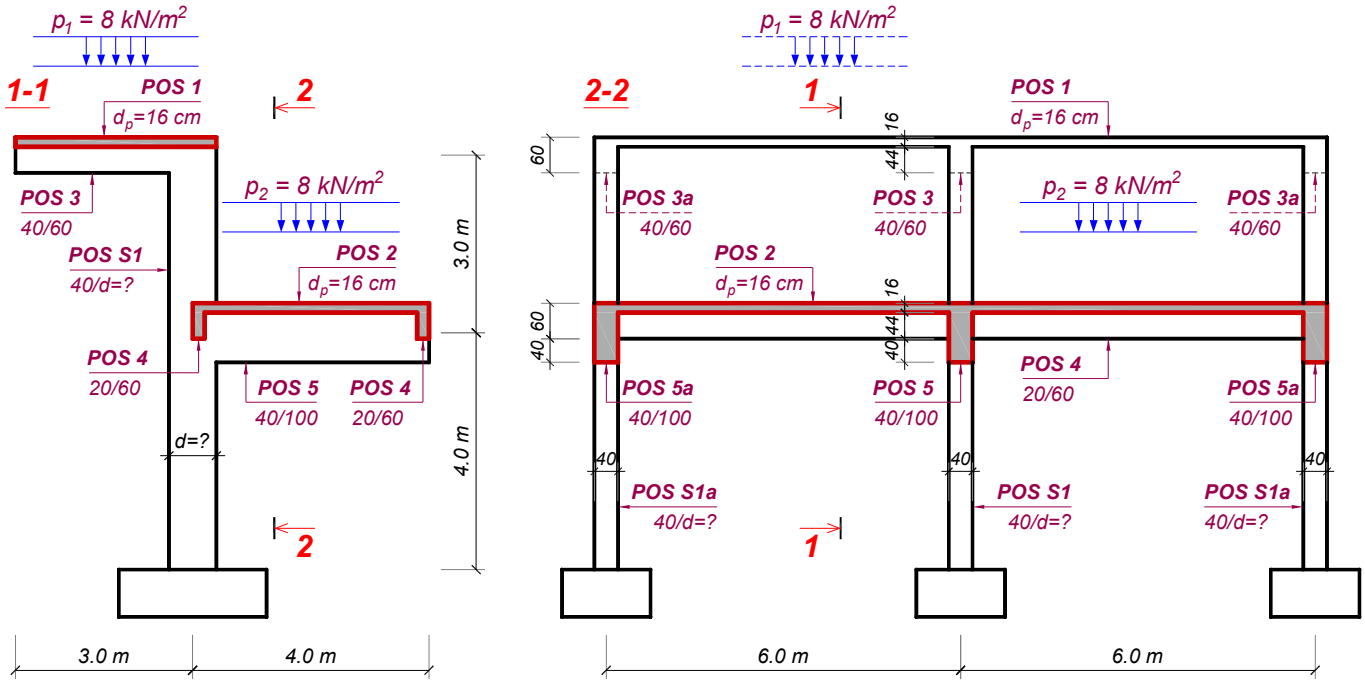
GRAĐEVINSKI FAKULTET U BEOGRADU *pismeni ispit*
Modul za konstrukcije *16.06.2009.*

PROJEKTOVANJE I GRAĐENJE BETONSKIH KONSTRUKCIJA 1
NOVI NASTAVNI PLAN



Konstrukcija prikazana na skici je, pored sopstvene težine elemenata, opterećena i jednako raspodeljenim povremenim opterećenjem p = 8 kN/m² koje se <u>može naći u proizvoljnom položaju na pločama POS 1 i POS 2</u>. Potrebno je prema MERODAVNIM uticajima, zavisno od položaja povremenog opterećenja:

