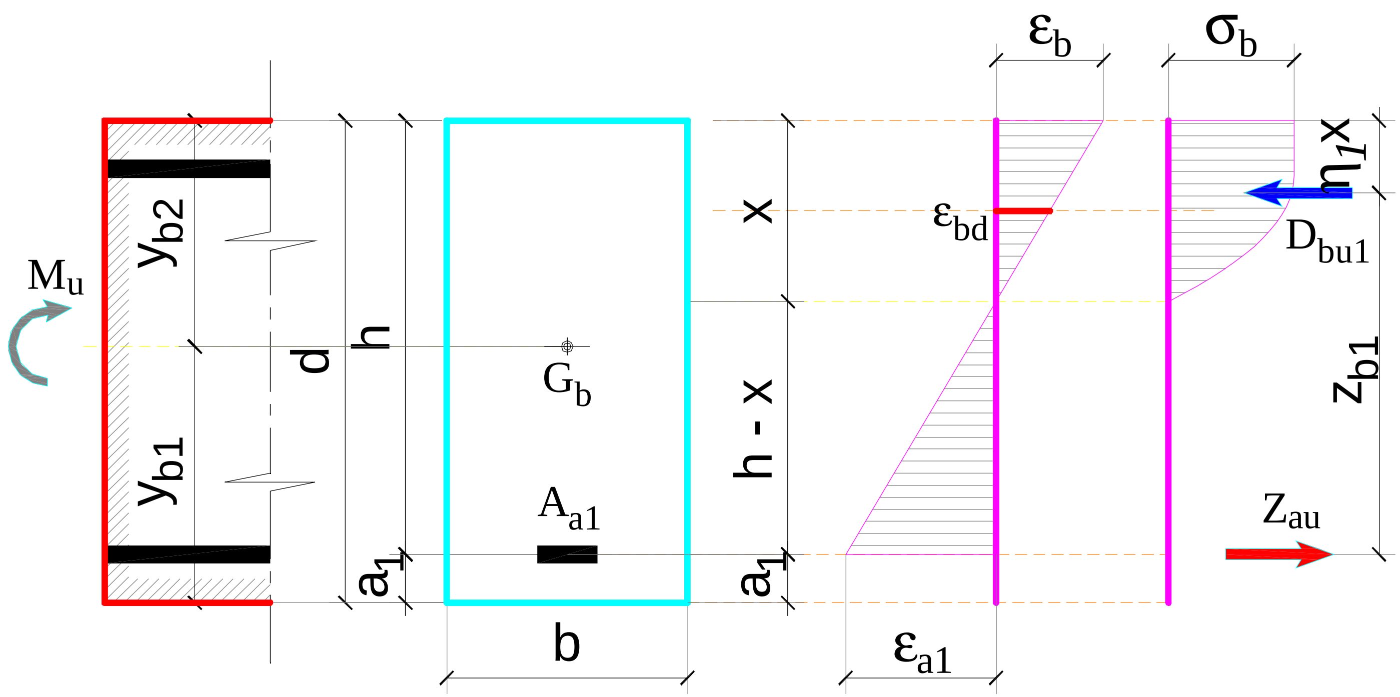
PRORAČUN PRESEKA ZA GRANIČNE UTICAJE TRANSVERZALNIH SILA

- **PRORAČUN** PREMA **TEORIJI GRANIČNIH STANJA**

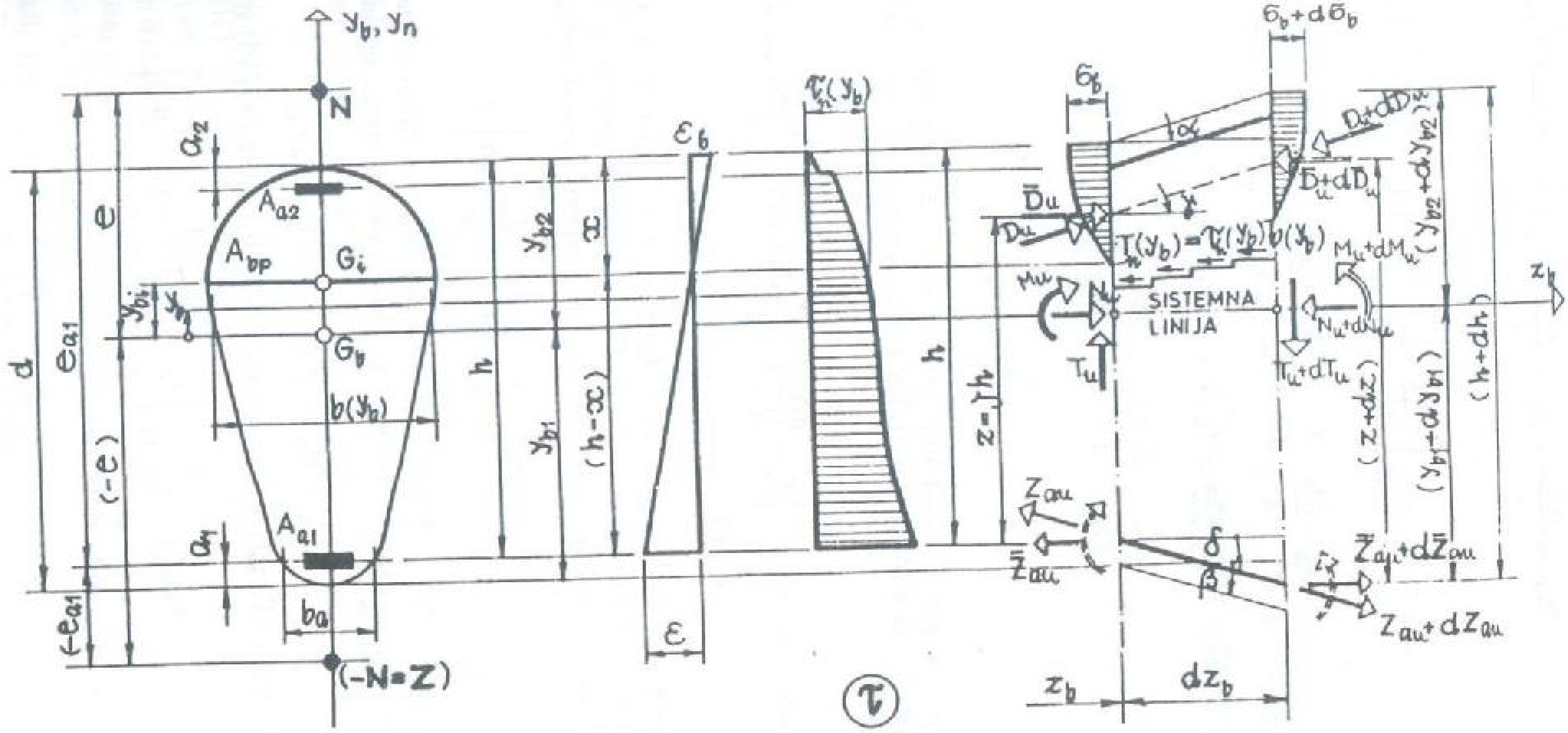
- **SAVIJANJE** AB PRESEKA POPREČNIM SILAMA
  → KONTROLA **GLAVNIH NAPONA ZATEZANJA**

