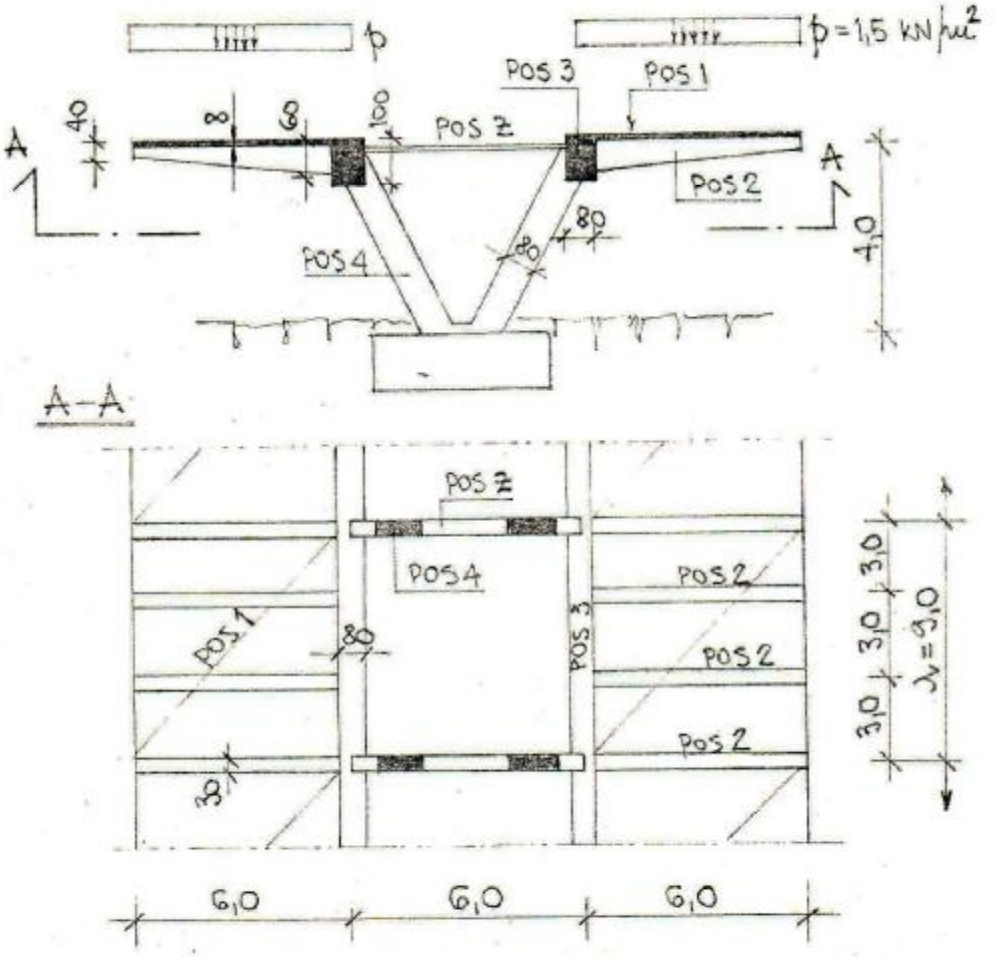
GRADJEVINSKI FAKULTET U BEOGRADU
BETONSKE KONSTRUKCIJE I
TEORIJA BETONSKIH KONSTRUKCIJA

Pismeni ispit
04.02.1991.g.



$p = 1{,}5\ kN/m^2$
$\lambda = 9{,}0\ m$
MB 40
RA 400/500

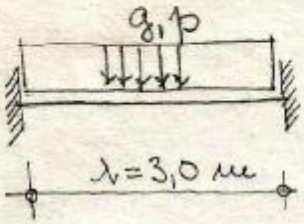
Pos 1: d = 8 cm
Pos 2: b = 30 cm
d_1 = 40 cm
d_2 = 60 cm
Pos 3: 80/100 cm
Pos 4: 30/ 80 cm

Za neko srednje polje konstrukcije na skici potrebno je:

1. Sračunati statičke uticaje i dimenzionisati gredu POS 2 prema M i Q.
2. Nacrtati dijagrame sila u presecima za gredu POS 3 a zatim izvršiti osiguranje od glavnih napona zatezanja na osloncu.
3. Nacrtati dijagrame presečnih sila za POS 4, POS Z usled stalnog i povremenog opterećenja (pri proračunu statičkih uticaja usvojiti površinu armature zatege Aa = 25 cm^2)
4. Dimenzionisati POS 4 u karakterističnim presecima prema uticajima M, N, Q sračunatim u prethodnoj tački.
5. Sa silom sračunatom u tački 3. dimenzionisati POS Z. Za usvojeni presek i armaturu odrediti razmak i karakterističnu širinu prslina.
6. Nacrtati plan armature za POS 2, POS 4, POS Z i prikazati karakteristične poprečne preseke, uz propisno kotiranje i pozicioniranje armature.

