

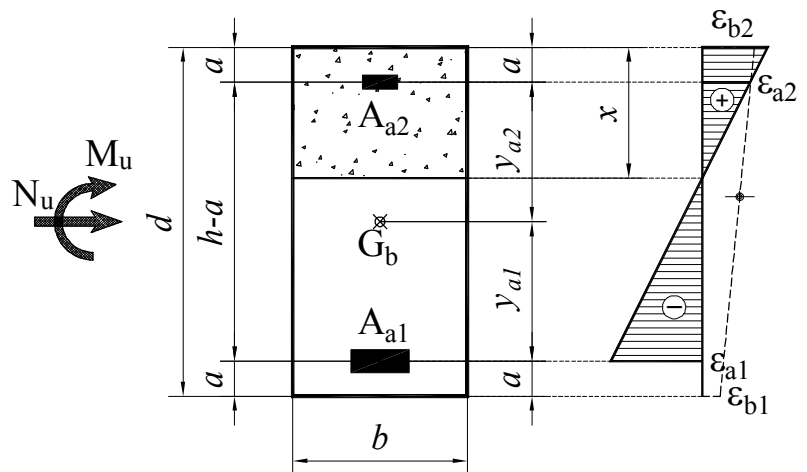
## KONSTRUKCIJA DIJAGRAMA INTERAKCIJE

Konstruisati dijagram interakcije za pravougaoni presek opterećena na pravo savijanje, varirajući mehanički koeficijent armiranja *zategnutom* armaturom u granicama 0 - 80%, sa korakom od 10%. Podaci za proračun:

$$a/d = 0.075$$

$$A_{a2}/A_{a1} = 1/3$$

$$GA\ 240/360$$



Na skici desno je prikazan poprečni presek i označene potrebne geometrijske veličine. Pri određivanju bezdimenzionih koeficijenata usvojeno je da su **pozitivni** moment savijanja koji zateže **donju** ivicu nosača i normalna sila **pritiska**. Uvode se sledeće oznake:

$$A_{a2} = k \times A_{a1} = 0.33 \times A_{a1}$$

$$a_1 = a_2 = a = \alpha \times d = 0.075 \times d$$

Dilatacija armature pri dostizanju granice tečenja (zatezanje), odnosno gnječenja (pritisk) je:

$$GA\ 240/360 \Rightarrow \varepsilon_v = \frac{\sigma_v}{E_a} = \frac{240}{210 \times 10^3} = 1.143\%$$

Površinu donje ("zategnute"), odnosno gornje ("pritisnute") armature u preseku moguće je izraziti u funkciji dimenzija preseka i mehaničkog koeficijenta armiranja.

$$A_{a1} = \mu_1 \times b \times d = \bar{\mu}_1 \times b \times d \times \frac{f_B}{\sigma_v}$$

$$A_{a2} = \mu_2 \times b \times d = \bar{\mu}_2 \times b \times d \times \frac{f_B}{\sigma_v} = k \times \bar{\mu}_1 \times b \times d \times \frac{f_B}{\sigma_v}$$

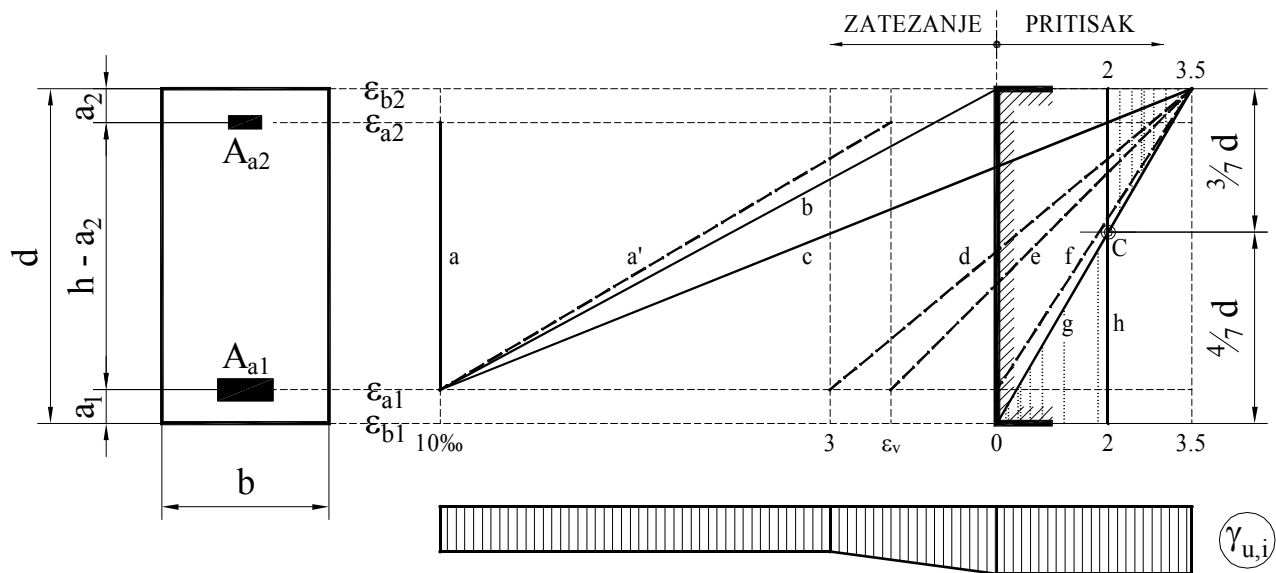
Termine "pritisnuta" i "zategnuta" armatura treba uslovno shvatiti, jer je kod ekscentričnog naprezanja u fazi malog ekscentriciteta čitav presek pritisnut, odnosno zategnut. Stoga se indeksi "1" odnose na donju (jače zategnutu) a indeksi "2" na gornju (jače pritisnutu) armaturu u preseku.