$$\sigma_{1,2} = \frac{\sigma_b}{2} \pm \sqrt{\frac{\sigma_b{}^2}{4} + \tau^2}$$



$$\sigma_{1,2} = \frac{\sigma_b}{2} \pm \sqrt{\frac{\sigma_b{}^2}{4} + \tau^2} \longrightarrow \sigma_{1,2} = \pm\tau$$

$$-\sigma_{2,max} = \tau_{max} = \frac{T}{b_{min} \cdot z} \qquad -\sigma_{2,max} = \tau_{max} = \frac{T \cdot S_i}{b_{min} \cdot I_i}$$



Slika 87/1 Potrebni podaci za određivanje smičućih napona $\tau(y_b)$ armiranobetonskog preseka sa prslinom - promenljiva visina elementa nosača

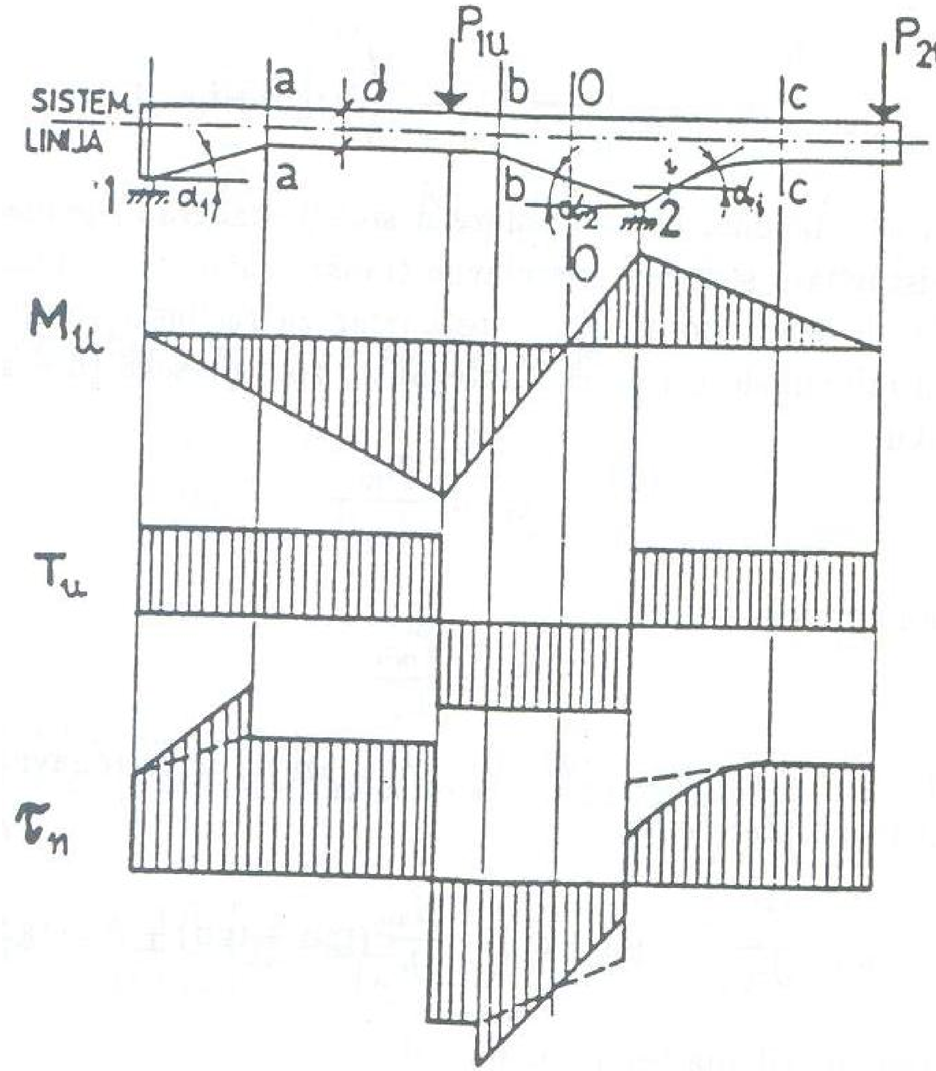
$$T_{mu} = T_u \quad \mathbf{m} \quad \frac{M_{au}}{h} \times (tg\alpha + tg\beta) \; + \; N_u \times tg\beta \; - \; \frac{dN_u}{dz_b} \times (z - y_{b1} + a_1)$$