Sve dimenzije su zadate i ne smeju se menjati. Podatke potrebne za proračun usvojiti prema važećem Pravilniku.

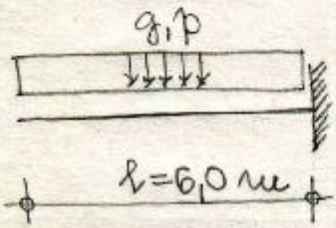
Pos 1 – ПЛОЧА



$g = 0{,}08 \cdot 25{,}0 = 2{,}0$ kN/m²
$p = 1{,}5$ kN/m²

$R_{g1} = 2{,}0 \cdot 3{,}0/2 = 3{,}0$ kN/m'
$R_{p1} = 1{,}5 \cdot 3{,}0/2 = 2{,}25$ kN/m'

Pos 2 – КОНЗОЛНА ГРЕДА



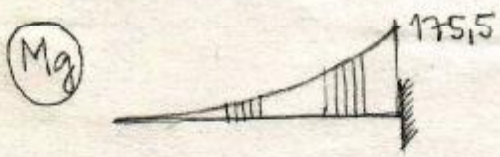
– СОПСТВЕНА ТЕЖИНА: $\frac{0{,}40+0{,}60}{2} \cdot 0{,}30 \cdot 25{,}0 = 3{,}75$ kN/m'
– РЕАКЦИЈА ПЛОЧЕ: $2R_{g1} = 2 \cdot 3{,}0 = 6{,}00$ kN/m'

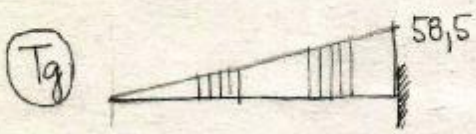
$g = 9{,}75$ kN/m'

– РЕАКЦИЈА ПЛОЧЕ: $2R_{p1} = 2 \cdot 2{,}25 =$ $p = 4{,}50$ kN/m'

а) СТАЛНО ОПТЕРЕЋЕЊЕ

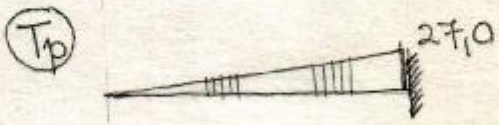
$M_g = 9{,}75 \cdot 6{,}0^2/2 = 175{,}5$ kNm
$R_{g2} = 9{,}75 \cdot 6{,}0 = 58{,}5$ kN





б) ПОВРЕМЕНО ОПТЕРЕЋЕЊЕ

$M_p = 4{,}50 \cdot 6{,}0^2/2 = 81{,}0$ kNm
$R_{p2} = 4{,}50 \cdot 6{,}0 = 27{,}0$ kN



ДИМЕНЗИОНИСАЊЕ

$b/d/h = 30/60/54$ cm

$M_{Ed} = 1{,}6 \cdot 175{,}5 + 1{,}8 \cdot 81{,}0 = 426{,}6$ kNm

$$K_r = \frac{54{,}0}{\sqrt{\frac{426{,}6}{0{,}30 \cdot 2{,}55}}} = 2{,}287 \longrightarrow \varepsilon_b/\varepsilon_a = 3{,}5/9{,}7\,‰$$

$\zeta = 21{,}464\,‰$

$$A_a = 21{,}464 \cdot \frac{30{,}0 \cdot 54{,}0}{100} \cdot \frac{2{,}55}{40} = 22{,}17 \text{ cm}^2$$