Položaj težišta armature u odnosu na težište betonskog preseka određen je kao:

$$y_{a1} = y_{a2} = \frac{d}{2} - a = (0.5 - \alpha) \times d = 0.425 \times d = y_a$$

Prilikom konstruisanja dijagrama interakcije potrebno je odrediti koje su to karakteristične vrednosti dilatacija betona i armature, koje u potpunosti opisuju karakteristična naponsko-deformacijska stanja preseka napregnutog na složeno pravo savijanje. Vrednosti za koje će tačke dijagrama biti sračunate su karakteristične linije sa dijagrama mogućih dilatacija (skica dole). Pored toga, konstrukcijom će

biti obuhvaćene i još dve karakteristične linije:  $\varepsilon_{b2}/\varepsilon_{a1} = 3.5/2\text{‰}$  i  $\varepsilon_{b2}/\varepsilon_{a1} = 3.5/1\text{‰}$ , koje treba da omoguće preciznije određivanje koeficijenata sigurnosti u oblasti u kojoj se ovi linearno menjaju.



$\varepsilon_{a1} = \varepsilon_{a2} = 10\text{‰}$  (linija "a")

Čitav presek je zategnut. Kako se nosivost betonskog dela preseka zanemaruje u proračunu, spoljašnje uticaje prihvata samo armatura. Sledi:

$$\varepsilon_{a1} > \varepsilon_v \Rightarrow \sigma_{a1} = \sigma_v \Rightarrow Z_{au1} = A_{a1} \times \sigma_{a1} = \bar{\mu}_1 \times b \times d \times \frac{f_B}{\sigma_v} \times \sigma_v = \bar{\mu}_1 \times b \times d \times f_B$$

$$\varepsilon_{a2} > \varepsilon_v \Rightarrow \sigma_{a2} = \sigma_v \Rightarrow Z_{au2} = A_{a2} \times \sigma_{a2} = \bar{\mu}_2 \times b \times d \times \frac{f_B}{\sigma_v} \times \sigma_v = k \times \bar{\mu}_1 \times b \times d \times f_B$$

Vrednosti spoljašnjih sila koje izazivaju željeno stanje dilatacija preseka slede iz uslova ravnoteže normalnih sila:

$$\Sigma N = 0: \quad N_u = -Z_{au1} - Z_{au2}$$

$$N_u = -(1+k) \times \bar{\mu}_1 \times b \times d \times f_B \quad / (b \times d \times f_B)$$

$$n_u = \frac{N_u}{b \times d \times f_B} = -(1+k) \times \bar{\mu}_1 = -1.333 \times \bar{\mu}_1$$

odnosno momenata savijanja u odnosu na težište betonskog dela preseka:

$$\Sigma M = 0: \quad M_u = (Z_{au1} - Z_{au2}) \times y_a$$

$$M_u = (1-k) \times \bar{\mu}_1 \times b \times d \times f_B \times (0.5 - \alpha) \times d \quad / (b \times d^2 \times f_B)$$

$$m_u = \frac{M_u}{b \times d^2 \times f_B} = (1-k) \times \bar{\mu}_1 \times (0.5 - \alpha) = (1-0.33) \times \bar{\mu}_1 \times (0.5 - 0.075) = 0.283 \times \bar{\mu}_1$$

Na potpuno istovetan način biće određene vrednosti bezdimenzionih veličina  $n_u$ ,  $m_u$  za sve druge karakteristične linije **b** do **h** sa gornje skice:

**$\varepsilon_{b2}/\varepsilon_{a1} = 0/10\%$  (linija "b")**

Ovo je granica između malog i velikog ekscentriciteta u zoni zatezanja (neutralna linija je na gornjoj ivici preseka). I u ovom slučaju čitav betonski deo preseka je zategnut, pa računski presek sačinjavaju donja i gornja armatura  $A_{a1}$  i  $A_{a2}$ .

Određivanje dilatacije i napona gornje armature:

$$\varepsilon_{a2} = \frac{a_2}{h} \times \varepsilon_{a1} = \frac{a}{d-a} \times \varepsilon_{a1} = \frac{\alpha}{1-\alpha} \times \varepsilon_{a1}$$

$$\varepsilon_{a2} = \frac{0.075}{1-0.075} \times 10 = 0.81\% < \varepsilon_v$$

$$\sigma_{a2} = 0.81 \times 10^{-3} \times 210 \times 10^3 = 170.3 \text{ MPa}$$

$$\frac{\sigma_{a2}}{\sigma_v} = \frac{170.3}{240} = 0.709$$

$$\sigma_{a1} = \sigma_v \Rightarrow Z_{au1} = A_{a1} \times \sigma_{a1} = \bar{\mu}_1 \times b \times d \times f_B$$

$$\sigma_{a2} < \sigma_v \Rightarrow Z_{au2} = A_{a2} \times \sigma_{a2} = k \times \bar{\mu}_1 \times b \times d \times f_B \times \frac{\sigma_{a2}}{\sigma_v}$$

$$\Sigma N = 0: \quad N_u = -Z_{au1} - Z_{au2}$$

$$N_u = - \left( 1 + k \times \frac{\sigma_{a2}}{\sigma_v} \right) \times \bar{\mu}_1 \times b \times d \times f_B \quad / (b \times d \times f_B)$$

$$n_u = - \left( 1 + k \times \frac{\sigma_{a2}}{\sigma_v} \right) \times \bar{\mu}_1 = - (1 + 0.33 \times 0.709) \times \bar{\mu}_1 = - 1.236 \times \bar{\mu}_1$$

$$\Sigma M = 0: \quad M_u = (Z_{au1} - Z_{au2}) \times y_a$$

$$M_u = \left( 1 - k \times \frac{\sigma_{a2}}{\sigma_v} \right) \times \bar{\mu}_1 \times b \times d \times f_B \times (0.5 - \alpha) \times d \quad / (b \times d^2 \times f_B)$$

$$m_u = \left( 1 - k \times \frac{\sigma_{a2}}{\sigma_v} \right) \times \bar{\mu}_1 \times (0.5 - \alpha) = (1 - 0.33 \times 0.709) \times \bar{\mu}_1 \times (0.5 - 0.075) = 0.324 \times \bar{\mu}_1$$

