

ZADATAK 2

1. Odrediti visinu i potrebnu površinu armature za presek pravougaonog oblika, opterećen momentima savijanja M_g i M_p usled stalnog, odnosno povremenog opterećenja.

$$b = 30 \text{ cm}$$

$$M_g = 60,8 \text{ kNm}$$

$$M_p = 39 \text{ kNm}$$

2. Odrediti potrebnu površinu armature za presek pravougaonog oblika poznatih dimenzija, opterećen momentima savijanja M_g i M_p . Odrediti potrebnu površinu armature za slučajeve da je isti presek, pored zadatih momenata savijanja, opterećen i silama zatezanja, odnosno pritiska.

$$b = 20 \text{ cm}$$

$$M_g = 29,6 \text{ kNm}$$

$$Z_g = -48,3 \text{ kN}$$

$$N_g = 64,4 \text{ kN}$$

$$d = 40 \text{ cm}$$

$$M_p = 26 \text{ kNm}$$

$$Z_p = -42,6 \text{ kN}$$

$$N_p = 56,9 \text{ kN}$$

3. Odrediti potrebnu površinu armature za pravougaoni presek poznatih dimenzija, opterećen zadatim momentima savijanja i silama pritiska.

$$b = 20 \text{ cm}$$

$$M_g = 47,4 \text{ kNm}$$

$$N_g = 146,6 \text{ kN}$$

$$d = 40 \text{ cm}$$

$$M_p = 41,9 \text{ kNm}$$

$$N_p = 129,4 \text{ kN}$$

4. Odrediti potrebnu površinu armature za **T** presek poznatih dimenzija, opterećen граниčnim momentom savijanja M_u .

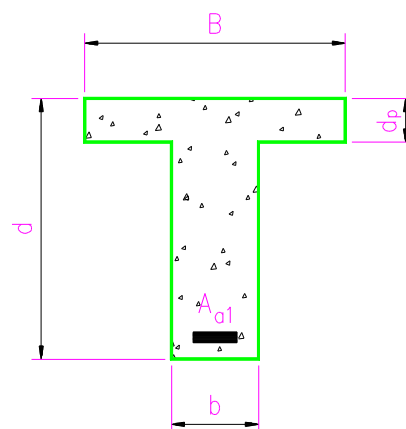
$$b = 20 \text{ cm}$$

$$B = 105 \text{ cm}$$

$$d = 40 \text{ cm}$$

$$d_p = 10 \text{ cm}$$

$$M_u = 94,16 \text{ kNm}$$



za sve zadatke: **MB 30 GA 240/360**
Sve proračunate poprečne preseke nacrtati u razmeri 1:10.

u Beogradu, 30/11/2004.

Predmetni nastavnik:

asistent:

overa: _____

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$$\text{MB 30} \quad \Rightarrow \quad f_B = 20.5 \text{ MPa} = 2.05 \text{ kN/cm}^2$$

$$\text{GA 240/360} \quad \Rightarrow \quad \sigma_v = 240 \text{ MPa} = 24 \text{ kN/cm}^2$$

Zadatak 1. Odrediti visinu i potrebnu površinu armature za presek pravougaonog oblika, opterećen momentima savijanja M_g i M_p usled stalnog, odnosno povremenog opterećenja.

$$b = 30 \text{ cm} \quad M_g = 60.8 \text{ kNm} \quad M_p = 39 \text{ kNm}$$

$$M_u = 1.6 \times M_g + 1.8 \times M_p = 1.6 \times 60.8 + 1.8 \times 39 = 167.5 \text{ kNm}$$

$$\text{usvojeno: } \varepsilon_b/\varepsilon_a = 3.0/10\text{‰} \Rightarrow k = 2.479 ; \bar{\mu} = 17.949\%$$

$$h_{\text{potr.}} = 2.479 \times \sqrt{\frac{167.5 \times 10^2}{30 \times 2.05}} = 40.9 \text{ cm}$$

