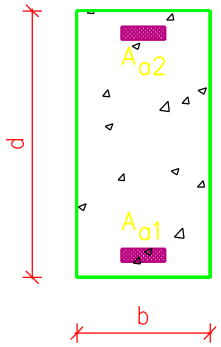
GRAĐEVINSKI FAKULTET U BEOGRADU
ODSEK ZA KONSTRUKCIJE
TEORIJA BETONSKIH KONSTRUKCIJA
školska 2004/2005. godina

ZADATAK 3

1. Za presek pravougaonog oblika prikazan na skici, potrebno je:
   1a. Pravilno rasporediti zadatu količinu armature i nacrtati presek u razmeri 1:10.
   1b. Odrediti moment loma M_u ukoliko se zanemari nosivost pritisnute armature (A_{a2}=0) za sledeća tri slučaja:
   - presek je opterećen na čisto savijanje (M_u = ?, N_u = 0)
   - presek je ekscentrično pritisnut (M_u = ?, zadata sila N_u)
   - presek je ekscentrično zategnut (M_u = ?, zadata sila Z_u)

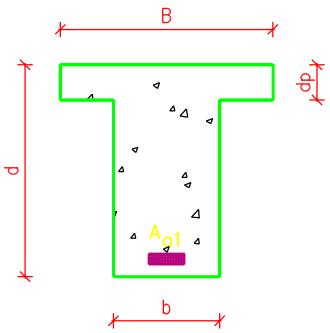
   1c. Sprovesti proračun definisan u tački 1b. uz uzimanje u obzir nosivosti ukupne zadate armature u preseku. Uporediti odgovarajuće rezultate proračuna 1b. i 1c.

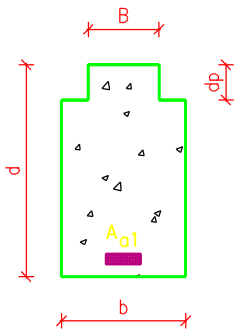
A_{a1} = 6Ø18
A_{a2} = 2Ø18
b = 30 cm
d = 50 cm
MB 25
GA 240/360
N_u = 312 kN
Z_u = -150 kN



2. Za presek prikazan na donjoj skici, potrebno je:

   2a. Pravilno rasporediti zadatu količinu armature i nacrtati presek u razmeri 1:10.
   2b. Odrediti moment loma preseka prema zadatim podacima. Izvršiti poređenje sa rezultatom odgovarajućeg proračuna iz primera 1b.





A_{a1} = 6Ø18

B = 45 cm
b = 30 cm
d = 50 cm
d_p = 10 cm
N_u = 312 kN
MB 25
GA 240/360

u Beogradu, 01/12/2004

Predmetni nastavnik:

asistent:

overa:_____

Prof. dr Mihajlo Đurđević, dipl.građ.inž., s.r.

148 194 119 ® 150 204 119 Ø 209

MB 25 $\Rightarrow$ $f_B = 17.25$ MPa = 1.725 kN/cm^2

GA 240/360 $\Rightarrow$ $\sigma_v = 240$ MPa = 24 kN/cm^2

$A_{a1} = 15.27$ cm^2 (6Ø18) ; $A_{a2} = 5.09$ cm^2 (2Ø18) (varijanta sa $A_{a2} \neq 0$)

$$a_1 = \frac{4\times 4.5 + 2\times 9.5}{6} = 6.2 \text{ cm} \Rightarrow h = 50 - 6.2 = 43.8 \text{ cm} \quad ; \quad a_2 = 4.5 \text{ cm}$$

Zadatak 1b. ***Zanemarenje nosivosti armature u pritisnutoj zoni preseka***

$\Sigma N = 0$: $\quad \bar{\mu} = \alpha_b \times s = \dfrac{A_a \times \sigma_v + N_u}{b \times h \times f_B}$