УСВОЈЕНО $\boxed{6R\emptyset 22}$ $(22{,}81 \text{ cm}^2)$

КОНТРОЛА ГЛАВНИХ НАПОНА ЗАТЕЗАЊА

$T_{sr} = 1{,}6 \cdot 58{,}5 + 1{,}8 \cdot 27{,}0 = 142{,}2 \text{ kN}$

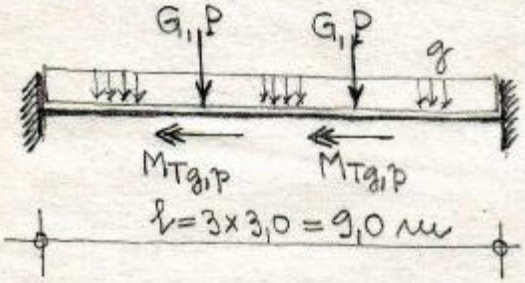
$$\operatorname{tg}\beta = \frac{d_2 - d_1}{l} = \frac{60{,}0 - 40{,}0}{600} = \frac{1}{30}$$

$$T_{tmu} = T_{sr} - \frac{M_{sr}}{h} \cdot \operatorname{tg}\beta = 142{,}2 - \frac{426{,}6}{0{,}54} \cdot \frac{1}{30} = 115{,}87 \text{ kN}$$

$$\tau_{tr} = \frac{T_{tmu}}{\beta_0 \cdot 0{,}9 \cdot h} = \frac{115{,}87}{30{,}0 \cdot 0{,}9 \cdot 54{,}0} = 0{,}079 \text{ kN/cm}^2 < \tau_{tc} = 0{,}13 \text{ kN/cm}^2$$

НИЈЕ ПОТРЕБНО ОСИГУРАЊЕ АРМАТУРОМ ОД ГЛАВНИХ НАПОНА ЗАТЕЗАЊА.

POS 3 – КОНТИНУАЛНА ГРЕДА



– СОПСТВЕНА ТЕЖИНА: $0{,}80 \cdot 1{,}00 \cdot 25{,}0 = \boxed{g = 20{,}0 \text{ kN/m}'}$

– КОНЦ. ОПТ. ОД POS 2: $R_{g2} = \boxed{G = 58{,}5 \text{ kN}}$, $R_{p2} = \boxed{P = 27{,}0 \text{ kN}}$

– КОНЦ. МОМЕНТИ ТОРЗИЈЕ ОД POS 2: $M_{g2} = \boxed{M_{Tg} = 175{,}5 \text{ kNm}}$, $M_{p2} = \boxed{M_{Tp} = 81{,}0 \text{ kNm}}$

а) СТАЛНО ОПТЕРЕЋЕЊЕ

$$M_g^o = \frac{2}{9} \cdot G \cdot l + \frac{g \cdot l^2}{12} = \frac{2}{9} \cdot 58{,}5 \cdot 9{,}0 + \frac{20{,}0 \cdot 9{,}0^2}{12} = 252{,}0 \text{ kNm}$$

$$M_g^p = \frac{1}{9} \cdot G \cdot l + \frac{g \cdot l^2}{24} = \frac{M_g^o}{2} = 126{,}0 \text{ kNm}$$

$$T_g = 58{,}5 + 20{,}0 \cdot 9{,}0/2 = 148{,}5 \text{ kN}$$

б) ПОВРЕМЕНО ОПТЕРЕЋЕЊЕ

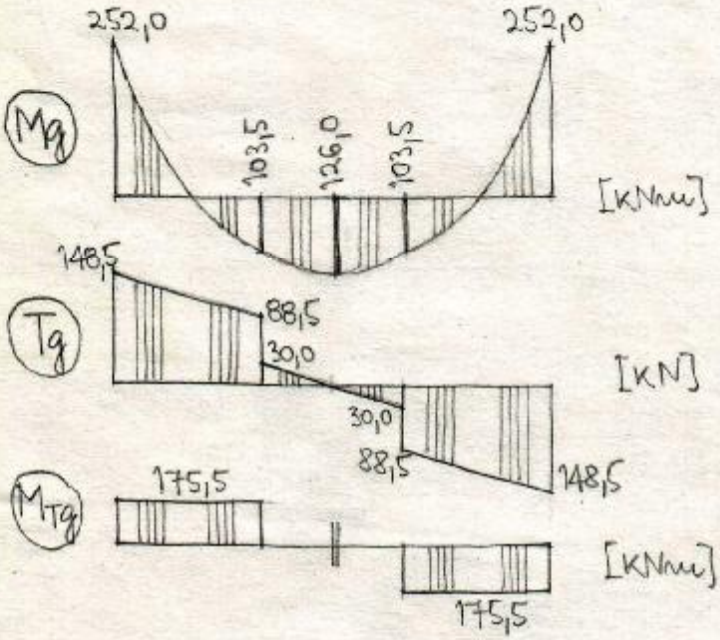
$M_p^o = \frac{2}{9} \cdot 27{,}0 \cdot 9{,}0 = 54{,}0$ kNm