1. *Dimenzionisati **POS 1** u karakterističnim presecima. Šematski prikazati usvojeni raspored armature (osnova, presek).*
2. *Dimenzionisati **POS 2** u karakterističnim presecima. Šematski prikazati usvojeni raspored armature (osnova, presek).*
3. *Dimenzionisati **POS 3** (40/60 cm), odnosno **POS 5** (40/100 cm).*
4. *Dimenzionisati **POS S1**. Uticaj izvijanja i sopstvenu težinu stuba zanemariti.*
5. *Nacrtati šemu armiranja rama **POS 3, POS 5, POS S1** (izgled, karakteristični poprečni preseci dimenzionisanih elemenata, sa svim neophodnim oznakama i kotama).*

<u>Napomene:</u>

- *u svim dimenzionisanim presecima obezbediti da dilatacija zategnute armature bude minimalno $\varepsilon_{a1} \geq 5‰$*
- *zadate dimenzije poprečnog preseka ne menjati*
- *za uzengije koristiti isključivo prečnike **RØ8** i **RØ10**.*

Kvalitet materijala: ***MB 30 , RA 400/500***

POS 1 *– ploča* d_p *= 16 cm*

Ploča je sistema kontinualnog nosača preko dva jednaka raspona L = 6.0 m.

$g = 0.16 \times 25 = 4.0\ kN/m^2$; $p = 8\ kN/m^2$

gornja zona

Maksimalni moment savijanja u preseku nad osloncem se javlja kada se povremeno opterećenje nalazi u oba polja:

$q_u = 1.6 \times 4.0 + 1.8 \times 8.0 = 20.8\ kN/m^2$

$M_u = 20.8 \times 6.0^2/8 = 93.6\ kNm/m$

MB 30 $\Rightarrow$ $f_B = 20.5\ MPa$; RA 400/500 $\Rightarrow$ $\sigma_v = 400\ MPa$

$a = 3.0\ cm \Rightarrow h = d - a = 16 - 3 = 13\ cm$

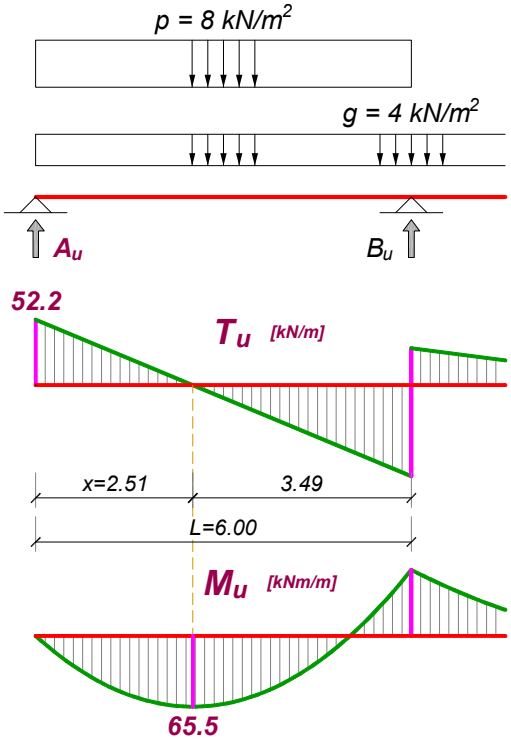
$$k = \frac{13}{\sqrt{\frac{93.6}{2.05}}} = 1.924 \Rightarrow \varepsilon_b/\varepsilon_a = 3.5/5.24‰ \ ; \ \bar{\mu} = 32.416\%$$

$$A_a = 32.416 \times \frac{100 \times 13}{100} \times \frac{2.05}{40} = 21.60\ cm^2/m \Rightarrow \textbf{RØ19/12.5}\ (22.68\ cm^2/m)$$

$A_{ap} = 0.2 \times 21.60 = 4.32\ cm^2/m \Rightarrow$ **RØ12/25** $(4.52\ cm^2/m)$

donja zona

Maksimalni moment u donjoj zoni se javlja u slučaju da je samo razmatrano polje opterećeno povremenim opterećenjem. Nije neophodno sračunati oslonački moment savijanja ili srednju reakciju oslonca (nisu merodavne, tj. maksimalne vrednosti), kao ni maksimalne momente savijanja usled pojedinačnih opterećenja (maksimumi nisu u istom preseku), već samo krajnje reakcije oslonaca i, pomoću njih, maksimalni M_u.