$$A_{a,\text{potr.}} = 17.949 \times \frac{30 \times 40.9}{100} \times \frac{2.05}{24} = 18.82 \text{ cm}^2 \quad \Rightarrow \quad \text{usvojeno: } \mathbf{6\text{Ø}20} \text{ (18.85 cm}^2\text{)}$$

$$a_1 = \frac{4 \times 4.5 + 2 \times 9.5}{6} = 6.2 \text{ cm}$$

$$d_{\text{potr.}} = h_{\text{potr.}} + a_1 = 40.9 + 6.2 = 47.1 \text{ cm} \quad \Rightarrow \quad \text{usvojeno: } \mathbf{b/d = 30/50 \text{ cm}}$$

Zadatak 2. Odrediti potrebnu površinu armature za presek pravougaonog oblika poznatih dimenzija, opterećen momentima savijanja M_g i M_p . Odrediti potrebnu površinu armature za slučajeve da je isti presek, pored zadatih momenata savijanja, opterećen i silama zatezanja, odnosno pritiska.

$$b = 20 \text{ cm} \quad M_g = 29.6 \text{ kNm} \quad Z_g = -48.3 \text{ kN} \quad N_g = 64.4 \text{ kN}$$

$$d = 40 \text{ cm} \quad M_p = 26 \text{ kNm} \quad Z_p = -42.6 \text{ kN} \quad N_p = 56.9 \text{ kN}$$

a. čisto savijanje

$$\text{pretp. } a_1 = 6.5 \text{ cm} \Rightarrow h = 40 - 6.5 = 33.5 \text{ cm}$$

$$M_u = 1.6 \times 29.6 + 1.8 \times 26 = 94.2 \text{ kNm}$$

$$k = \frac{h}{\sqrt{\frac{M_u}{b \times f_B}}} = \frac{33.5}{\sqrt{\frac{94.2 \times 10^2}{20 \times 2.05}}} = 2.211 \Rightarrow \varepsilon_b/\varepsilon_a = 3.5/8.692\text{‰} ; \bar{\mu} = 23.239\%$$

$$A_{a,\text{potr.}} = 23.239 \times \frac{20 \times 33.5}{100} \times \frac{2.05}{24} = 13.30 \text{ cm}^2 \quad \Rightarrow \quad \text{usvojeno: } \mathbf{5\text{Ø}20} \text{ (15.71 cm}^2\text{)}$$

$$a_1 = \frac{3 \times 4.5 + 2 \times 9.5}{5} = 6.5 \text{ cm} \quad \Rightarrow \quad h_{\text{stv.}} = 40 - 6.5 = 33.5 \text{ cm} = h_{\text{pretp.}}$$

b. savijanje sa silom zatezanja

$$\text{pretp. } a_1 = 6.5 \text{ cm} \Rightarrow h = 40 - 6.5 = 33.5 \text{ cm}$$

$$M_u = 94.2 \text{ kNm} ; Z_u = 1.6 \times 48.3 + 1.8 \times 42.6 = 153.96 \text{ kN}$$

$$M_{\text{au}} = M_u + N_u \times \left(\frac{d}{2} - a_1 \right) = 94.2 - 153.96 \times \left(\frac{40}{2} - 6.5 \right) \times 10^{-2} = 73.4 \text{ kNm}$$

$$k = \frac{h}{\sqrt{\frac{M_{au}}{b \times f_B}}} = \frac{33.5}{\sqrt{\frac{73.4 \times 10^2}{20 \times 2.05}}} = 2.504 \Rightarrow \varepsilon_b/\varepsilon_a = 2.938/10\text{‰} ; \bar{\mu} = 17.554\%$$

$$A_{a, \text{potr.}} = 17.554 \times \frac{20 \times 33.5}{100} \times \frac{2.05}{24} + \frac{153.96}{24} = 10.04 + 6.42 = 16.46 \text{ cm}^2$$

usvojeno: **6Ø20** (18.85 cm²)