$M_p^p = \frac{1}{9} \cdot 27{,}0 \cdot 9{,}0 = 27{,}0$ kNm

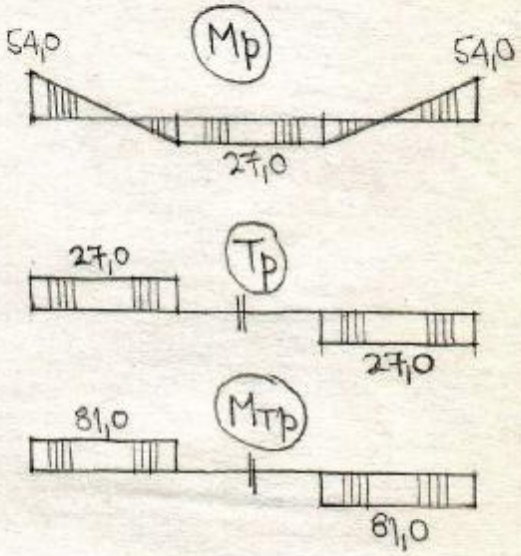
$T_p = 27{,}0$ kN

ДИЈАГРАМИ СИЛА У ПРЕСЕЦИМА

а) СТАЛНО ОПТЕРЕЋЕЊЕ



б) ПОВРЕМЕНО ОПТЕРЕЋЕЊЕ



ОСИГУРАЊЕ ОД ГЛАВНИХ НАПОНА ЗАТЕЗАЊА

$b/d/h = 80/100/95$ cm

$z \approx 0{,}9 \cdot h = 0{,}9 \cdot 95{,}0 = 85{,}5$ cm

$b_o \approx (b - 2 \cdot 4{,}0) = 72{,}0$ cm

$d_o \approx (d - 2 \cdot 4{,}0) = 92{,}0$ cm

$A_{bo} = b_o \cdot d_o = 72{,}0 \cdot 92{,}0 = 6624{,}0$ cm^2

$O_{bo} = 2 \cdot (b_o + d_o) = 2 \cdot (72{,}0 + 92{,}0) = 328{,}0$ cm

$\delta \leq \frac{b_o}{8} = \frac{72{,}0}{8} = 9{,}0$ cm $\rightarrow$ УСВОЈЕНО $\delta = 9{,}0$ cm

$T_{Ru} = 1{,}6 \cdot 148{,}5 + 1{,}8 \cdot 27{,}0 = 286{,}2$ kN

$M_{TRu} = 1{,}6 \cdot 175{,}5 + 1{,}8 \cdot 81{,}0 = 426{,}6$ kNm

$\tau_{n,T} = \frac{286{,}2}{80{,}0 \cdot 85{,}5} = 0{,}042$ kN/cm^2

$\tau_{n,M_T} = \frac{426{,}6 \cdot 10^2}{2 \cdot 6624{,}0 \cdot 9{,}0} = 0{,}358$ kN/cm^2

$\Big\}$ $\tau_n = 0{,}400 \frac{kN}{cm^2}$ $> 3\tau_c = 0{,}39$ $< 5\tau_c$

$\tau_n > 3\tau_c \rightarrow T_{Rw} = M_{TRw} = 0$

$\tau_{Rd} = \tau_{rd,T} = 0{,}042$ kN/cm²

$M_{TRd} = M_{Td} = 426{,}6$ kNm

УСВОЈЕНО: $\alpha = 90°$; $\theta = 45°$

а) УЗЕНГИЊЕ ЗА УТИЦАЈ Т:

$$a_w^{(1)} = \frac{8 \cdot \tau_{Rd}}{m \cdot \delta_v} \cdot e_w = \frac{80{,}0 \cdot 0{,}042}{4 \cdot 40{,}0} \cdot e_w = 0{,}021 \cdot e_w$$

б) УЗЕНГИЊЕ ЗА УТИЦАЈ M_T:

$$a_w^{(1)} = \frac{M_{TRd}}{2 \cdot A_{b0} \cdot \delta_v} \cdot e_w = \frac{426{,}6 \cdot 10^2}{2 \cdot 6624{,}0 \cdot 40{,}0} \cdot e_w = 0{,}081 \cdot e_w$$