$$M_{au} = M_u + N_u \times (y_{b1} - a_1) = M_u + N_u \times y_{a1}$$

$$y_{a1} = y_{b1} - a_1$$

$$T_{mu} = T_u \quad \mathbf{m} \quad \frac{M_u}{h} \times (tg\alpha + tg\beta) \; + \; N_u \times \left(\frac{y_{b1}}{h} + tg\beta \right) \; - \; \frac{dN_u}{dz_b} \times (z - y_{a1})$$

$$T_{mu} = T_u \quad \mathbf{m} \quad \frac{M_u}{h} \times (tg\alpha + tg\beta)$$



Slika 87/2 Naponi smicanja τ_n duž grednog nosača sa vutama

– Preseci od oslonca 1 do a^l

$$\tau_n = \frac{1}{bz}\left[T_u + \frac{M_u}{h}\mathrm{tg}\alpha_1\right].$$

– Preseci $a^d - b^l$ i $c^d - P_{2u}$

$$\tau_n = \frac{T_u}{bz} \quad (d = const.)$$

– Preseci $b^d - 0^l$

$$\tau_n = \frac{1}{bz}\left[T_u + \frac{M_u}{h}\mathrm{tg}\alpha_2\right].$$

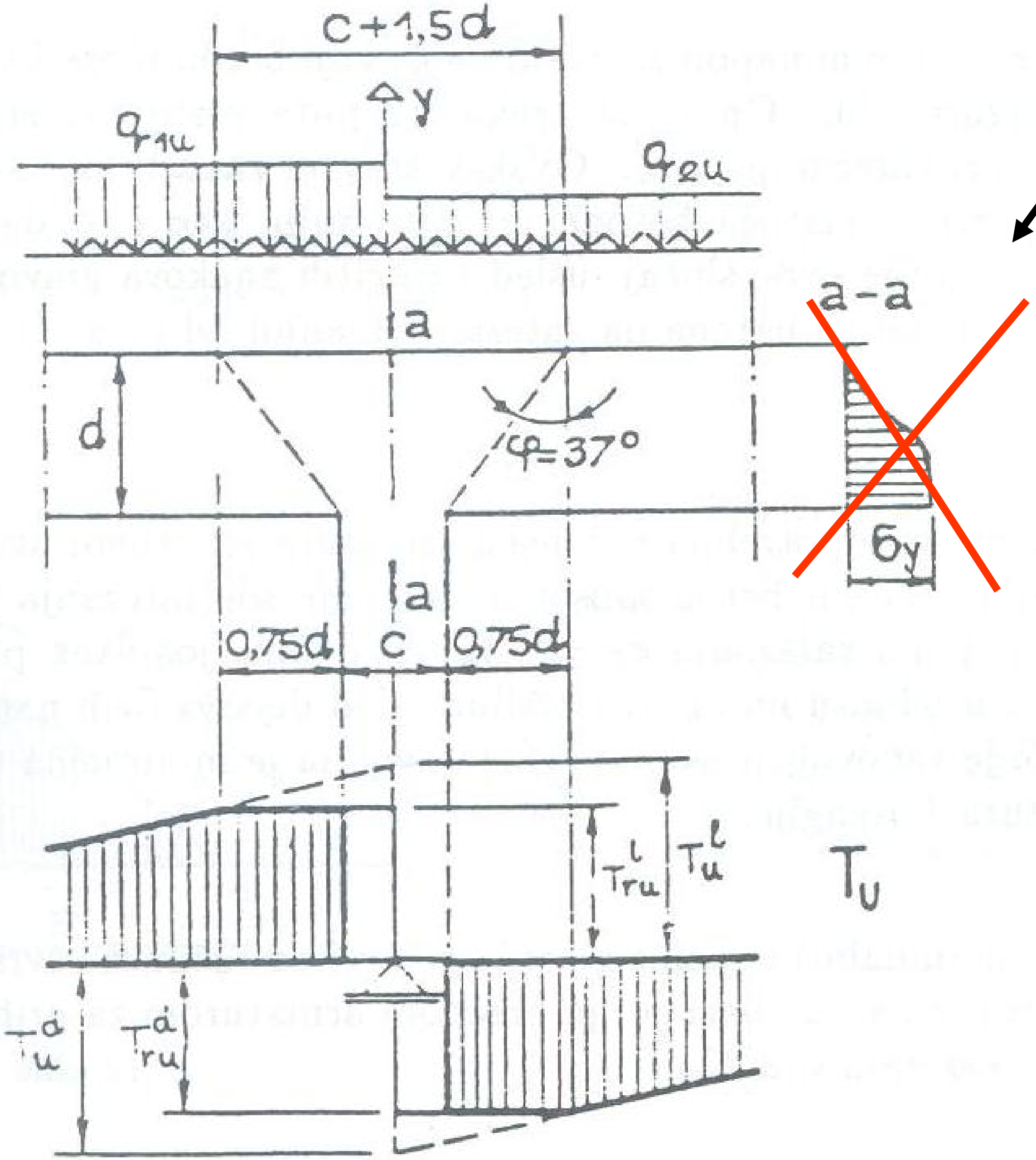
– Preseci $0^d - 2^l$