$\Sigma M_{a1} = 0$: $\quad M_u = \left(\dfrac{h}{k}\right)^2 \times b \times f_B - N_u \times \left(\dfrac{d}{2} - a_1\right)$

a. čisto savijanje

$$\bar{\mu} = \frac{15.27\times 24}{30\times 43.8\times 1.725} = 0.162 = 16.154\% \Rightarrow \varepsilon_b/\varepsilon_{a1} = 2.725/10‰ \quad ; \quad k = 2.599 \; ; \; s = 0.214$$

$$M_u = \left(\frac{43.8}{2.599}\right)^2 \times 30 \times 1.725 = 14714 \text{ kNcm} = 147.14 \text{ kNm}$$

b. savijanje sa silom pritiska

$$\bar{\mu} = \frac{15.27\times 24 + 312}{30\times 43.8\times 1.725} = 0.299 = 29.908\% \Rightarrow \varepsilon_b/\varepsilon_{a1} = 3.5/5.95‰ \quad ; \quad k = 1.986 \; ; \; s = 0.370$$

$$M_u = \left(\frac{43.8}{1.986}\right)^2 \times 30 \times 1.725 - 312 \times \left(\frac{50}{2} - 6.2\right) = 19333 \text{ kNcm} = 193.33 \text{ kNm}$$

c. savijanje sa silom zatezanja

$$\bar{\mu} = \frac{15.27\times 24 - 150}{30\times 43.8\times 1.725} = 0.095 = 9.541\% \Rightarrow \varepsilon_b/\varepsilon_{a1} = 1.8/10‰ \quad ; \quad k = 3.321 \; ; \; s = 0.153$$

$$M_u = \left(\frac{43.8}{3.321}\right)^2 \times 30 \times 1.725 + 150 \times \left(\frac{50}{2} - 6.2\right) = 11840 \text{ kNcm} = 118.4 \text{ kNm}$$

Zadatak 1c. ***Uticaj nosivosti armature u pritisnutoj zoni preseka***

Najpre se određuje položaj neutralne linije ***s*** iz uslova ravnoteže normalnih sila:

$\Sigma N = 0$: $\quad D_{bu1} + D_{au} - Z_{au} - N_u = 0$

$$s \le 0.259 = 7/27 \Rightarrow \varepsilon_{a1} = 10‰ \quad ; \quad \varepsilon_b = \frac{s}{1-s} \times \varepsilon_{a1}$$

$$s \ge 0.259 = 7/27 \Rightarrow \varepsilon_b = 3.5‰ \quad ; \quad \varepsilon_{a1} = \frac{1-s}{s} \times \varepsilon_b$$

$$\alpha_b = \frac{\varepsilon_b}{12} \times (6 - \varepsilon_b) \text{ za } \varepsilon_b \le 2‰ \quad ; \quad \alpha_b = \frac{3\varepsilon_b - 2}{3\varepsilon_b} \text{ za } 2‰ \le \varepsilon_b \le 3.5‰$$

$D_{bu} = \alpha_b \times b \times x \times f_B = \alpha_b \times s \times b \times h \times f_B \quad (s = x/h)$

$$\varepsilon_{a2} = \frac{x - a_2}{x} \times \varepsilon_b \quad \Rightarrow \quad \sigma_{a2} = E_a \times \varepsilon_{a2} \le \sigma_v \quad \Rightarrow \quad D_{au} = A_{a2} \times \sigma_{a2}$$

$$\sigma_{a1} = E_a \times \varepsilon_{a1} \le \sigma_v \quad \Rightarrow \quad Z_{au} = A_{a1} \times \sigma_{a1}$$

Kada se nakon niza iteracija pronađe položaj neutralne linije, sračunava se položaj sile pritiska u betonu i krak unutrašnjih sila:

$$\eta = \frac{8 - \varepsilon_b}{4 \times (6 - \varepsilon_b)} \text{ za } \varepsilon_b \le 2‰ \; ; \text{ odnosno } \eta = \frac{\varepsilon_b \times (3\varepsilon_b - 4) + 2}{2\varepsilon_b \times (3\varepsilon_b - 2)} \text{ za } 2‰ \le \varepsilon_b \le 3.5‰$$

$$z_b = h - \eta \times x = h \times (1 - \eta \times s)$$

a zatim, iz uslova ravnoteže momenata savijanja u odnosu na težište zategnute armature, određuje i traženi moment loma:

$$\Sigma M_{a1} = 0: \quad D_{bu} \times z_b + D_{au} \times (h - a_2) = M_{au}$$

$$M_u = M_{au} - N_u \times \left(\frac{d}{2} - a_1 \right)$$

Rezultati proračuna prikazani su tabelarno.