СПОЉА: $a_w^{(1)} = (0{,}021 + 0{,}081) \cdot e_w = 0{,}102 \cdot e_w$

URØ12: $e_w \leq \frac{1{,}13}{0{,}102} = 11{,}1$ cm

УНУТРА: $a_w^{(1)} = 0{,}021 \cdot e_w$

URØ8: $e_w \leq \frac{0{,}503}{0{,}021} = 23{,}9$ cm

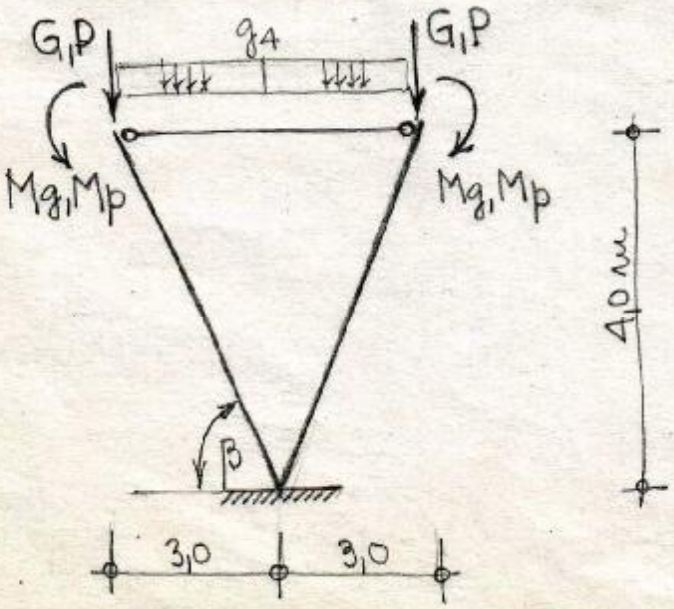
УСВОЈЕНО:

URØ12/10	→ СПОЉНА (Т+M_T)
URØ8/20	→ УНУТРАШЊА (Т)

$$\Sigma A_a = \frac{M_{Td}}{2 \cdot A_{b0} \cdot \delta_v} \cdot \sigma_{b0} = \frac{426{,}6 \cdot 10^2}{2 \cdot 6624{,}0 \cdot 40{,}0} \cdot 328{,}0 = 26{,}41 \text{ cm}^2$$

УСВОЈЕНО: 18 RØ14 (27,71 cm²)

POS 4, POS Ž



$$\operatorname{tg}\beta = \frac{4{,}0}{3{,}0} = 1{,}33$$

$\beta = 53{,}13°$

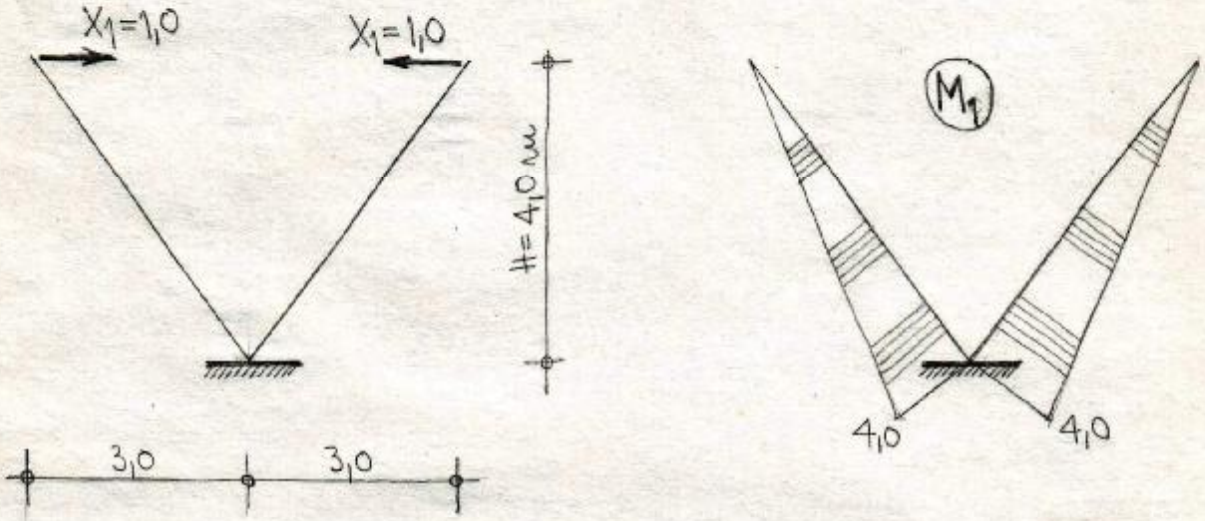
$\cos\beta = 0{,}60$

$\sin\beta = 0{,}80$

– СОПСТВЕНА ТЕЖИНА POS 4: $\frac{0,30 \cdot 0,80 \cdot 25,0}{0,60} = \boxed{g = 10,0 \text{ kN/m'}}$