$$\tau_n = \frac{1}{bz}\left[T_u - \frac{M_u}{h}\mathrm{tg}\alpha_2\right].$$

– Preseci $2^d - c^l$

$$\tau_n = \frac{1}{bz}\left[T_u - \frac{M_u}{h}\mathrm{tg}\alpha_i\right],$$

BAB 87



Slika 87/3 Dijagram transverzalnih sila (u blizini oslonca) iz koga se određuju naponi smicanja

- Napon smicanja: $$\tau_n(y_b) = \frac{T_{mu}}{b(y_b)\cdot z}$$

- Računska čvrstoća betona pri smicanju, τ_r:

MB	15	20	30	40	50	60
τ_r [MPa]	0.6	0.8	1.1	1.3	1.5	1.6

- Mogući slučajevi:

1) $\tau_n \leq \tau_r$ konstruktivna poprečna armatura

2) $\tau_n > \tau_r$ proračunska poprečna armatura za prihvatanje uticaja od dejstva transverzalnih sila T_{mu}

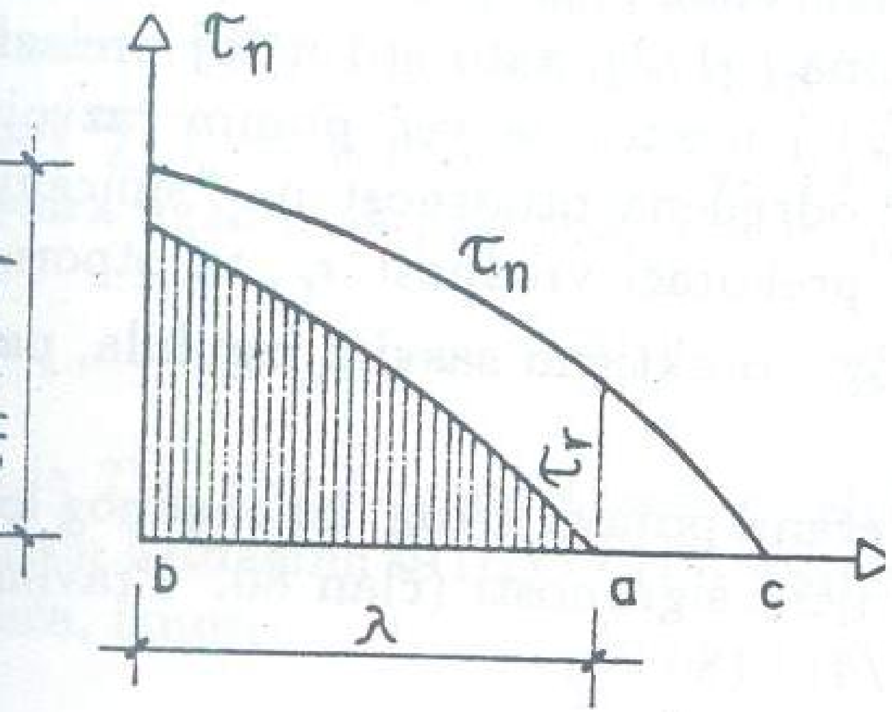
1) $\tau_r < \tau_n < 3\tau_r$ potrebna površina armature se određuje na osnovu $\mathbf{T_{Ru}}$