a. čisto savijanje

s	ε_b	ε_{a1}	α_b	D_{bu}	ε_{a2}	σ_{a2}	D_{au}	σ_{a1}	Z_{au}	ΣN_u
(-)	(‰)	(‰)	(-)	(kN)	(‰)	(MPa)	(kN)	(MPa)	(kN)	(kN)
0.200	2.5	10	0.733	332.7	1.217	240.0	122.1	240.0	366.4	88.4
0.150	1.765	10	0.623	211.9	0.557	117.0	59.5	240.0	366.4	-95.0
0.180	2.195	10	0.696	284.3	0.943	198.1	100.8	240.0	366.4	18.7
0.175	**2.123**	**10**	**0.686**	**272.5**	**0.879**	**184.5**	**93.9**	**240.0**	**366.4**	**0.0**

$$\varepsilon_b = 2.123‰ \Rightarrow \quad \eta = \frac{2.123 \times (3 \times 2.123 - 4) + 2}{2 \times 2.123 \times (3 \times 2.123 - 2)} = 0.379$$

$z_b = 43.8 \times (1 - 0.379 \times 0.175) = 40.92$ cm

$\Sigma M_{a1} = 0$: $272.5 \times 40.92 + 93.9 \times (43.8 - 4.5) = M_{au} = 14850$ kNcm $= 148.5$ kNm

$N_u = 0 \Rightarrow M_u = M_{au} = 148.5$ kNm

b. savijanje sa silom pritiska

s	ε_b	ε_{a1}	α_b	D_{bu}	ε_{a2}	σ_{a2}	D_{au}	σ_{a1}	Z_{au}	ΣN_u
(-)	(‰)	(‰)	(-)	(kN)	(‰)	(MPa)	(kN)	(MPa)	(kN)	(kN)
0.350	3.5	6.5	0.810	642.7	2.473	240.0	122.1	240.0	366.4	86.4
0.300	3.5	8.167	0.810	550.9	2.302	240.0	122.1	240.0	366.4	-5.4
0.305	3.5	7.975	0.810	560.1	2.322	240.0	122.1	240.0	366.4	3.8
0.303	**3.5**	**8.053**	**0.810**	**556.3**	**2.314**	**240.0**	**122.1**	**240.0**	**366.4**	**0.0**

$\varepsilon_b = 3.5‰ \Rightarrow \quad \eta = \dfrac{3.5\times(3\times3.5-4)+2}{2\times3.5\times(3\times3.5-2)} = 0.416$

$z_b = 43.8\times(1 - 0.416\times0.303) = 38.31$ cm

$\Sigma M_{a1} = 0$: $556.3\times38.31 + 93.9\times(43.8 - 4.5) = M_{au} = 26120$ kNcm = 261.2 kNm

$N_u = 312$ kN $\Rightarrow \quad M_u = 261.2 - 312\times\left(\dfrac{0.50}{2} - 0.062\right) = 202.4$ kNm

c. savijanje sa silom zatezanja

s	ε_b	ε_{a1}	α_b	D_{bu}	ε_{a2}	σ_{a2}	D_{au}	σ_{a1}	Z_{au}	ΣN_u
(-)	(‰)	(‰)	(-)	(kN)	(‰)	(MPa)	(kN)	(MPa)	(kN)	(kN)
0.150	1.765	10	0.623	211.9	0.557	117.0	59.5	240.0	366.4	55.0
0.130	1.494	10	0.561	165.5	0.314	66.0	33.6	240.0	366.4	-17.4
0.140	1.628	10	0.593	188.4	0.434	91.2	46.4	240.0	366.4	18.3
0.135	**1.559**	**10**	**0.577**	**176.6**	**0.373**	**78.3**	**39.8**	**240.0**	**366.4**	**0.0**

$\varepsilon_b = 1.559‰ \Rightarrow \quad \eta = \dfrac{8-1.559}{4\times(6-1.559)} = 0.363$

$z_b = 43.8\times(1 - 0.363\times0.135) = 41.69$ cm

$\Sigma M_{a1} = 0$: $176.6\times41.69 + 39.8\times(43.8 - 4.5) = M_{au} = 8929$ kNcm = 89.29 kNm

$Z_u = 150$ kN $\Rightarrow \quad M_u = 89.29 - (-150)\times\left(\dfrac{0.50}{2} - 0.062\right) = 117.5$ kNm