$A_g = 3/8 \times 4.0 \times 6.0 = 9.0\ kN/m$

$A_p = 7/16 \times 8.0 \times 6.0 = 21.0\ kN/m$

$A_u = 1.6 \times 9.0 + 1.8 \times 21.0 = 52.2\ kN/m$

$x = A_u/q_u = 52.2/20.8 = 2.51\ m$

$$M_u = 52.2 \times 2.51 - \frac{20.8 \times 2.51^2}{2} = 65.5\ kNm/m$$

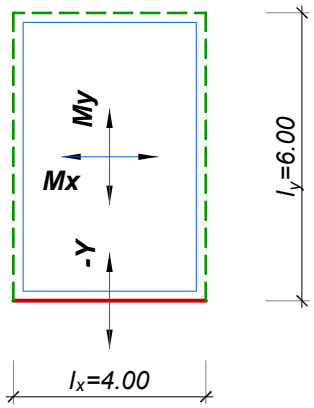
$$k = \frac{13}{\sqrt{\frac{65.5}{2.05}}} = 2.300 \Rightarrow \varepsilon_b/\varepsilon_a = 3.5/9.852‰ \ ; \ \bar{\mu} = 21.220\%$$

$$A_a = 21.220 \times \frac{100 \times 13}{100} \times \frac{2.05}{40} = 14.14\ cm^2/m \Rightarrow \textbf{RØ16/12.5}\ (16.08\ cm^2/m)$$

$A_{ap} = 0.2 \times 14.14 = 2.83\ cm^2/m \Rightarrow$ **RØ10/25** $(3.14\ cm^2/m)$

POS 2 *– ploča* d_p *= 16 cm*

gornja zona



Slično ploči POS 1, maksimalni moment savijanja u gornjoj zoni se javlja kada su oba polja opterećena povremenim opterećenjem:

$l_y/l_x = 6.0/4.0 = 1.5$

$Q_u = 20.8 \times 6.0 \times 4.0 = 499.2$ kNm

$-Y_u = 0.075 \times Q_u = 0.075 \times 499.2 = 37.4$ kNm/m

$a = 2.5$ cm $\Rightarrow$ $h = d - a = 16 - 2.5 = 13.5$ cm

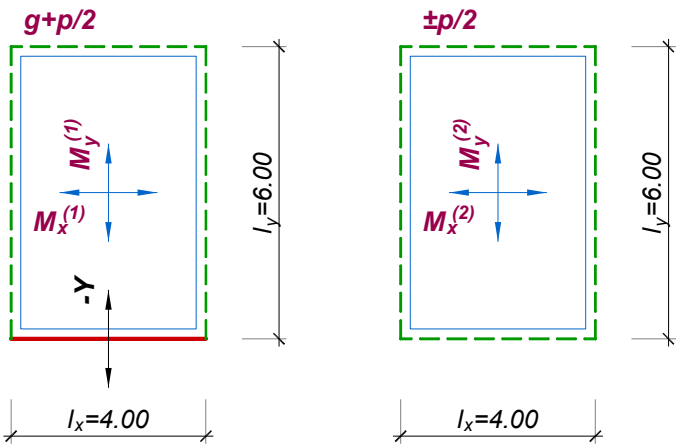
$$k = \frac{13.5}{\sqrt{\frac{37.4}{2.05}}} = 3.159 \quad \Rightarrow \quad \varepsilon_b/\varepsilon_a = 1.94/10‰ \quad ; \quad \bar{\mu} = 10.668\%$$

$$A_a = 10.668 \times \frac{100 \times 13.5}{100} \times \frac{2.05}{40} = 7.38 \text{ cm}^2/\text{m} \quad \Rightarrow \quad \textbf{RØ12/15} \ (7.53 \text{ cm}^2/\text{m})$$

$A_{ap} = 0.2 \times 7.38 = 1.48 \text{ cm}^2/\text{m} > 0.085 \times 16 = 1.36$ $\Rightarrow$ **RØ8/30** $(1.67 \text{ cm}^2/\text{m})$

donja zona

Maksimalni moment u donjoj zoni se javlja kad povremeno opterećenje deluje samo u jednom polju. Da bismo mogli koristiti tablice za proračun uticaja, moramo izvršiti dekompoziciju opterećenja na simetrični i antimetrični deo (+p/2 deluje na čitavoj ploči, ±p/2 deluje u suprotnim smerovima u dva susedna polja).