– УТИЦАЈИ СА POS 2 И POS 3:

$G = 2R_{g3} + R_{g2} = 2 \cdot 148,5 + 58,5 = 355,5 \text{ kN}$
$P = 2R_{p3} + R_{p2} = 2 \cdot 27,0 + 27,0 = 81,0 \text{ kN}$
$M_g = 2M_{Tg3} + M_{g2} = 2 \cdot 175,5 + 175,5 = 526,5 \text{ kNm}$
$M_p = 2M_{Tp3} + M_{p2} = 2 \cdot 81,0 + 81,0 = 243,0 \text{ kNm}$



$J_B = \frac{30 \cdot 80^3}{12} = 1280\,000 \text{ cm}^4$
$A_a = 25,0 \text{ cm}^2$

$\frac{E_B \cdot J_B}{E_a \cdot A_a} = \frac{34,0 \cdot 1280000 \cdot 10^{-8}}{210,0 \cdot 25,0 \cdot 10^{-4}} = 0,829$

$E_B J_B \cdot \delta_{11} = \int_{S_1} M_1 \cdot M_1 \, ds + \frac{E_B J_B}{E_a A_a} \cdot \int_{S_2} Z_1 \cdot Z_1 \cdot ds$

$E_B J_B \delta_{11} = 2 \cdot \frac{5,0}{3} \cdot 4,0^2 + 0,829 \cdot 6,0 \cdot 1,0^2 = 53,3 + 4,97 = 58,31$

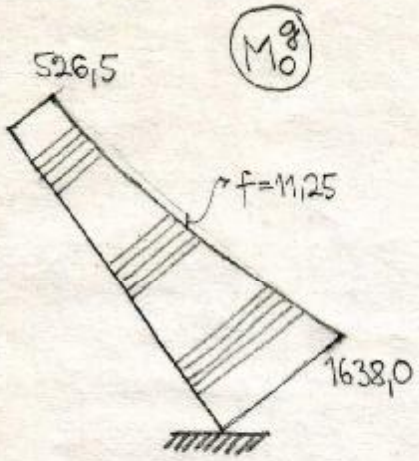
$\boxed{E_B \cdot J_B \cdot \delta_{11} = 58,31}$

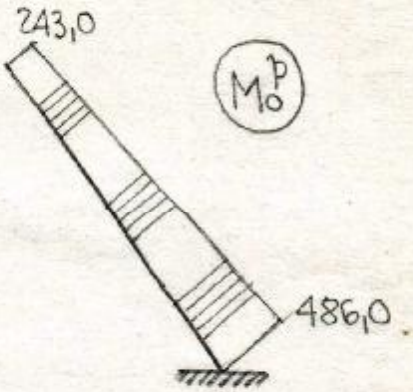
$-E_B J_B \delta_{10}^g = 2 \cdot \frac{5,0}{6} \cdot 4,0 \cdot (2 \cdot 1638,0 + 526,5) - 2 \cdot \frac{5,0}{3} \cdot 4,0 \cdot 11,25 = 25200$

$X_1^g = \frac{-E_B J_B \delta_{10}^g}{E_B J_B \delta_{11}} = \frac{25200,0}{58,31} = 432,19 \text{ kN}$

$\boxed{X_1^g = 432,19 \text{ kN}}$

РАДИ ПРЕГЛЕДНОСТИ, ПРИКАЗАНЕ СУ ПОЛОВИНЕ ДИЈАГРАМА :



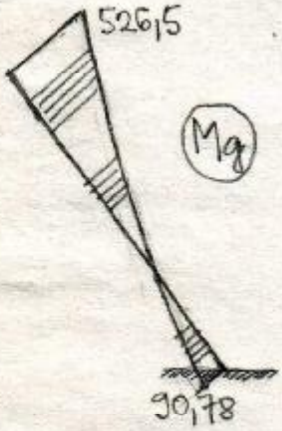


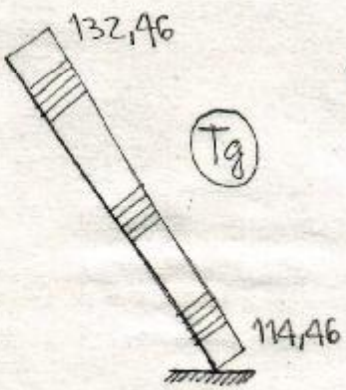
$$-E_8 J_8 \delta_{10}^p = 2 \cdot \frac{5,0}{6} \cdot 4,0 \cdot (2 \cdot 486,0 + 243,0) = 8100$$