Zadatak 2. ***Određivanje nosivosti T preseka***

Najpre se određuje položaj neutralne linije ***s*** iz uslova ravnoteže normalnih sila:

$\Sigma N = 0$: $D_{bu1} - D_{bu2} - Z_{au} - N_u = 0$

$s \le 0.259 = 7/27 \Rightarrow \varepsilon_{a1} = 10‰ \;;\; \varepsilon_b = \dfrac{s}{1-s}\times\varepsilon_{a1}$

$s \ge 0.259 = 7/27 \Rightarrow \varepsilon_b = 3.5‰ \;;\; \varepsilon_{a1} = \dfrac{1-s}{s}\times\varepsilon_b$

$\sigma_{a1} = E_a\times\varepsilon_{a1} \le \sigma_v \Rightarrow Z_{au} = A_{a1}\times\sigma_{a1}$

$\varepsilon_{bd} = \dfrac{\mathbf{x}-\mathbf{d_p}}{\mathbf{x}}\times\varepsilon_b$

$D_{bu1} = \alpha_{b1}\times B\times x\times f_B = \alpha_{b1}\times s\times B\times h\times f_B \quad (s = x/h)$

$D_{bu2} = \alpha_{b2}\times(B - b)\times(x - d_p)\times f_B = \alpha_{b2}\times(B - b)\times(s - \delta)\times h\times f_B \quad (\delta = d_p/h)$

Koeficijenti punoće naponskog dijagrama betona α_{b1}, α_{b2} se određuju za dilatacije ε_b i ε_{bd} respektvino (dilatacije gornje, odnosno donje ivice ploče) iz odgovarajućih izraza:

$\alpha_b = \dfrac{\varepsilon_b}{12}\times(6-\varepsilon_b)$ za $\varepsilon_b \le 2‰$; $\alpha_b = \dfrac{3\varepsilon_b - 2}{3\varepsilon_b}$ za $2‰ \le \varepsilon_b \le 3.5‰$

Kada se nakon niza iteracija pronađe položaj neutralne linije, sračunava se položaj sila D_{bu1}, D_{bu2} i kraci unutrašnjih sila u odnosu na težište zategnute armature:

$$z_{b1} = h - \eta_1 \times x = h \times (1 - \eta_1 \times s)$$

$$z_{b2} = h - d_p - \eta_2 \times (x - d_p) = h \times [(1 - \delta - \eta_2 \times (s - \delta)]$$

Bezdimenzioni koeficijent η je funkcija dilatacije betona i sračunava se iz izraza:

$$\eta = \frac{8 - \varepsilon_b}{4 \times (6 - \varepsilon_b)} \quad \text{za} \quad \varepsilon_b \le 2‰ \quad ; \quad \text{odnosno} \quad \eta = \frac{\varepsilon_b \times (3\varepsilon_b - 4) + 2}{2\varepsilon_b \times (3\varepsilon_b - 2)} \quad \text{za} \quad 2‰ \le \varepsilon_b \le 3.5‰$$

Zatim se iz uslova ravnoteže $\Sigma M_{a1} = 0$ određuje i traženi moment loma:

$$\Sigma M_{a1} = 0: \quad D_{bu} \times z_{b1} - D_{bu2} \times z_{b2} = M_{au}$$

$$M_u = M_{au} - N_u \times \left(\frac{d}{2} - a_1 \right)$$

Rezultati proračuna prikazani su tabelarno.

$$\delta = \frac{d_p}{h} = \frac{10}{43.8} = 0.228$$

s	ε_b	ε_{a1}	α_{b1}	D_{bu1}	ε_{bd}	α_{b2}	D_{bu2}	σ_{a1}	Z_{au}	ΣN_u
(-)	(‰)	(‰)	(-)	(kN)	(‰)	(-)	(kN)	(MPa)	(kN)	(kN)
0.228	2.956	10	0.774	601.2	0.000	0.000	0.0	240.0	366.4	-77.3
0.3	3.5	8.167	0.810	826.3	0.838	0.361	29.4	240.0	366.4	118.5
0.24	3.158	10	0.789	644.2	0.156	0.076	1.0	240.0	366.4	-35.2
0.255	3.423	10	0.805	698.7	0.361	0.169	5.2	240.0	366.4	15.1
0.2504	**3.341**	**10**	**0.800**	**682.0**	**0.297**	**0.141**	**3.6**	**240.0**	**366.4**	**0.0**

$$\varepsilon_b = 3.341‰ \Rightarrow \quad \eta_1 = \frac{3.341 \times (3 \times 3.341 - 4) + 2}{2 \times 3.341 \times (3 \times 3.341 - 2)} = 0.413$$