$q_{u1} = 1.6 \times 4.0 + 1.8 \times 8.0/2 = 13.6$ kN/m

$Q_{u1} = 13.6 \times 6.0 \times 4.0 = 326.4$ kN

$M_{xu}^{(1)} = 0.044 \times 326.4 = 14.36$ kNm/m

$M_{yu}^{(1)} = 0.028 \times 326.4 = 9.14$ kNm/m

$q_{u2} = \pm 1.8 \times 8.0/2 = \pm 7.2$ kN/m

$Q_{u2} = \pm 7.2 \times 6.0 \times 4.0 = 172.8$ kN

$M_{xu}^{(2)} = 0.052 \times 172.8 = 8.99$ kNm/m

$M_{yu}^{(2)} = 0.029 \times 172.8 = 5.01$ kNm/m

$M_{xu} = M_{xu}^{(1)} + M_{xu}^{(2)} = 14.36 + 8.99 = 23.34$ kNm/m

$M_{yu} = M_{yu}^{(1)} + M_{yu}^{(2)} = 9.14 + 5.01 = 14.15$ kNm/m

Kako je za oslonački moment savijanja Y, kao najveći u ploči, usvojena veća statička visina, to je učinjeno i u donjoj zoni, mada je M_x *veći od* M_y*. Nije obavezno, bitno je samo da se proračun i armatura u preseku slažu.*

$$h_y = 13.5 \text{ cm} \quad \Rightarrow \quad A_{ay} = \frac{14.15 \times 10^2}{0.9 \times 13.5 \times 40} = 2.92 \text{ cm}^2/\text{m} \quad \Rightarrow \quad \textbf{RØ 8/15} \ (3.35 \text{ cm}^2/\text{m})$$

$h_x = h_y - Ø_y/2 - Ø_x/2 \approx 13.5 - 1 = 12.5$ cm

$$h_x = 12.5 \text{ cm} \quad \Rightarrow \quad A_{ax} = \frac{23.34 \times 10^2}{0.9 \times 12.5 \times 40} = 5.19 \text{ cm}^2/\text{m} \quad \Rightarrow \quad \textbf{RØ 10/15} \ (5.24 \text{ cm}^2/\text{m})$$

***POS 3** – greda b/ d = 40/60 cm*

*Konzolna greda, raspona **L=3.0 m**, opterećena opterećenjem od POS 1. Maksimalno opterećenje na POS 3 se dobija kada se povremeno opterećenje nalazi u oba polja POS 1.*

sopstvena težina POS 3 $0.4\times0.6\times25.0$ = 6.0 kN/m

od POS 1: $B_g = 1.25\times4.0\times6.0$ = 30.0 kN/m

ukupno, stalno opterećenje: ***g* = 36.0 kN/m**

od POS 1: $B_p = 1.25\times8.0\times6.0 =$ ***p* = 60.0 kN/m**

$M_g = 36.0\times3.0^2/2 = 162$ kNm ; $T_g = 36.0\times3.0 = 108$ kN

$M_p = 60.0\times3.0^2/2 = 270$ kNm ; $T_p = 60.0\times3.0 = 180$ kN

Dimenzionisanje

$M_u = 1.6\times162 + 1.8\times270 = 745.2$ kNm

pretp. $a_1 = 7$ cm $\Rightarrow$ b/d/h = 40/60/53 cm

$$k = \frac{53}{\sqrt{\frac{745.2\times10^2}{40\times2.05}}} = 1.758 \quad \Rightarrow \quad \varepsilon_a < 5‰$$