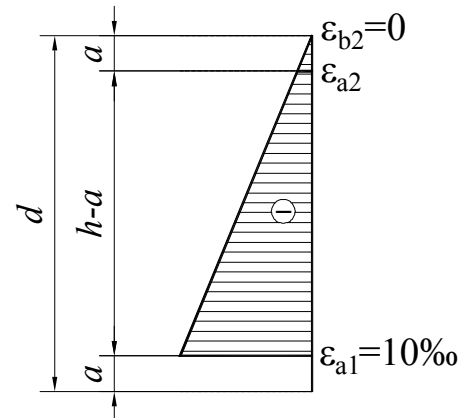
Linije "a" i "b" se na dijagramu interakcije poklapaju ukoliko je dostignuta granica razvlačenja u gornjoj armaturi, dakle, zavisno od njenog položaja u preseku, ukoliko je zadovoljena relacija:

$$\varepsilon_{a2} = \frac{\alpha}{1-\alpha} \times \varepsilon_{a1} \Rightarrow \alpha \geq \frac{\varepsilon_v}{\varepsilon_{a1} + \varepsilon_v}$$

Dakle, u slučaju glatke, odnosno rebraste armature, linije "a" i "b" će se poklopiti za:

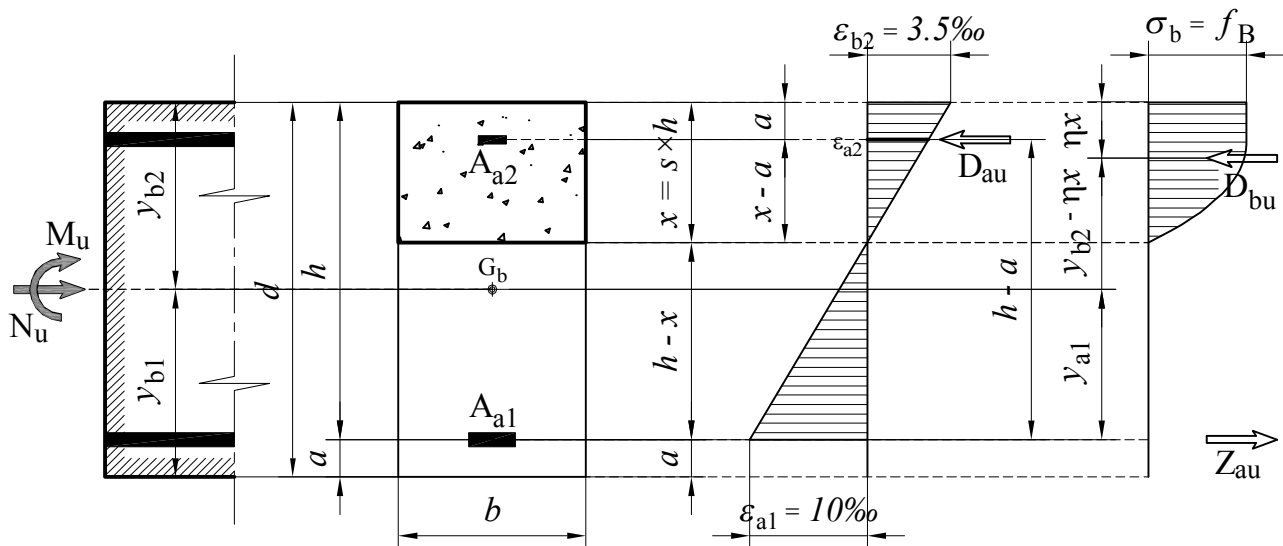
$$\text{GA 240/360: } \varepsilon_v = 1.143\% \Rightarrow \alpha \geq 1.143 / (10 + 1.143) = 0.103$$

$$\text{RA 400/500: } \varepsilon_v = 1.905\% \Rightarrow \alpha \geq 1.905 / (10 + 1.905) = 0.160$$



$\epsilon_{b2}/\epsilon_{a1} = 3.5/10\%$  (linija "c")

Presek je napregnut na složeno savijanje u fazi velikog ekscentriciteta - neutralna linija je u poprečnom preseku. Odgovarajući proračunski model prikazan je na donjoj skici.



Položaj neutralne linije u preseku određen je bezdimenzionom veličinom  $s$ :

$$s = \frac{\epsilon_{b2}}{\epsilon_{b2} + \epsilon_{a1}} = \frac{3.5}{3.5 + 10} = 0.259 \Rightarrow x = s \times h = s \times (1 - \alpha) \times d$$

Koeficijent punoće naponskog dijagrama betona  $\alpha_b$  je funkcija dilatacije betona  $\epsilon_{b2}$  i može se očitati iz tabele za dimenzionisanje pravougaonih preseka ili odrediti iz analitičkog izraza:

$$\alpha_b = \frac{3 \times \epsilon_{b2} - 2}{3 \times \epsilon_{b2}} = \frac{3 \times 3.5 - 2}{3 \times 3.5} = 0.810$$

Sila pritiska u betonu određena je izrazom:

$$D_{bu} = \alpha_b \times b \times x \times f_B = \alpha_b \times b \times s \times (1 - \alpha) \times d \times f_B$$

$$D_{bu} = 0.810 \times 0.259 \times (1 - 0.075) \times b \times d \times f_B = 0.194 \times b \times d \times f_B$$

Sa dijagrama dilatacija je očito da je dilatacija pritisnute armature:

$$\epsilon_{a2} = \frac{x - a}{x} \times \epsilon_{b2} = \frac{s \times (1 - \alpha) - \alpha}{s \times (1 - \alpha)} \times \epsilon_{b2} = \frac{0.259 \times (1 - 0.075) - 0.075}{0.259 \times (1 - 0.075)} \times 3.5 = 2.405\% > \epsilon_v$$

Kako su i gornja i donja armatura ušle u prag tečenja, sledi da su odgovarajuće sile:

$$\epsilon_{a2} > \epsilon_v \Rightarrow \sigma_{a2} = \sigma_v \Rightarrow D_{au} = A_{a2} \times \sigma_{a2} = k \times \bar{\mu}_1 \times b \times d \times f_B$$

$$\epsilon_{a1} > \epsilon_v \Rightarrow \sigma_{a1} = \sigma_v \Rightarrow Z_{au} = A_{a1} \times \sigma_{a1} = \bar{\mu}_1 \times b \times d \times f_B$$

$$\Sigma N = 0: \quad N_u = D_{bu} + D_{au} - Z_{au}$$

$$N_u = \left\{ \alpha_b \times s \times (1 - \alpha) + \left[ k \times \frac{\sigma_{a2}}{\sigma_v} - \frac{\sigma_{a1}}{\sigma_v} \right] \times \bar{\mu}_1 \right\} \times b \times d \times f_B \quad / (b \times d \times f_B)$$

$$n_u = 0.194 + (0.33 \times 1 - 1) \times \bar{\mu}_1 = \mathbf{0.194 - 0.67 \times \bar{\mu}_1}$$