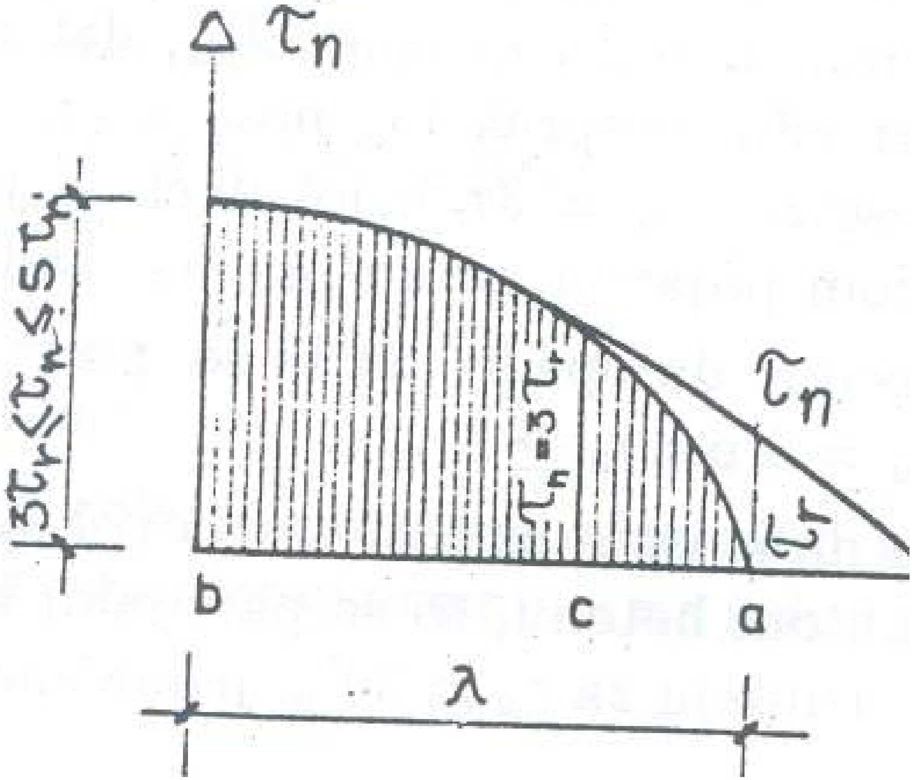
$$T_{Ru} = T_{mu} - T_{bu}$$

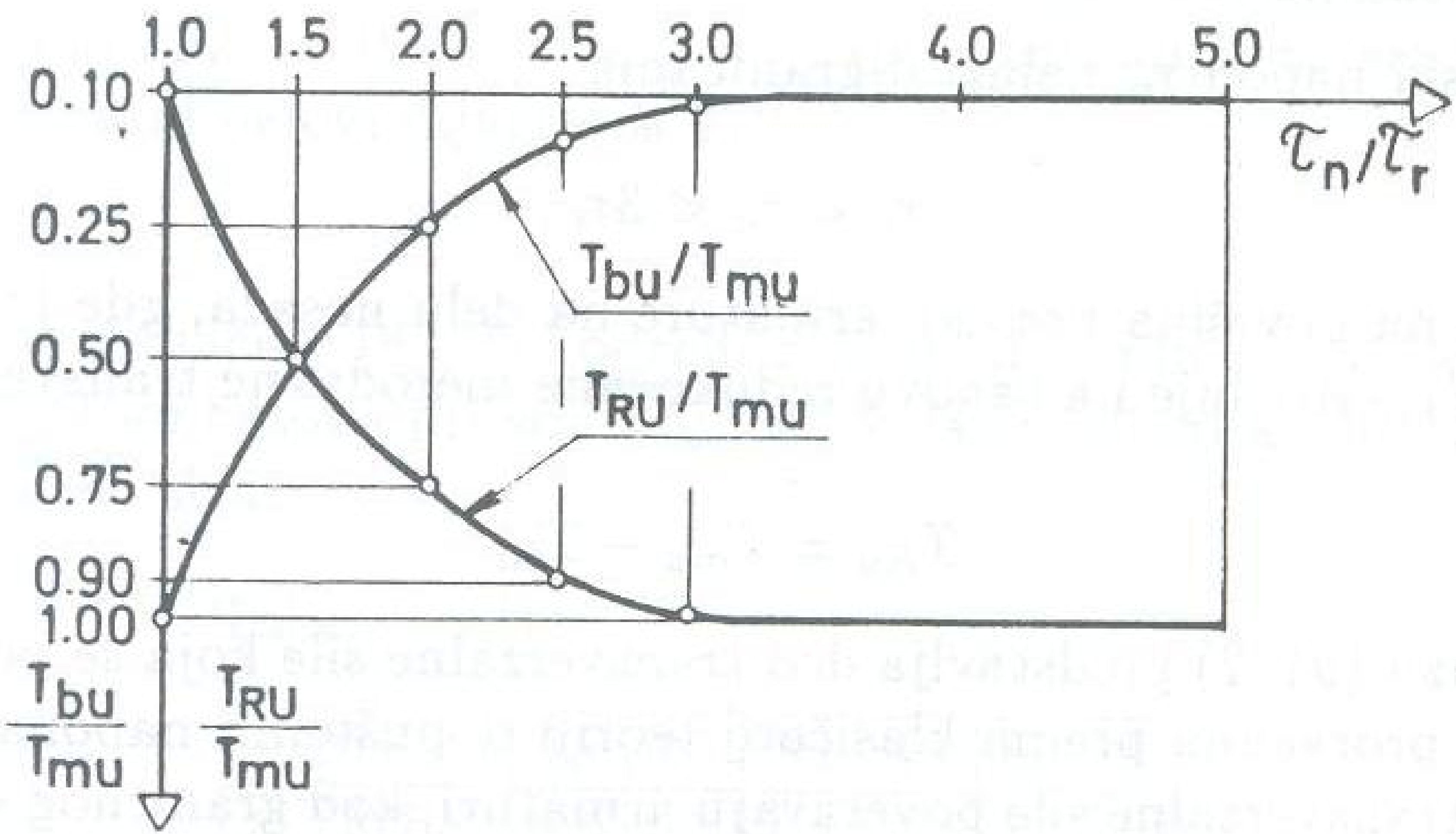
$$T_{bu} = \frac{1}{2} \cdot \left(3\tau_r - \tau_n\right) \cdot b \cdot z$$

2) $3\tau_r < \tau_n < 5\tau_r$ beton ne učestvuje u prijemu uticaja od transverzalnih sila