Kako bi dilataciju zategnute armature bila zadržana na zahtevanom nivou ($\varepsilon_{a1} \geq 5‰$), pristupa se dvojnom armiranju.

$$usv.\varepsilon_{a1}^{*} = 5‰ \Rightarrow k^{*} = 1.903 \ ; \ \bar{\mu} = 33.333\%$$

$$M_{bu} = \left(\frac{53}{1.903}\right)^2 \times 0.40 \times 2.05 = 636.3 \text{ kNm}$$

$$\Delta M_u = 745.2 - 636.6 = 108.9 \text{ kNm}$$

pretp. $a_2 = 5$ cm $\Rightarrow A_{a2} = \frac{108.9\times10^2}{(53-5)\times40} = 5.67$ cm^2 $\Rightarrow$ ***2RØ25*** *(9.82 cm^2)*

$$A_{a1} = 33.333\times\frac{40\times53}{100}\times\frac{2.05}{40} + 5.67 = 41.89 \text{ cm}^2 \quad \Rightarrow \quad \textbf{9RØ25} \ (44.18 \text{ cm}^2)$$

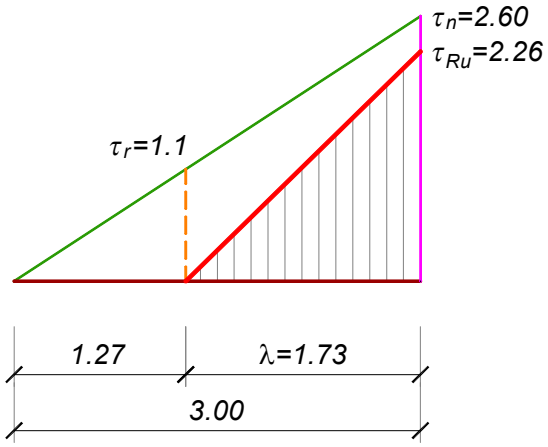
$T_u = 1.6\times108 + 1.8\times180 = 496.8$ kN

usvojeno: $z = 0.9\times53 = 47.7$ cm

$$\tau_n = \frac{496.8}{40\times47.7} = 0.26\frac{kN}{cm^2} \begin{cases} > \tau_r = 1.1 MPa \\ < 3\tau_r \end{cases}$$

$$\lambda = 3.0\times\left(1-\frac{0.11}{0.26}\right) = 1.73 m$$

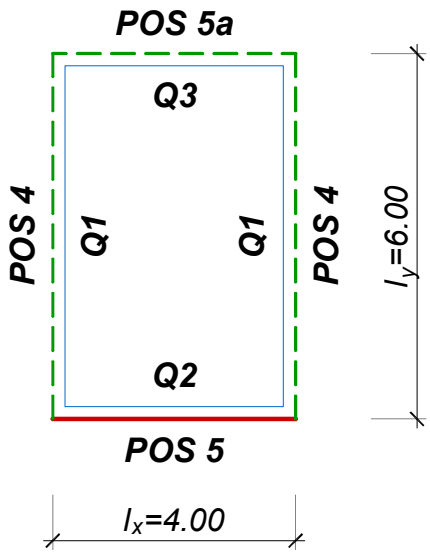
$$\tau_{Ru} = \frac{3}{2}\times(0.26-0.11) = 0.226\frac{kN}{cm^2}$$



$$e_u = \frac{4\times0.785}{40\times0.226}\times40 = 13.9 \text{ cm} \quad \Rightarrow \quad \text{usvojeno } \textbf{URØ10/12.5} \ (m=4)$$

***POS 5** – greda b/ d = 40/100 cm*

*Konzolna greda, raspona **L=4.0 m**, opterećena raspodeljenim opterećenjem od POS 2 i koncentrisanim silama od grede POS 4. Maksimalno opterećenje na POS 5 se dobija kada se povremeno opterećenje nalazi u oba polja POS 2.*