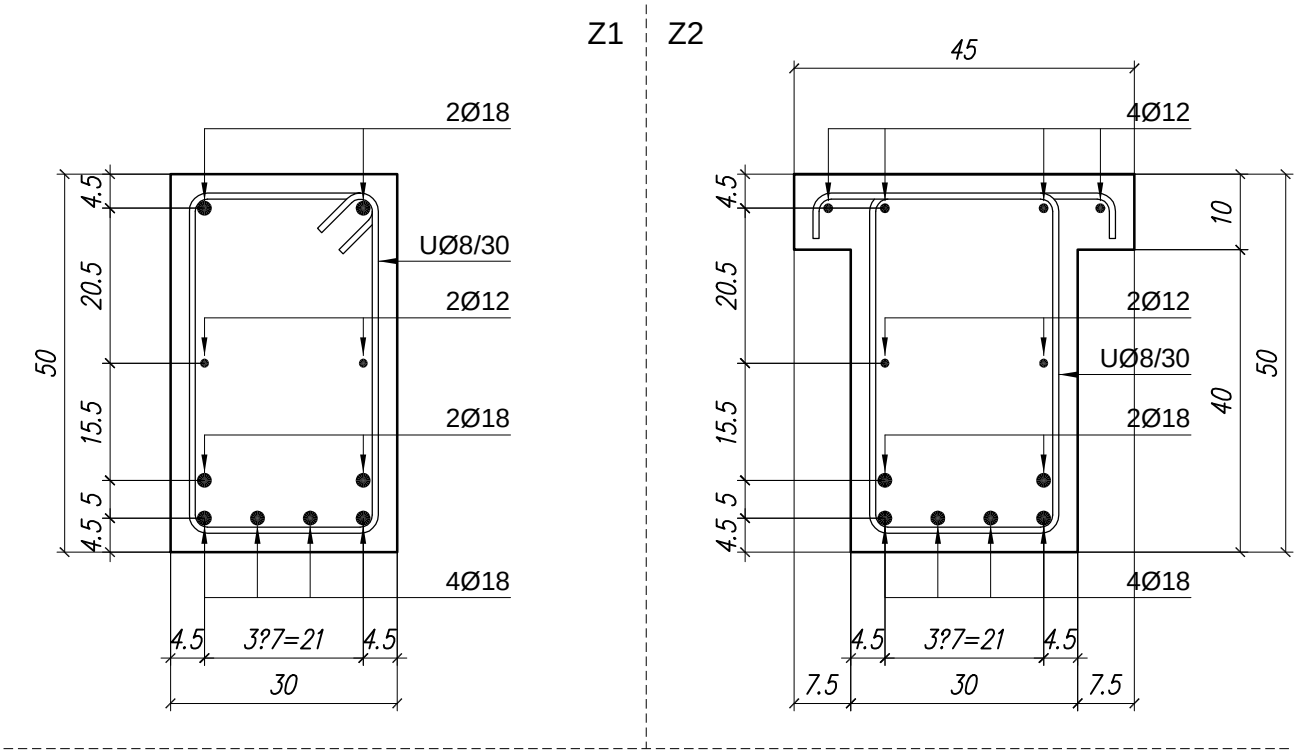
$$\varepsilon_{bd} = 0.297‰ \Rightarrow \quad \eta_2 = \frac{8 - 0.297}{4 \times (6 - 0.297)} = 0.338$$