$$a_1 = \frac{3 \times 4.5 + 3 \times 9.5}{6} = 7 \text{ cm} \Rightarrow h_{\text{stv.}} = 40 - 7 = 33 \text{ cm} \approx h_{\text{pretp.}} = 33.5 \text{ cm}$$

c. savijanje sa silom pritiska

$$\text{pretp. } a_1 = 5 \text{ cm} \Rightarrow h = 40 - 5 = 35 \text{ cm}$$

$$M_u = 94.2 \text{ kNm} ; N_u = 1.6 \times 64.4 + 1.8 \times 56.9 = 205.46 \text{ kN}$$

$$M_{au} = M_u + N_u \times \left(\frac{d}{2} - a_1 \right) = 94.2 + 205.46 \times \left(\frac{40}{2} - 5 \right) \times 10^{-2} = 125.0 \text{ kNm}$$

$$k = \frac{h}{\sqrt{\frac{M_{au}}{b \times f_B}}} = \frac{35}{\sqrt{\frac{125.0 \times 10^2}{20 \times 2.05}}} = 2.005 \Rightarrow \varepsilon_b/\varepsilon_a = 3.5/6.173\text{‰} ; \bar{\mu} = 29.292\%$$

$$A_{a, \text{potr.}} = 29.292 \times \frac{20 \times 35}{100} \times \frac{2.05}{24} - \frac{205.46}{24} = 17.51 - 8.56 = 8.95 \text{ cm}^2$$

usvojeno: **3Ø20** (9.42 cm²)

$$a_1 = 4.5 \text{ cm} \Rightarrow h_{\text{stv.}} = 40 - 4.5 = 35.5 \text{ cm} > h_{\text{pretp.}} = 33.5 \text{ cm}$$

Zadatak 3. Odrediti potrebnu površinu armature za pravougaoni presek poznatih dimenzija, opterećen zadatim momentima savijanja i silama pritiska.

$$b = 20 \text{ cm}$$

$$M_g = 47.4 \text{ kNm}$$

$$N_g = 146.6 \text{ kN}$$

$$d = 40 \text{ cm}$$

$$M_p = 41.9 \text{ kNm}$$

$$N_p = 129.4 \text{ kN}$$

$$M_u = 1.6 \times 47.4 + 1.8 \times 41.9 = 151.3 \text{ kNm}$$

$$N_u = 1.6 \times 146.6 + 1.8 \times 129.4 = 467.5 \text{ kN}$$

$$\text{pretp. } a_1 = 7 \text{ cm} \Rightarrow h = 40 - 7 = 33 \text{ cm}$$

$$M_{au} = 151.3 + 467.5 \times \left(\frac{40}{2} - 7 \right) \times 10^{-2} = 212.0 \text{ kNm}$$

$$k = \frac{33}{\sqrt{\frac{212.0 \times 10^2}{20 \times 2.05}}} = 1.451 \Rightarrow \varepsilon_a < 3\text{‰} \Rightarrow \text{dvojno armiranje}$$

$$\text{Usvojeno: } \varepsilon_b/\varepsilon_{a1}^* = 3.5/3\text{‰} \Rightarrow k^* = 1.719 ; \bar{\mu}^* = 43.590\%$$

$$M_{abu} = \left(\frac{h}{k^*} \right)^2 \times b \times f_B = \left(\frac{33}{1.719} \right)^2 \times 20 \times 2.05 = 15100 \text{ kNcm} = 151.0 \text{ kNm}$$

$$\Delta M_{au} = M_{au} - M_{abu} = 212.0 - 151.0 = 61.0 \text{ kNm}$$

pretp. $a_2 = 4.5 \text{ cm}$

$$A_{a2} = \frac{\Delta M_{au}}{\sigma_v \times (h - a_2)} = \frac{61.0 \times 10^2}{24 \times (33 - 4.5)} = 8.92 \text{ cm}^2$$

$$A_{a1} = \bar{\mu} * \times \frac{b \times h}{100} \times \frac{f_B}{\sigma_v} - \frac{N_u}{\sigma_v} + A_{a2} = 43.590 \times \frac{20 \times 33}{100} \times \frac{2.05}{24} - \frac{467.5}{24} + 8.92 = 14.01 \text{ cm}^2$$

usvojeno: **5Ø20** (15.71 cm²) – zategnuta armatura
3Ø20 (9.42 cm²) – pritisnuta armatura