$G = 4.0\times4.0\times6.0 = 96.0$ kN

$G_1 = 0.273\times96.0 = 26.2$ kN - na POS 4

$G_2 = 0.262\times96.0 = 25.2$ kN - na POS 5

$P = 8.0\times4.0\times6.0 = 192.0$ kN

$P_1 = 0.273\times192.0 = 52.4$ kN - na POS 4

$P_2 = 0.262\times192.0 = 50.3$ kN - na POS 5

sopstvena težina POS 5	$0.4\times1.0\times25.0$	= 10.00 kN/m
od POS 2:	$2\times G_2 / l_x = 2\times25.2/4.0$	= 12.58 kN/m
ukupno, stalno opterećenje:	**g**	**= 22.58 kN/m**
od POS 2:	$2\times P_2 / l_x = 50.3/4.0$ = **p**	**= 25.15 kN/m**

Reakcije POS 4

Potrebno je sračunati reakciju srednjeg oslonca POS 4:

sopstvena težina POS 4	$0.2\times0.6\times25.0$	= 3.00 kN/m
od POS 2:	$G_1 / l_y = 26.2/6.0$	= 4.37 kN/m
ukupno, stalno opterećenje:	**g**	**= 7.37 kN/m**
od POS 2:	$P_1 / l_y = 52.4/6.0$ = **p**	**= 8.74 kN/m**

$B_g = 1.25\times7.37\times6.0 = 55.8$ kN ; $B_p = 1.25\times8.74\times6.0 = 65.5$ kN

Dimenzionisanje POS 5

$M_g = 22.58\times4.0^2/2 + 55.8\times4.0 = 401.6$ kNm

$M_p = 25.15\times4.0^2/2 + 65.5\times4.0 = 463.3$ kNm

$M_u = 1.6\times401.6 + 1.8\times463.3 = 1476.6$ kNm

pretp. $a_1 = 7$ cm $\Rightarrow$ b/d/h = 40/100/93 cm

$$k = \frac{93}{\sqrt{\frac{1476.6\times10^2}{40\times2.05}}} = 2.192 \quad \Rightarrow \quad \varepsilon_b/\varepsilon_a = 3.5/8.451‰ \quad ; \quad \bar{\mu} = 23.708\%$$

$$A_a = 23.708\times\frac{40\times93}{100}\times\frac{2.05}{40} = 45.20\ cm^2 \quad \Rightarrow \quad \textbf{10RØ25}\ (49.09\ cm^2)$$

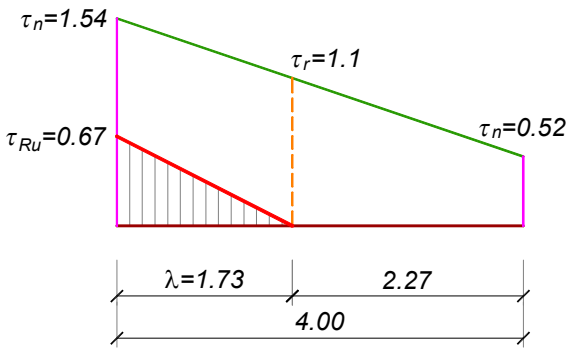
$T_g = 22.58\times4.0 + 55.8 = 145.6$ kN

$T_p = 25.15\times4.0 + 65.5 = 166.1$ kN

$T_{u,max} = 1.6\times145.6 + 1.8\times166.1 = 516.9$ kN

$$\tau_n^{max} = \frac{516.9}{40 \times 0.9 \times 93} = 0.154 \frac{kN}{cm^2} \begin{cases} > \tau_r = 1.1 MPa \\ < 3\tau_r \end{cases}$$

$$\tau_{Ru} = 1.5 \times (0.154 - 0.11) = 0.067\ kN/cm^2$$



$T_{u,min}$ = 1.6×55.8 + 1.8×65.5 = 172.7 kN

$$\tau_n^{min} = \frac{172.7}{40 \times 0.9 \times 93} = 0.052 \frac{kN}{cm^2} < \tau_r$$

$$\lambda = 4.0 \times \frac{0.154 - 0.11}{0.154 - 0.052} = 1.73\ m$$

$$\mu_{uz} \geq 0.2\% \Rightarrow e_u \leq \frac{2 \times 0.785}{40 \times 0.2 \times 10^{-2}} = 19.6\ cm \quad \Rightarrow \quad \text{usvojeno } \textbf{URØ10/15} \text{ (m=2)}$$

$$\tau_{u,u} = \frac{2 \times 0.785}{40 \times 15} \times 40 = 0.105\ kN/cm^2 > 0.067\ kN/cm^2 = {}_{max.}\tau_{Ru}$$