Položaj sile pritiska u betonu u odnosu na gornju ivicu preseka određen je koeficijentom  $\eta$ , koji se može očitati iz tabele za dimenzionisanje pravougaonih preseka ili odrediti iz analitičkog izraza:

$$\eta = \frac{3 \times \varepsilon_{b2}^2 - 4 \times \varepsilon_{b2} + 2}{2 \times \varepsilon_{b2} \times (3 \times \varepsilon_{b2} - 2)} = \frac{3 \times 3.5^2 - 4 \times 3.5 + 2}{2 \times 3.5 \times (3 \times 3.5 - 2)} = 0.416$$

pa je krak sile pritiska  $D_{bu}$  u odnosu na težište betonskog preseka određen izrazom:

$$z_b = y_{b2} - \eta \times x = [0.5 - \eta \times s \times (1 - \alpha)] \times d = [0.5 - 0.416 \times 0.259 \times (1 - 0.075)] \times d = 0.400 \times d$$

Uslov ravnoteže momenata savijanja u odnosu na težište betonskog dela preseka:

$$\Sigma M = 0: \quad M_u = D_{bu} \times z_b + (D_{au} + Z_{au}) \times y_a$$

$$M_u = \alpha_b \times s \times (1 - \alpha) \times b \times d \times f_B \times [0.5 - \eta \times s \times (1 - \alpha)] \times d + \\ + \left( k \times \frac{\sigma_{a2}}{\sigma_v} + \frac{\sigma_{a1}}{\sigma_v} \right) \times \bar{\mu}_1 \times b \times d \times f_B \times (0.5 - \alpha) \times d \quad / (b \times d^2 \times f_B)$$

$$m_u = \alpha_b \times s \times (1 - \alpha) \times [0.5 - \eta \times s \times (1 - \alpha)] + \left( k \times \frac{\sigma_{a2}}{\sigma_v} + \frac{\sigma_{a1}}{\sigma_v} \right) \times \bar{\mu}_1 \times (0.5 - \alpha)$$

$$m_u = 0.810 \times 0.259 \times (1 - 0.075) \times [0.5 - 0.416 \times 0.259 \times (1 - 0.075)] + \\ + (0.33 \times 1 + 1) \times \bar{\mu}_1 \times (0.5 - 0.075)$$

$$m_u = 0.194 \times 0.400 + (0.33 \times 1 + 1) \times 0.425 \times \bar{\mu}_1 = \mathbf{0.078 + 0.567 \times \bar{\mu}_1}$$

### $\varepsilon_{b2}/\varepsilon_{a1} = 3.5/3\text{‰}$ (linija "d")

Proračunski model i postupak su potpuno isti kao za liniju "c", pa se zamenom odgovarajućih numeričkih vrednosti dobija:

$$s = \frac{3.5}{3.5 + 3} = 0.538$$

$$\varepsilon_{b2} = 3.5\text{‰} \Rightarrow \alpha_b = 0.810$$

$$\varepsilon_{a2} = \frac{x - a}{x} \times \varepsilon_{b2} = \frac{s \times (1 - \alpha) - \alpha}{s \times (1 - \alpha)} \times \varepsilon_{b2} = \frac{0.538 \times (1 - 0.075) - 0.075}{0.538 \times (1 - 0.075)} \times 3.5 = 2.973\text{‰} > \varepsilon_v$$

$$\varepsilon_{a2} > \varepsilon_v \Rightarrow \sigma_{a2} = \sigma_v$$

$$\varepsilon_{a1} > \varepsilon_v \Rightarrow \sigma_{a1} = \sigma_v$$

$$n_u = \alpha_b \times s \times (1 - \alpha) + \left[ k \times \frac{\sigma_{a2}}{\sigma_v} - \frac{\sigma_{a1}}{\sigma_v} \right] \times \bar{\mu}_1 = 0.810 \times 0.538 \times (1 - 0.075) + [0.33 \times 1 - 1] \times \bar{\mu}_1$$

$$n_u = \mathbf{0.403 - 0.67 \times \bar{\mu}_1}$$

$$\varepsilon_{b2} = 3.5\text{‰} \Rightarrow \eta = 0.416$$

$$m_u = \alpha_b \times s \times (1 - \alpha) \times [0.5 - \eta \times s \times (1 - \alpha)] + \left( k \times \frac{\sigma_{a2}}{\sigma_v} + \frac{\sigma_{a1}}{\sigma_v} \right) \times \bar{\mu}_1 \times (0.5 - \alpha)$$

$$m_u = 0.810 \times 0.538 \times (1 - 0.075) \times [0.5 - 0.416 \times 0.538 \times (1 - 0.075)] + \\ + (0.33 \times 1 + 1) \times \bar{\mu}_1 \times (0.5 - 0.075)$$

$$\mathbf{m_u = 0.403 \times 0.293 + (0.33 \times 1 + 1) \times 0.425 \times \bar{\mu}_1 = 0.118 + 0.567 \times \bar{\mu}_1}$$

$$\varepsilon_{b2}/\varepsilon_{a1} = 3.5/2\text{‰}$$

$$s = \frac{3.5}{3.5 + 2} = 0.636$$

$$\varepsilon_{b2} = 3.5\text{‰} \Rightarrow \alpha_b = 0.810$$

$$\varepsilon_{a2} = \frac{x - a}{x} \times \varepsilon_{b2} = \frac{s \times (1 - \alpha) - \alpha}{s \times (1 - \alpha)} \times \varepsilon_{b2} = \frac{0.636 \times (1 - 0.075) - 0.075}{0.636 \times (1 - 0.075)} \times 3.5 = 3.054\text{‰} > \varepsilon_v$$