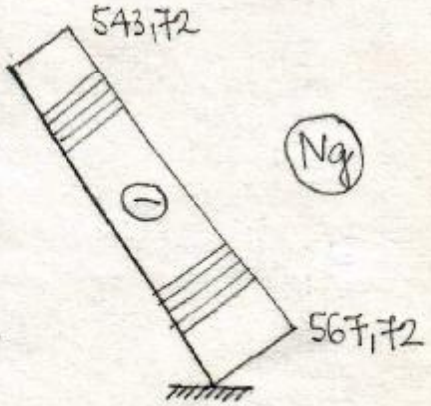
$$X_1^p = \frac{-E_8 J_8 \delta_{10}^p}{E_8 J_8 \delta_{11}} = \frac{8100,0}{58,31} = 138,92 \text{ kN}$$

$$\boxed{X_1^p = 138,92 \text{ kN}}$$

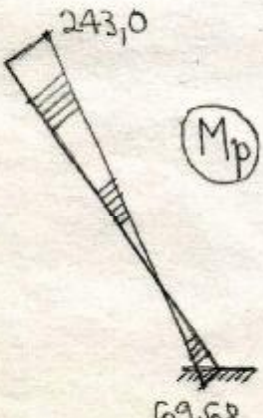
а) СТАЛНО ОПТЕРЕЋЕЊЕ

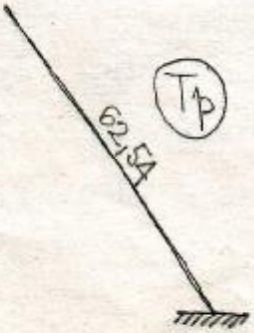






б) ПОВРЕМЕНО ОПТЕРЕЋЕЊЕ





РОЗ4 – ДИМЕНЗИОНИСАЊЕ

ПРЕСЕК ГОРЕ

$M_{Ed} = 1,6 \cdot 526,5 + 1,8 \cdot 243,0 = 1279,8$ kNm
$N_{Ed} = 1,6 \cdot 543,72 + 1,8 \cdot 148,15 = 1136,6$ kN
$b/d/h = 30/80/72$ cm
$M_{Aa} = 1279,8 + 1136,6 \cdot \left(\frac{0,80}{2} - 0,08\right) = 1643,5$ kNm

$$k = \frac{72,0}{\sqrt{\frac{1643,5}{0,30 \cdot 2,55}}} = 1,553 \longrightarrow \varepsilon_b/\varepsilon_{a1} = 3,5/1,25\,‰$$

$\varepsilon_{a1} < 3‰ \rightarrow$ ДВОСТРУКО АРМИРАЊЕ

УСВОЈЕНО: $\varepsilon_b/\varepsilon_{a1} = 3,5/4,5\,‰ \rightarrow k = 1,858$, $\bar{\mu} = 35,416\,‰$

$$M_{Rbu} = \left(\frac{72,0}{1,858}\right)^2 \cdot 0,30 \cdot 2,55 = 1148,8 \text{ kNm}$$

$\Delta M_{Ed} = 1643,5 - 1148,8 = 494,7$ kNm

ПРЕТПОСТАВЉАМО: $a_2 = 5,0$ cm

$$A_{a1} = 35,416 \cdot \frac{30,0 \cdot 72,0}{100} \cdot \frac{2,55}{40} + \frac{494,7 \cdot 10^2}{(72,0-5,0) \cdot 40} - \frac{1136,6}{40} = 38,81 \text{ cm}^2$$

УСВОЈЕНО **8RØ25** ($39,27$ cm^2)

$$A_{a2} = \frac{494,7 \cdot 10^2}{(72,0-5,0) \cdot 40} = 18,46 \text{ cm}^2$$

УСВОЈЕНО **4RØ25** ($19,64$ cm^2)