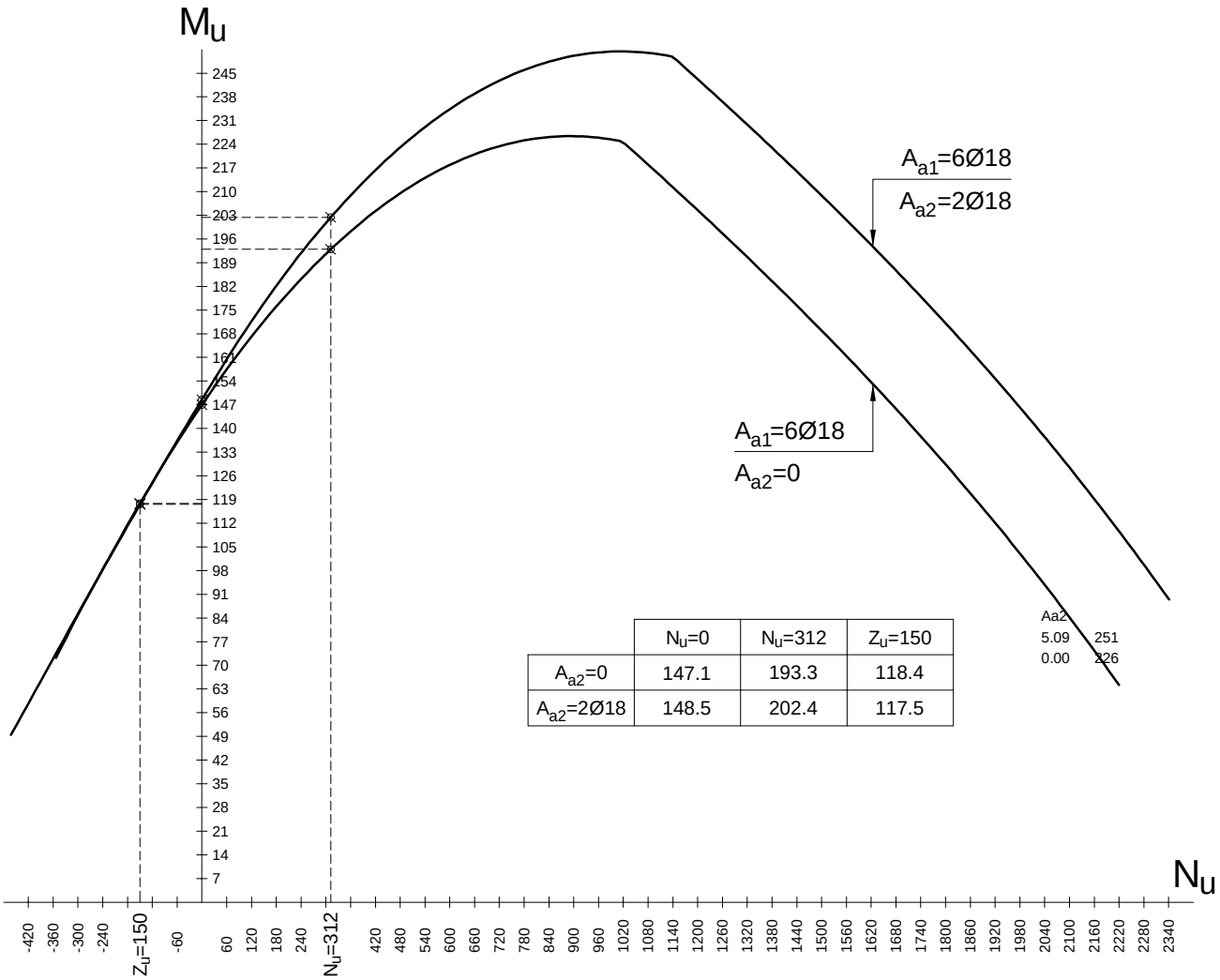
$$z_{b1} = 43.8 \times (1 - 0.413 \times 0.2504) = 39.3 \text{ cm}$$

$$z_{b2} = 43.8 \times [(1 - 0.228 - 0.338 \times (0.2504 - 0.228)] = 33.5 \text{ cm}$$

$$\Sigma M_{a1} = 0: \quad 682.0 \times 39.3 - 3.6 \times 33.5 = M_{au} = 26686 \text{ kNcm} = 266.86 \text{ kNm}$$

$$N_u = 312 \text{ kN} \Rightarrow \quad M_u = 266.86 - 312 \times \left(\frac{0.50}{2} - 0.062 \right) = 208.1 \text{ kNm}$$





	N_u=0	N_u=312	Z_u=150
A_{a2}=0	147.1	193.3	118.4
A_{a2}=2Ø18	148.5	202.4	117.5