$$\varepsilon_{a2} > \varepsilon_v \Rightarrow \sigma_{a2} = \sigma_v$$

$$\varepsilon_{a1} = 2\text{‰} > \varepsilon_v \Rightarrow \sigma_{a1} = \sigma_v$$

$$n_u = \alpha_b \times s \times (1 - \alpha) + \left[ k \times \frac{\sigma_{a2}}{\sigma_v} - \frac{\sigma_{a1}}{\sigma_v} \right] \times \bar{\mu}_1 = 0.810 \times 0.636 \times (1 - 0.075) + [0.33 \times 1 - 1] \times \bar{\mu}_1$$

$$\mathbf{n_u = 0.477 - 0.67 \times \bar{\mu}_1}$$

$$\varepsilon_{b2} = 3.5\text{‰} \Rightarrow \eta = 0.416$$

$$m_u = \alpha_b \times s \times (1 - \alpha) \times [0.5 - \eta \times s \times (1 - \alpha)] + \left( k \times \frac{\sigma_{a2}}{\sigma_v} + \frac{\sigma_{a1}}{\sigma_v} \right) \times \bar{\mu}_1 \times (0.5 - \alpha)$$

$$m_u = 0.810 \times 0.636 \times (1 - 0.075) \times [0.5 - 0.416 \times 0.636 \times (1 - 0.075)] + \\ + (0.33 \times 1 + 1) \times \bar{\mu}_1 \times (0.5 - 0.075)$$

$$\mathbf{m_u = 0.477 \times 0.255 + (0.33 \times 1 + 1) \times 0.425 \times \bar{\mu}_1 = 0.122 + 0.567 \times \bar{\mu}_1}$$

$$\varepsilon_{b2}/\varepsilon_{a1} = 3.5/\varepsilon_v \text{ (linija "e")}$$

$$s = \frac{3.5}{3.5 + 1.143} = 0.754 \quad ; \quad \varepsilon_{b2} = 3.5\text{‰} \Rightarrow \alpha_b = 0.810$$

$$\varepsilon_{a2} = \frac{x - a}{x} \times \varepsilon_{b2} = \frac{s \times (1 - \alpha) - \alpha}{s \times (1 - \alpha)} \times \varepsilon_{b2} = \frac{0.754 \times (1 - 0.075) - 0.075}{0.754 \times (1 - 0.075)} \times 3.5 = 3.124\text{‰} > \varepsilon_v$$

$$\varepsilon_{a2} > \varepsilon_v \quad \Rightarrow \quad \sigma_{a2} = \sigma_v$$

$$\varepsilon_{a1} = \varepsilon_v \quad \Rightarrow \quad \sigma_{a1} = \sigma_v$$

$$n_u = \alpha_b \times s \times (1 - \alpha) + \left[ k \times \frac{\sigma_{a2}}{\sigma_v} - \frac{\sigma_{a1}}{\sigma_v} \right] \times \bar{\mu}_1 = 0.810 \times 0.754 \times (1 - 0.075) + [0.33 \times 1 - 1] \times \bar{\mu}_1$$

$$n_u = 0.564 - 0.67 \times \bar{\mu}_1$$

$$\varepsilon_{b2} = 3.5\text{‰} \Rightarrow \eta = 0.416$$

$$m_u = \alpha_b \times s \times (1 - \alpha) \times [0.5 - \eta \times s \times (1 - \alpha)] + \left( k \times \frac{\sigma_{a2}}{\sigma_v} + \frac{\sigma_{a1}}{\sigma_v} \right) \times \bar{\mu}_1 \times (0.5 - \alpha)$$

$$m_u = 0.810 \times 0.754 \times (1 - 0.075) \times [0.5 - 0.416 \times 0.754 \times (1 - 0.075)] + (0.33 \times 1 + 1) \times \bar{\mu}_1 \times (0.5 - 0.075)$$

$$m_u = 0.564 \times 0.210 + (0.33 \times 1 + 1) \times 0.425 \times \bar{\mu}_1 = 0.119 + 0.567 \times \bar{\mu}_1$$

$$\varepsilon_{b2}/\varepsilon_{a1} = 3.5/1\text{‰}$$

$$s = \frac{3.5}{3.5 + 1} = 0.778 \quad ; \quad \varepsilon_{b2} = 3.5\text{‰} \Rightarrow \alpha_b = 0.810$$

$$\varepsilon_{a2} = \frac{x - a}{x} \times \varepsilon_{b2} = \frac{s \times (1 - \alpha) - \alpha}{s \times (1 - \alpha)} \times \varepsilon_{b2} = \frac{0.778 \times (1 - 0.075) - 0.075}{0.778 \times (1 - 0.075)} \times 3.5 = 3.135\text{‰} > \varepsilon_v$$

$$\varepsilon_{a2} > \varepsilon_v \quad \Rightarrow \quad \sigma_{a2} = \sigma_v$$

$$\varepsilon_{a1} = 1\text{‰} \quad \Rightarrow \quad \sigma_{a1} = 1 \times 10^{-3} \times 210 \times 10^3 = 210 \text{ MPa} \Rightarrow \frac{\sigma_{a1}}{\sigma_v} = \frac{210}{240} = 0.875$$

$$n_u = \alpha_b \times s \times (1 - \alpha) + \left[ k \times \frac{\sigma_{a2}}{\sigma_v} - \frac{\sigma_{a1}}{\sigma_v} \right] \times \bar{\mu}_1 = 0.810 \times 0.778 \times (1 - 0.075) + [0.33 \times 1 - 0.875] \times \bar{\mu}_1$$

$$n_u = 0.582 - 0.542 \times \bar{\mu}_1$$

$$\varepsilon_{b2} = 3.5\text{‰} \Rightarrow \eta = 0.416$$

$$m_u = \alpha_b \times s \times (1 - \alpha) \times [0.5 - \eta \times s \times (1 - \alpha)] + \left( k \times \frac{\sigma_{a2}}{\sigma_v} + \frac{\sigma_{a1}}{\sigma_v} \right) \times \bar{\mu}_1 \times (0.5 - \alpha)$$

$$m_u = 0.810 \times 0.778 \times (1 - 0.075) \times [0.5 - 0.416 \times 0.778 \times (1 - 0.075)] + (0.33 \times 1 + 0.875) \times \bar{\mu}_1 \times (0.5 - 0.075)$$

$$m_u = 0.582 \times 0.201 + (0.33 \times 1 + 0.875) \times 0.425 \times \bar{\mu}_1 = 0.117 + 0.514 \times \bar{\mu}_1$$