POS S1 *– stub b/ d = 40/d=?*

stalno opterećenje

$G^{POS\ 3}$ = 108 kN

$M_g{}^{POS\ 3}$ = 162 kNm

$G^{POS\ 4,5}$ = 55.3 + 145.6 = 200.8 kN

$M_g{}^{POS\ 4}$ = 401.6 kNm

povremeno opterećenje na POS 1:

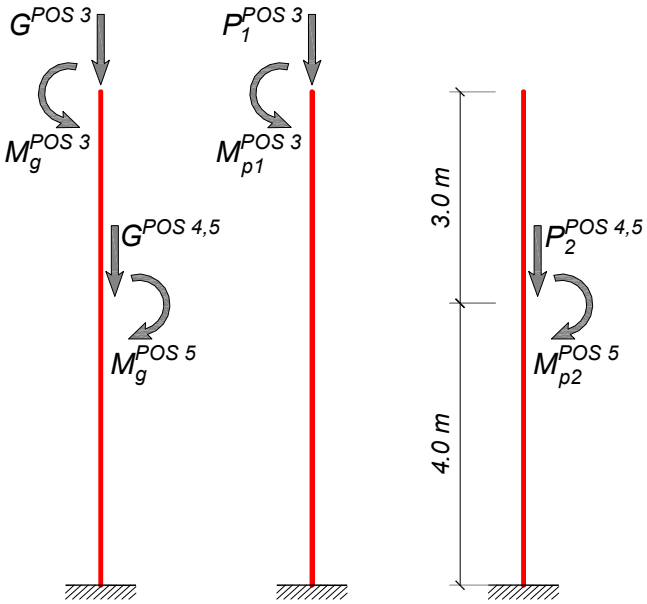
$P_1{}^{POS\ 3}$ = 180 kN

$M_{p1}{}^{POS\ 3}$ = 270 kNm

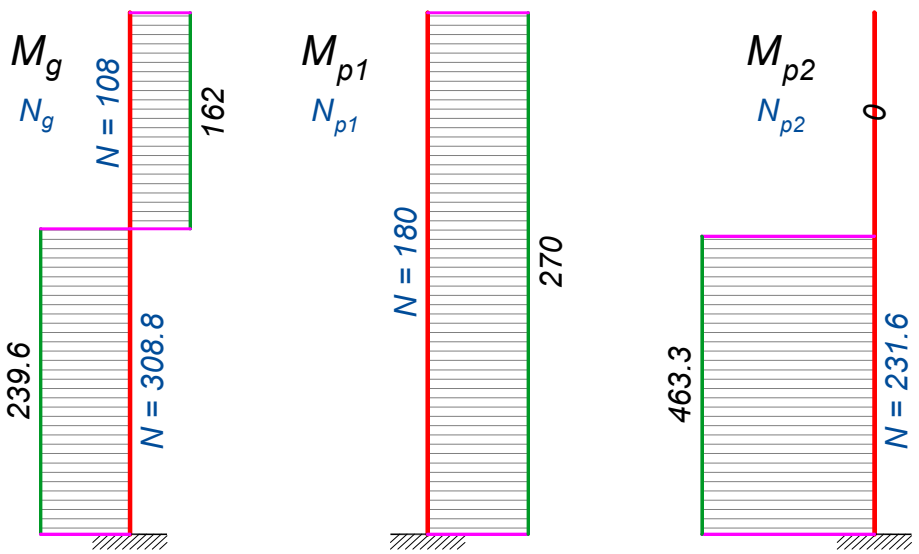
povremeno opterećenje na POS 2:

$P_2{}^{POS\ 4,5}$ = 65.5 + 166.1 = 231.6 kN

$M_{p2}{}^{POS\ 4}$ = 463.3 kNm



Dijagrami presečnih sila



Dimenzionisanje donjeg dela stuba, leva ivica

$M_u = 1.6\times239.6 + 1.8\times463.3 = 1217.4$ kNm (G, P_2)

$N_u = 1.6\times308.8 + 1.8\times231.6 = 911.1$ kN (G, P_2)

$usv.\varepsilon^*_{a1} = 5‰ \Rightarrow k^* = 1.903 \;;\; \bar{\mu} = 33.333\%$

$$M_{au} = M_u \quad \Rightarrow \quad h^I = 1.903\times\sqrt{\frac{1217.4\times10^2}{40\times2.05}} = 73.3 \text{ cm} \quad \Rightarrow \quad d^I = 80 \text{ cm}$$

$$M_{au} = 1217.4 + 911.1\times\left(0.8/2 - 0.07\right) = 1518.0 \text{ kNm}$$

$$h^{II} = 1.903\times\sqrt{\frac{1518.0\times10^2}{40\times2.05}} = 81.9 \text{ cm} \quad \Rightarrow \quad d^{II} = 90 \text{ cm}$$

$$M_{au} = 1217.4 + 911.1\times\left(0.9/2 - 0.07\right) = 1563.6 \text{ kNm}$$

$$h^{II} = 1.903\times\sqrt{\frac{1563.6\times10^2}{40\times2.05}} = 83.1 \text{ cm} \quad \Rightarrow \text{ usvojeno } \mathbf{d = 90 \text{ cm}}$$

$$A_a = 33.333\times\frac{40\times83.1}{100}\times\frac{2.05}{40} - \frac{911.1}{40} = 34.00 \text{ cm}^2 \quad \Rightarrow \quad \mathbf{7RØ25}\ (34.36 \text{ cm}^2)$$

Gornji deo stuba, desna ivica

$M_u = 1.6\times162 + 1.8\times270 = 745.2$ kNm (G, P_1)

$N_u = 1.6\times108 + 1.8\times180 = 496.8$ kN

$$M_{au} = 745.2 + 496.8\times\left(0.9/2 - 0.05\right) = 943.9 \text{ kNm}$$

$$k = \frac{85}{\sqrt{\frac{943.9\times10^2}{40\times2.05}}} = 2.505 \quad \Rightarrow \quad \varepsilon_b/\varepsilon_a = 2.908/10‰ \quad ; \quad \bar{\mu} = 17.362\%$$

$$A_a = 17.362\times\frac{40\times85}{100}\times\frac{2.05}{40} - \frac{496.8}{40} = 17.83 \text{ cm}^2 \quad \Rightarrow \quad \mathbf{4RØ25}\ (19.63 \text{ cm}^2)$$

Donji deo stuba, desna ivica

$M_u = 1.0\times(-239.6) + 1.8\times270 = 246.2$ kNm (G, P_1)

$N_u = 1.0\times308.8 + 1.8\times180 = 632.8$ kN

$$M_{au} = 246.2 + 632.8\times\left(0.9/2 - 0.05\right) = 499.5 \text{ kNm}$$

$$k = \frac{85}{\sqrt{\frac{499.5\times10^2}{40\times2.05}}} = 3.444 \quad \Rightarrow \quad \varepsilon_b/\varepsilon_a = 1.701/10‰ \quad ; \quad \bar{\mu} = 8.857\%$$

$$A_a = 8.857\times\frac{40\times85}{100}\times\frac{2.05}{40} - \frac{632.8}{40} \approx 0$$

$$A_{a,min} = 0.2\times10^{-2}\times40\times90 = 7.2 \text{ cm}^2 \quad \Rightarrow \quad \mathbf{2RØ25}\ (9.82 \text{ cm}^2)$$