Zadatak 4. Odrediti potrebnu površinu armature za **T** presek poznatih dimenzija, opterećen graničnim momentom savijanja M_u . Podaci za proračun:

$$B = 105 \text{ cm} \qquad d = 40 \text{ cm} \qquad M_u = 94.16 \text{ kNm}$$

$$b = 20 \text{ cm} \qquad d_p = 10 \text{ cm}$$

$B/b = 105/20 \text{ cm} > 5$ - može se koristiti približan postupak sa zanemarenjem nosivosti rebra (ukoliko se neutralna linija nađe u rebu)

pretp. $a_1 = 6.5 \text{ cm} \Rightarrow h = 40 - 6.5 = 33.5 \text{ cm}$

$$\sigma_{bp} = \frac{M_{au}}{B \times d_p \times \left(h - \frac{d_p}{2} \right)} = \frac{94.16 \times 10^2}{105 \times 10 \times \left(33.5 - \frac{10}{2} \right)} = 0.31 \text{ kN/cm}^2 = 3.1 \text{ MPa}$$

Kako je $d_p = 10 \text{ cm} < 12 \text{ cm}$, to se računaska čvrstoća umanjuje za 10%, pa je:

$$f_B = 0.9 \times 20.5 = 18.45 \text{ MPa} = 1.845 \text{ kN/cm}^2$$

$$\varepsilon_{bp} = 2 \times \left(1 - \sqrt{1 - \frac{\sigma_{bp}}{f_B}} \right) = 2 \times \left(1 - \sqrt{1 - \frac{3.1}{18.45}} \right) = 0.179\% \quad ; \quad \varepsilon_a = 10\%$$

$$x_0 = \frac{\varepsilon_{bp}}{\varepsilon_{bp} + \varepsilon_a} \times \left(h - \frac{d_p}{2} \right) = \frac{0.179}{0.179 + 10} \times \left(33.5 - \frac{10}{2} \right) = 0.50 \text{ cm} < d_p/2 = 5 \text{ cm}$$

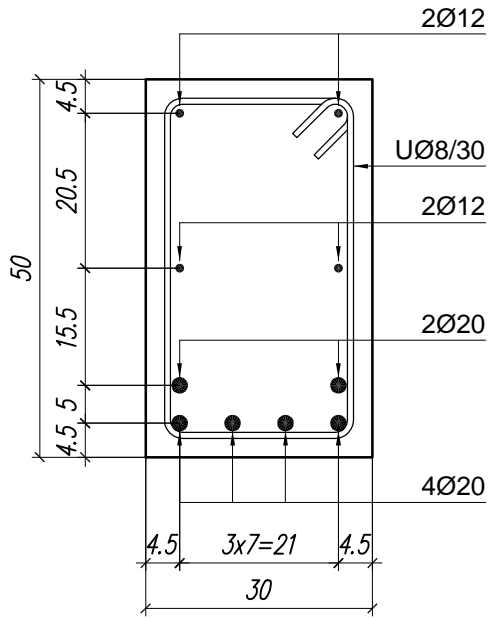
Neutralna linija se nalazi u ploči, pa se presek proračunava kao pravougaoni, širine $B = 105 \text{ cm}$:

$$k = \frac{h}{\sqrt{\frac{M_u}{B \times f_B}}} = \frac{33.5}{\sqrt{\frac{94.16 \times 10^2}{105 \times 1.845}}} = 4.805 \Rightarrow \varepsilon_b/\varepsilon_a = 1.105/10\% \quad ; \quad \bar{\mu} = 4.488\% \quad ; \quad s = 0.100$$