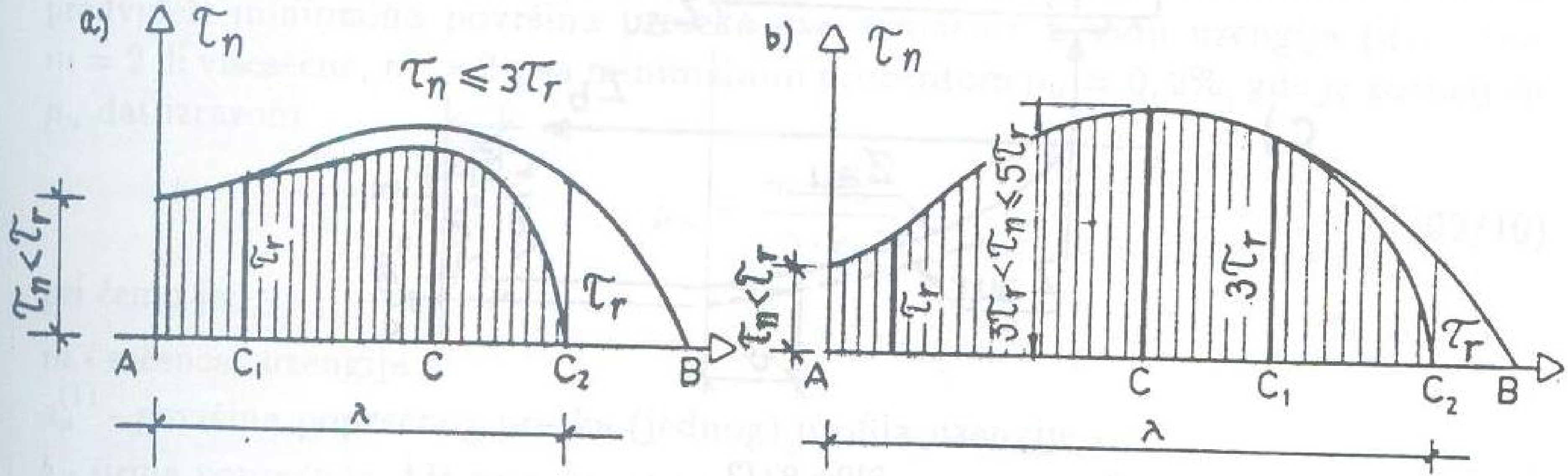
$$T_{Ru} = T_{mu} \qquad T_{bu} = 0$$


τn
τn<3τr
τn
τr
b
a
c
λ

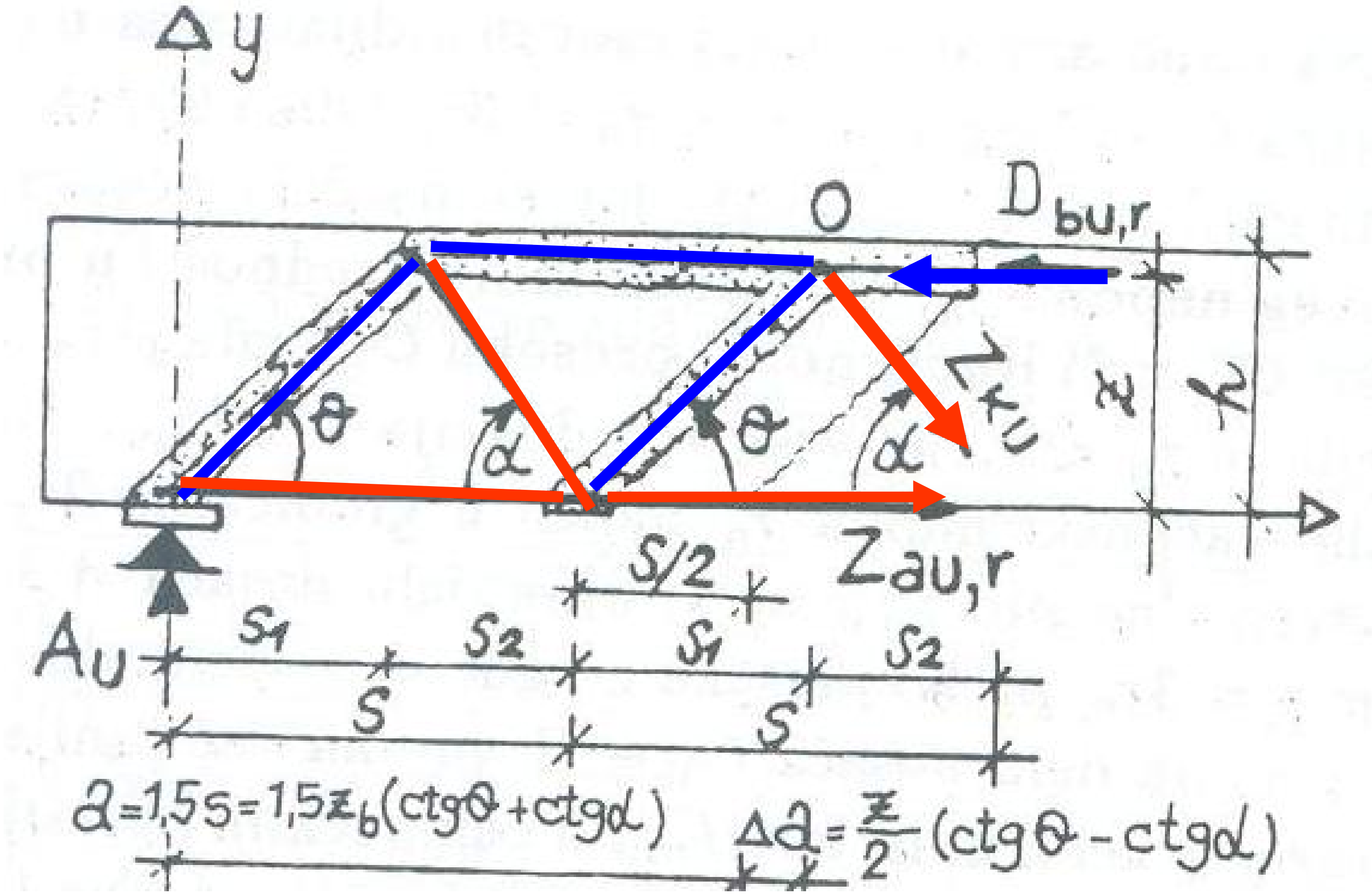

τn
3τr<τn≤5τr
τn=3τr
τn
τr
b
c
a
λ



1.0
1.5
2.0
2.5
3.0
4.0
5.0
0.10
0.25
0.50
0.75
0.90
1.00
τn/τr
Tbu/Tmu
TRU/Tmu
Tbu/Tmu
TRU/Tmu