**$\varepsilon_{b2}/\varepsilon_{a1} = 3.5/0\%$  (linija "f")**

$$s = \frac{3.5}{3.5+0} = 1 \quad ; \quad \varepsilon_{b2} = 3.5\text{‰} \Rightarrow \alpha_b = 0.810$$

$$\varepsilon_{a2} = \frac{x-a}{x} \times \varepsilon_{b2} = \frac{s \times (1-\alpha) - \alpha}{s \times (1-\alpha)} \times \varepsilon_{b2} = \frac{1 \times (1-0.075) - 0.075}{1 \times (1-0.075)} \times 3.5 = 3.216\text{‰} > \varepsilon_v$$

$$\varepsilon_{a2} > \varepsilon_v \quad \Rightarrow \quad \sigma_{a2} = \sigma_v$$

$$\varepsilon_{a1} = 0\text{‰} \quad \Rightarrow \quad \sigma_{a1} = 0$$

$$n_u = \alpha_b \times s \times (1-\alpha) + \left[ k \times \frac{\sigma_{a2}}{\sigma_v} - \frac{\sigma_{a1}}{\sigma_v} \right] \times \bar{\mu}_1 = 0.810 \times 1 \times (1-0.075) + [0.33 \times 1 - 0] \times \bar{\mu}_1$$

$$n_u = 0.749 + 0.33 \times \bar{\mu}_1$$

$$\varepsilon_{b2} = 3.5\text{‰} \Rightarrow \eta = 0.416$$

$$m_u = \alpha_b \times s \times (1-\alpha) \times [0.5 - \eta \times s \times (1-\alpha)] + \left( k \times \frac{\sigma_{a2}}{\sigma_v} + \frac{\sigma_{a1}}{\sigma_v} \right) \times \bar{\mu}_1 \times (0.5 - \alpha)$$

$$m_u = 0.810 \times 1 \times (1-0.075) \times [0.5 - 0.416 \times 1 \times (1-0.075)] + (0.33 \times 1 + 0) \times \bar{\mu}_1 \times (0.5 - 0.075)$$

$$m_u = 0.749 \times 0.115 + (0.33 \times 1 + 0) \times 0.425 \times \bar{\mu}_1 = 0.086 + 0.142 \times \bar{\mu}_1$$

 **$\varepsilon_{b2}/\varepsilon_{b1} = 3.5/0\%$  (linija "g")**

Ovo je granica između malog i velikog ekscentriciteta u zoni pritiska (neutralna linija je na donjoj ivici preseka). U ovom slučaju čitav presek je pritisnut, pa sledi:

$$s = \frac{x}{h} = \frac{d}{h} = \frac{d}{d-a} = \frac{1}{1-\alpha} = \frac{1}{1-0.075} = 1.081$$

$$\varepsilon_{b2} = 3.5\text{‰} \Rightarrow \alpha_b = 0.810$$

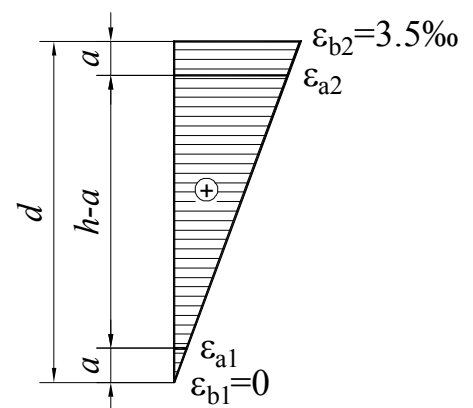
$$\varepsilon_{a2} = \frac{h}{d} \times \varepsilon_{b2} = \frac{d-a}{d} \times \varepsilon_{b2} = (1-\alpha) \times \varepsilon_{b2}$$

$$\varepsilon_{a2} = (1-0.075) \times 3.5 = 3.238\text{‰} > \varepsilon_v$$

$$\varepsilon_{a2} > \varepsilon_v \quad \Rightarrow \quad \sigma_{a2} = \sigma_v$$

$$\varepsilon_{a1} = \frac{a}{d} \times \varepsilon_{b2} = \alpha \times \varepsilon_{b2} = 0.075 \times 3.5 = 0.263\text{‰} \text{ (pritisak)}$$

$$\varepsilon_{a1} = 0.263\text{‰} \Rightarrow \sigma_{a1} = 0.263 \times 10^{-3} \times 210 \times 10^3 = 55.12 \text{ MPa} \Rightarrow \frac{\sigma_{a1}}{\sigma_v} = \frac{55.12}{240} = 0.23$$





S obzirom da su sve unutrašnje sile pritisci, uslov ravnoteže normalnih sila je oblika:

$$\Sigma N = 0: \quad N_u = D_{bu} + D_{au2} + D_{au1}$$

$$N_u = \left\{ \alpha_b \times s \times (1 - \alpha) + \left[ k \times \frac{\sigma_{a2}}{\sigma_v} + \frac{\sigma_{a1}}{\sigma_v} \right] \times \bar{\mu}_1 \right\} \times b \times d \times f_B \quad / (b \times d \times f_B)$$

$$n_u = \alpha_b \times s \times (1 - \alpha) + \left[ k \times \frac{\sigma_{a2}}{\sigma_v} + \frac{\sigma_{a1}}{\sigma_v} \right] \times \bar{\mu}_1$$

$$n_u = 0.810 \times 1.081 \times (1 - 0.075) + [0.33 \times 1 + 0.23] \times \bar{\mu}_1 = \mathbf{0.810 + 0.563 \times \bar{\mu}_1}$$

$$\varepsilon_{b2} = 3.5\text{‰} \Rightarrow \eta = 0.416$$

S obzirom na znak sile u donjoj armaturi, u ovom slučaju uslov ravnoteže momenata savijanja u odnosu na težište betonskog dela preseka je oblika:

$$\Sigma M = 0: \quad M_u = D_{bu} \times z_b + (D_{au2} - D_{au1}) \times y_a$$

$$M_u = \alpha_b \times s \times (1 - \alpha) \times b \times d \times f_B \times [0.5 - \eta \times s \times (1 - \alpha)] \times d +$$

$$+ \left( k \times \frac{\sigma_{a2}}{\sigma_v} - \frac{\sigma_{a1}}{\sigma_v} \right) \times \bar{\mu}_1 \times b \times d \times f_B \times (0.5 - \alpha) \times d \quad / (b \times d^2 \times f_B)$$

$$m_u = \alpha_b \times s \times (1 - \alpha) \times [0.5 - \eta \times s \times (1 - \alpha)] + \left( k \times \frac{\sigma_{a2}}{\sigma_v} - \frac{\sigma_{a1}}{\sigma_v} \right) \times \bar{\mu}_1 \times (0.5 - \alpha)$$

$$m_u = 0.810 \times 1.081 \times (1 - 0.075) \times [0.5 - 0.416 \times 1 \times (1 - 0.075)] +$$

$$+ (0.33 \times 1 - 0.23) \times \bar{\mu}_1 \times (0.5 - 0.075)$$

$$m_u = 0.810 \times 0.084 + (0.33 \times 1 - 0.23) \times 0.425 \times \bar{\mu}_1 = \mathbf{0.068 + 0.044 \times \bar{\mu}_1}$$

### $\varepsilon_{b2}/\varepsilon_{b1} = 2/2\text{‰}$ (linija "h")

Dilatacija čitavog preseka je konstantna i iznosi 2‰ (pritisk). U specijalnom slučaju simetrično armiranog preseka, radi se o centričnom pritisku.

$$\varepsilon_{b1} = \varepsilon_{b2} = 2\text{‰} \Rightarrow \sigma_b = f_B = \text{const.}$$

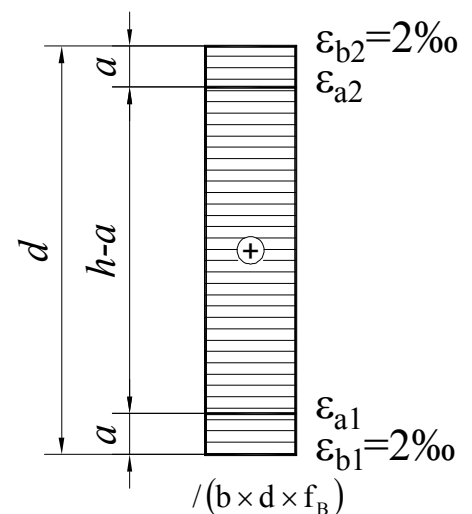
$$\varepsilon_{a2} = \varepsilon_{a1} = 2\text{‰} > \varepsilon_v \Rightarrow \sigma_{a2} = \sigma_{a1} = \sigma_q = |\sigma_v|$$

$$D_{bu} = A_b \times f_B = b \times d \times f_B$$

Uslov ravnoteže normalnih sila je oblika:

$$\Sigma N = 0: \quad N_u = D_{bu} + D_{au2} + D_{au1}$$

$$N_u = [1 + (k + 1) \times \bar{\mu}_1] \times b \times d \times f_B$$



$$n_u = 1 + (k + 1) \times \bar{\mu}_1$$

$$n_u = 1 + (0.33 + 1) \times \bar{\mu}_1 = 1 + 1.33 \times \bar{\mu}_1$$

Uslov ravnoteže momenata savijanja u odnosu na težište betonskog dela preseka je oblika:

$$\Sigma M = 0: \quad M_u = (D_{au2} - D_{au1}) \times y_a$$

$$M_u = (k - 1) \times \bar{\mu}_1 \times b \times d \times f_B \times (0.5 - \alpha) \times d \quad / (b \times d^2 \times f_B)$$

$$m_u = (k - 1) \times \bar{\mu}_1 \times (0.5 - \alpha)$$

$$m_u = (0.33 - 1) \times \bar{\mu}_1 \times (0.5 - 0.075)$$

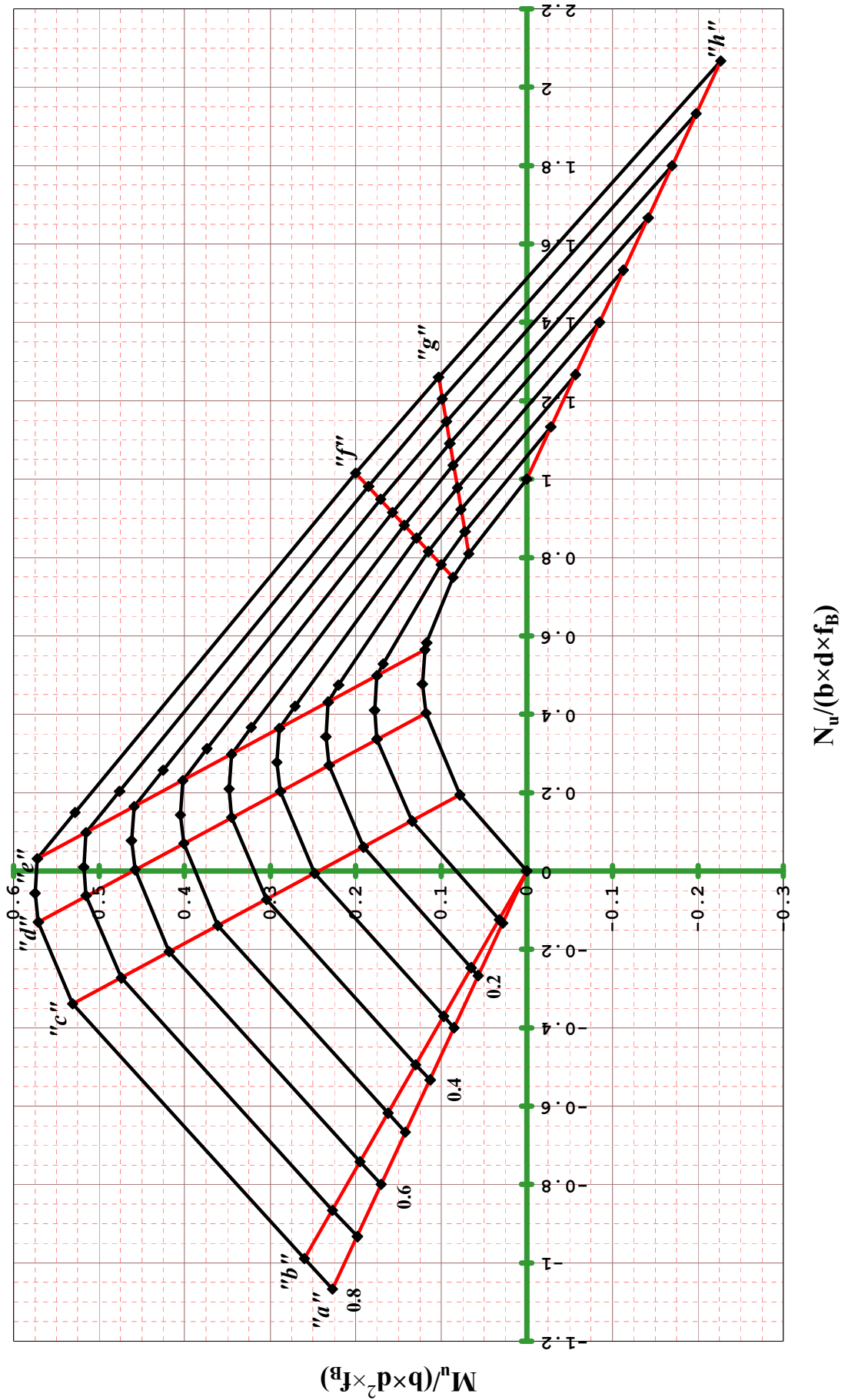
$$m_u = -0.67 \times 0.425 \times \bar{\mu}_1 = -0.283 \times \bar{\mu}_1$$