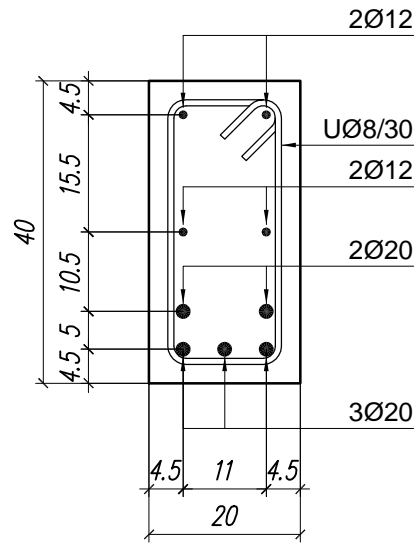
$x = s \times h = 0.100 \times 33.5 = 3.35 \text{ cm} < d_p = 10 \text{ cm}$ (kontrola položaja neutralne linije)

$$A_{a, \text{potr.}} = 4.488 \times \frac{105 \times 33.5}{100} \times \frac{1.845}{24} = 12.14 \text{ cm}^2 \Rightarrow \text{usvojeno: } \mathbf{5Ø18} \text{ (12.72 cm}^2\text{)}$$

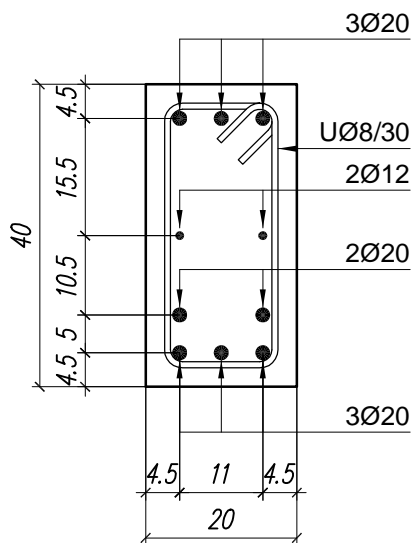
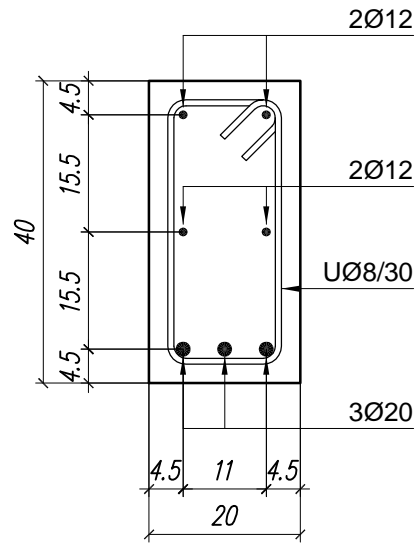
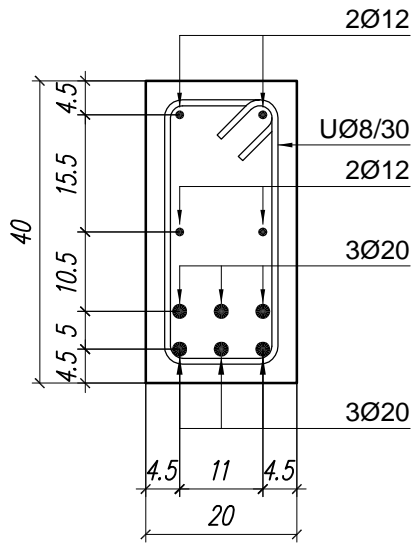
$$a_1 = \frac{3 \times 4.5 + 2 \times 9.5}{5} = 6.5 \text{ cm} \Rightarrow h_{stv.} = 40 - 6.5 = 33.5 \text{ cm} = h_{\text{pretp.}}$$



Z1 Z2a



Z2b Z2c



Z3 Z4