a)
τ_n
$\tau_n \leq 3\tau_r$
$\tau_n < \tau_r$
τ_r
τ_r
A
C_1
C
C_2
B
λ
b)
τ_n
$\tau_n < \tau_r$
τ_r
$3\tau_r < \tau_n \leq 5\tau_r$
$3\tau_r$
τ_r
A
C
C_1
C_2
B
λ

Slika 92/1



$$Z_{ku} = \frac{T_{Ru}}{\sin\alpha} \qquad Z_{ku}' = \frac{Z_{ku}}{s} = \frac{T_{Ru}}{z(\operatorname{ctg}\theta + \operatorname{ctg}\alpha)\sin\alpha}$$

$$s = z(\operatorname{ctg}\theta + \operatorname{ctg}\alpha)\sin\alpha$$

- Redukuvana sila smicanja na jedinicu dužine nosača:

$$\bar{T}_{Ru} = \frac{T_{Ru}}{z} = \tau_n \cdot b$$

- Potrebna površina preseka poprečne armature na jedinicu dužine nosača:

$$A_{ak}' = \frac{Z_{ku}'}{\sigma_v} = \frac{\bar{T}_{Ru}}{\sigma_v \left(\mathrm{ctg}\theta + \mathrm{ctg}\alpha\right) \sin\alpha}$$

- Ukupna rekukovana merodavna sila smicanja na dužini osiguranja (horizontalna sila veze):

$$H_{vu} = \int_{x=a}^{x=b} \overline{T}_{Ru}\,dx = \int_{x=a}^{x=b} \frac{T_{Ru}}{z}dx$$

- Ukupna potrebna površina preseka poprečne armature:

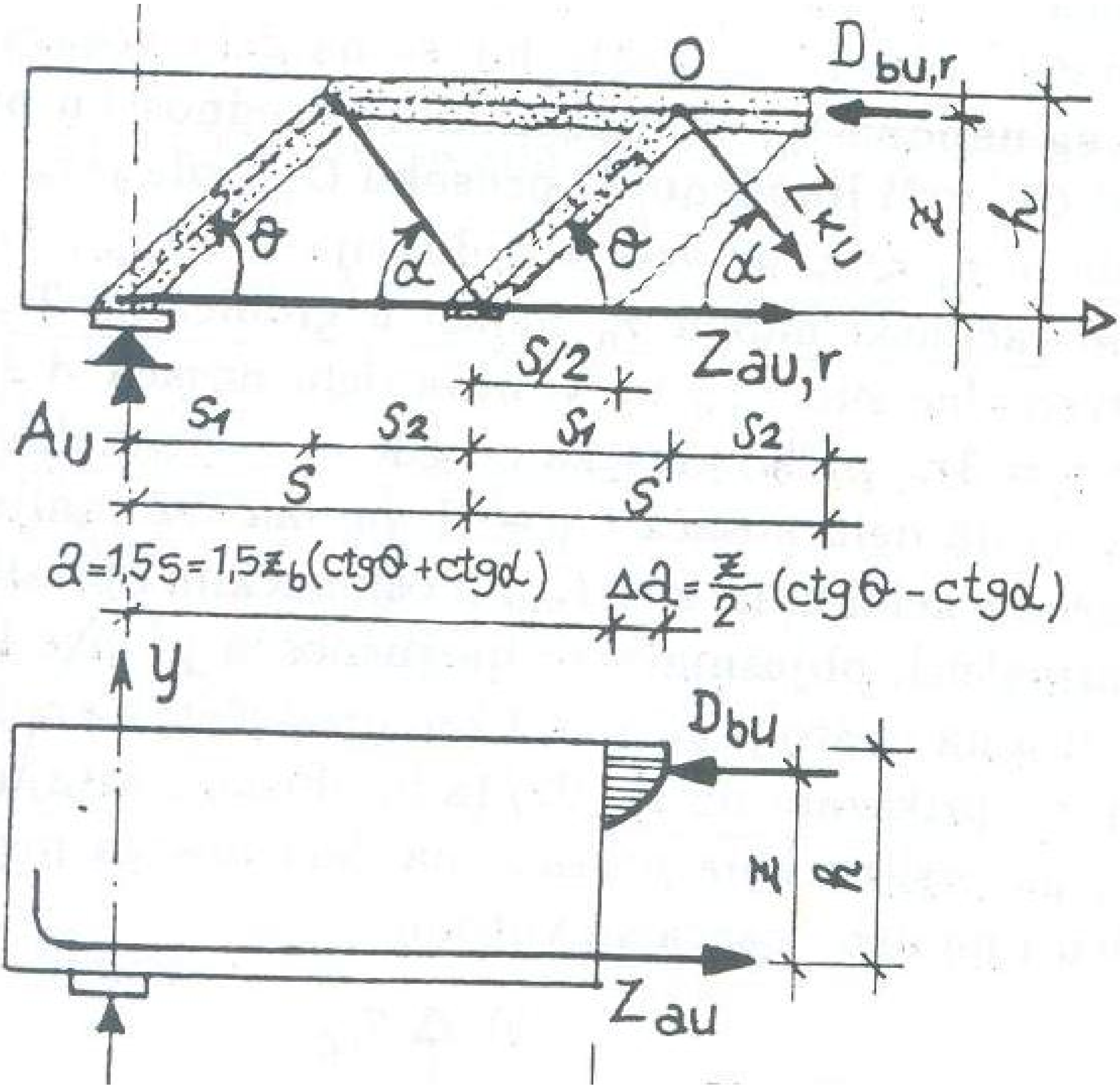
$$A_{ak} = \frac{1}{\sigma_v\,(\mathrm{ctg}\theta + \mathrm{ctg}\alpha)\,\mathbf{sin}\,\alpha} \int_{x=a}^{x=b} \frac{T_{Ru}}{z}dx = \frac{H_{vu}}{\sigma_v\,(\mathbf{cos}\,\theta + \mathbf{sin}\,\alpha \cdot \mathrm{ctg}\alpha)}$$