Tako su sračunate vrednosti koeficijenta  $m_u$  i  $n_u$  za karakteristične linije ("a" do "h") sa dijagrama mogućih dilatacija preseka. Variranjem mehaničkog koeficijenta armiranja u zadatim granicama dobijene su tačke, čijim spajanjem je konstruisan dijagram prikazan na narednoj skici.

LINIJA	$\epsilon_{b2}$	$\epsilon_{a1}$	$\epsilon_{b1}$	$n_u = A + B \times \mu_{1M}$		$m_u = C + D \times \mu_{1M}$	
				A	B	C	D
"a"	-10	-10		0	-1.333	0	0.283
"b"	0	-10		0	-1.236	0	0.324
"c"	3.5	-10		0.194	-0.667	0.078	0.567
"d"	3.5	-3		0.403	-0.667	0.118	0.567
	3.5	-2		0.477	-0.667	0.122	0.567
"e"	3.5	-1.143		0.564	-0.667	0.119	0.567
	3.5	-1		0.582	-0.542	0.117	0.514
"f"	3.5	0		0.749	0.333	0.086	0.142
"g"	3.5		0	0.81	0.563	0.068	0.044
"h"	2		2	1	1.333	0	-0.283

$\mu_{1M}$	0		0.1		0.2		0.3		0.4		0.5		0.6		0.7		0.8	
	$n_u$	$m_u$	$n_u$	$m_u$	$n_u$	$m_u$	$n_u$	$m_u$	$n_u$	$m_u$	$n_u$	$m_u$	$n_u$	$m_u$	$n_u$	$m_u$	$n_u$	$m_u$
"a"	0	0	-0.133	0.028	-0.267	0.057	-0.4	0.085	-0.533	0.113	-0.667	0.142	-0.8	0.17	-0.933	0.198	-1.067	0.227
"b"	0	0	-0.124	0.032	-0.247	0.065	-0.371	0.097	-0.495	0.13	-0.618	0.162	-0.742	0.195	-0.866	0.227	-0.989	0.26
"c"	0.194	0.078	0.127	0.134	0.061	0.191	-0.006	0.248	-0.073	0.304	-0.139	0.361	-0.206	0.418	-0.273	0.474	-0.339	0.531
"d"	0.403	0.118	0.337	0.175	0.27	0.231	0.203	0.288	0.137	0.345	0.07	0.401	0.003	0.458	-0.063	0.515	-0.13	0.571
	0.477	0.122	0.41	0.178	0.343	0.235	0.277	0.292	0.21	0.348	0.143	0.405	0.077	0.462	0.01	0.518	-0.057	0.575
"e"	0.564	0.119	0.498	0.175	0.431	0.232	0.364	0.289	0.298	0.345	0.231	0.402	0.164	0.459	0.098	0.515	0.031	0.572
	0.582	0.117	0.528	0.168	0.474	0.22	0.42	0.271	0.366	0.322	0.312	0.374	0.257	0.425	0.203	0.476	0.149	0.528
"f"	0.749	0.086	0.782	0.1	0.815	0.115	0.849	0.129	0.882	0.143	0.915	0.157	0.949	0.171	0.982	0.185	1.015	0.2
"g"	0.81	0.068	0.866	0.072	0.922	0.077	0.978	0.081	1.035	0.086	1.091	0.09	1.147	0.094	1.204	0.099	1.26	0.103
"h"	1	0	1.133	-0.028	1.267	-0.057	1.4	-0.085	1.533	-0.113	1.667	-0.142	1.8	-0.17	1.933	-0.198	2.067	-0.227

Poređenja radi, prikazan je i odgovarajući dijagram interakcije iz zbirke dijagrama autora Najdanovića, Alendara i Ješića, konstruisan pomoću znatno većeg broja tačaka. Oblik dijagrama je istovetan, ali se napominje da jedan kao rezultat daje površinu ZATEGNUTE, a drugi UKUPNE armature u preseku.



203. Dijagram za  
dimenzionisanje  $M_{xu}, N_u$   
GF - INK

$6_v = 24.0 \text{ KN/cm}^2$   
 $\bar{\mu}_{\max} = 0.4$

$\frac{m_y}{m_x} = \frac{M_y/b}{M_x/d} = 0.0$

$\frac{a}{d} = 0.075$