ПРЕСЕК ДОЛЕ

$M_{Ed} = 1,6 \cdot 90,178 + 1,8 \cdot 69,68 = 270,7$ kNm
$N_{Ed} = 1,6 \cdot 567,72 + 1,8 \cdot 148,15 = 1175,0$ kN
$b/d/h = 30/80/75$ cm
$M_{Aa} = 270,7 + 1175,0 \cdot \left(\frac{0,80}{2} - 0,05\right) = 681,9$ kNm

$$k = \frac{75,0}{\sqrt{\frac{681,9}{0,30 \cdot 2,55}}} = 2,512 \longrightarrow \varepsilon_b/\varepsilon_{a1} = 2,925/10\,‰$$

$\bar{\mu}_1 = 17,473\,‰$

$$A_a = 17,473 \cdot \frac{30,0 \cdot 75,0}{100} \cdot \frac{2,55}{40} - \frac{1175,0}{40} = -4,30 \text{ cm}^2$$

$\text{MIN. } A_{a1} = 0,20 \cdot 30,0 \cdot 80,0/100 = 6,0$ cm^2

УСВОЈЕНО **±2RØ25** ($\pm 9,82$ cm^2)

$T_{tc} = 1,6 \cdot 132,46 + 1,8 \cdot 62,54 = 324,51$ kN

$T_n = \frac{324,51}{30 \cdot 0,9 \cdot 72} = 0,167$ kN/cm² $> T_{tc} = 0,13$ kN/cm² $< 3T_{tc}$

$T_{bu} = \frac{1}{2} \cdot (3 \cdot 0,13 - 0,167) \cdot 30 \cdot 0,9 \cdot 72,0 = 216,8$ kN

$T_{RU} = 324,51 - 216,8 = 107,71$ kN

$T_{RU} = \frac{107,71}{30 \cdot 0,9 \cdot 72} = 0,055$ kN/cm²

УСВОЈЕНО: $m = 2$; $\alpha = 90°$; $\theta = 45°$; URϕ8

$e_u \leq \frac{m \cdot a_u^{(1)}}{b \cdot \text{MIN. } \mu_{uz.}} = \frac{2 \cdot 0,503}{30,0 \cdot 0,2 \cdot 10^{-2}} = 16,8$ cm

УСВОЈЕНО **URϕ8/15** НА ЧИТАВОЈ ДУЖИНИ СТУБА

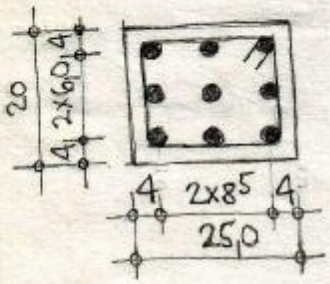
$T_{u,u} = \frac{2 \cdot 0,503}{30,0 \cdot 15,0} \cdot 40,0 = 0,089$ kN/cm² $> T_{RU} = 0,055$ kN/cm²

POS Z

$Z_u = 1,6 \cdot 432,19 + 1,8 \cdot 138,92 = 941,56$ kN

$A_a = \frac{941,56}{40,0} = 23,54$ cm²

УСВОЈЕНО **9Rϕ19** (25,52 cm²)



$A_n = \frac{210}{34} = 6,18$

$A_i = 25,0 \cdot 20,0 + 6,18 \cdot 25,52 = 657,6$ cm²

$f_{bz} = 0,7 \cdot 2,9 = 2,03$ MPa $= 0,203$ kN/cm²

$Z^* = 0,203 \cdot 657,6 = 133,5$ kN

$Z = Z_g + Z_p = 432,19 + 138,92 = 571,11$ kN $> Z^*$

$a_o = 4,0 - \frac{1,9}{2} = 3,05$ cm

$K_1 = 0,4$ (RA 400/500)

$K_2 = 0,25$ (ЗАТЕЗАЊЕ)

$\mu_{z,ef.} = \frac{25,52}{25 \cdot 20} = 0,051$

$e_\phi = 8,5$ cm

$l_{ps} = 2 \cdot (3,05 + \frac{8,5}{10}) + 0,4 \cdot 0,25 \cdot \frac{1,9}{0,051} = 11,52$ cm

$\sigma_a = \frac{571,11}{25,52} = 22,38$ kN/cm² $= 223,8$ MPa

$\xi_a = 1 - 1,0 \cdot 1,0 \cdot \left(\frac{133,5}{571,11}\right)^2 = 0,945$

$a_{pk} = 1,7 \cdot 0,945 \cdot \frac{223,8}{210 \cdot 10^3} \cdot 11,52 = 19,7 \cdot 10^{-3}$ cm

$a_{pk} = 0,197$ mm $< 0,2$ mm $= a_{pk,dop.}$