$$a_u^{(1)} = \frac{b \times \tau_{Ru}}{m \times \sigma_v} \times \frac{1}{(\cos\alpha + \sin\alpha \times \mathrm{ctg}\theta)} \times e_u$$

$$e_u = \frac{m \times a_u^{(1)}}{b \times \tau_{Ru}} \times (\cos\alpha + \sin\alpha \times \mathrm{ctg}\theta) \times \sigma_v$$

- Pri uglu nagiba $\theta<45^0$, dobija se manje poprečne armature nego pri $\theta=45^0$.

- Za isti ugao θ, poprečna armatura je veća kada je $\alpha=45^0$ nego $\alpha=90^0$.



O
$D_{bu,r}$
θ
α
θ
α
Z_{ku}
z
h
$Z_{au,r}$
s/2
A_u
s_1
s_2
s_1
s_2
s
s
$a = 1{,}5s = 1{,}5z_b(\mathrm{ctg}\,\theta + \mathrm{ctg}\,\alpha)$
$\Delta a = \frac{z}{2}(\mathrm{ctg}\,\theta - \mathrm{ctg}\,\alpha)$
y
D_{bu}
z
h
Z_{au}

- Za $\theta<45^0$, podužna sila zatezanja $Z_{au,r,}$ je veća od granične sile zatezanja grednog nosača Z_{au}

$$Z_{au,r} = -T_{mu} \cdot \left[\frac{a}{z} + \frac{1}{2} \cdot \left(ctg\theta - ctg\alpha \right) \right]$$

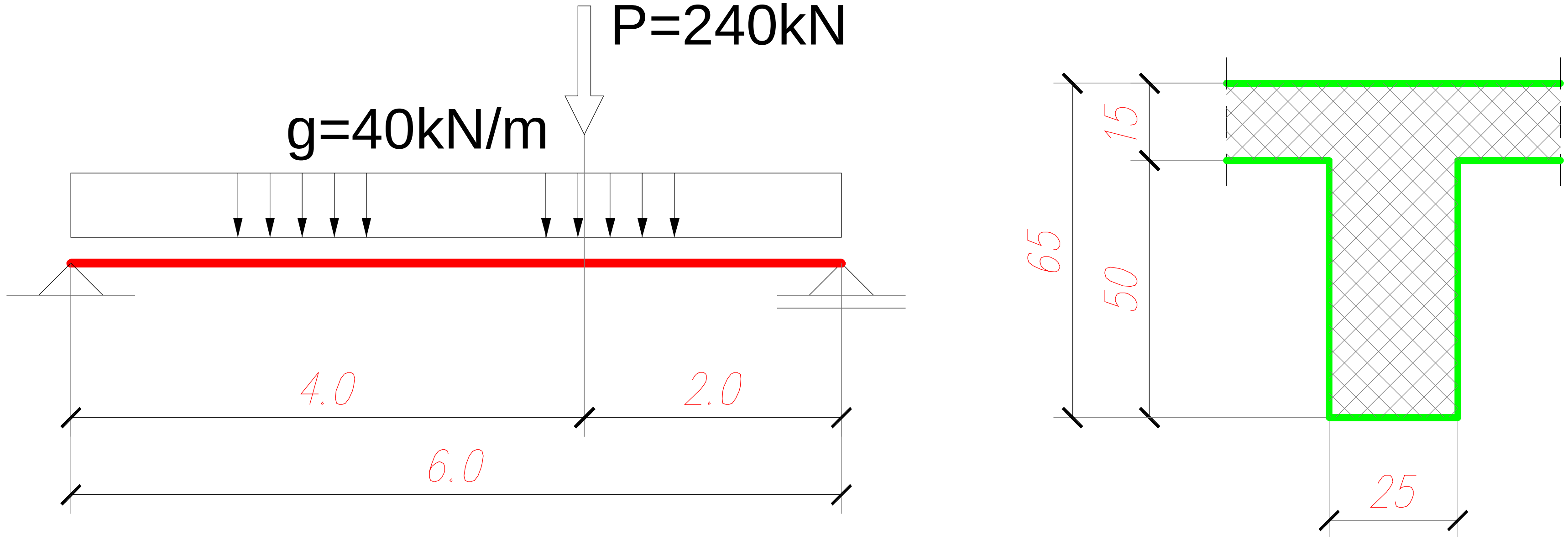
$$Z_{au} = -T_{mu} \cdot \frac{a}{z} = -\frac{M_u}{h}$$

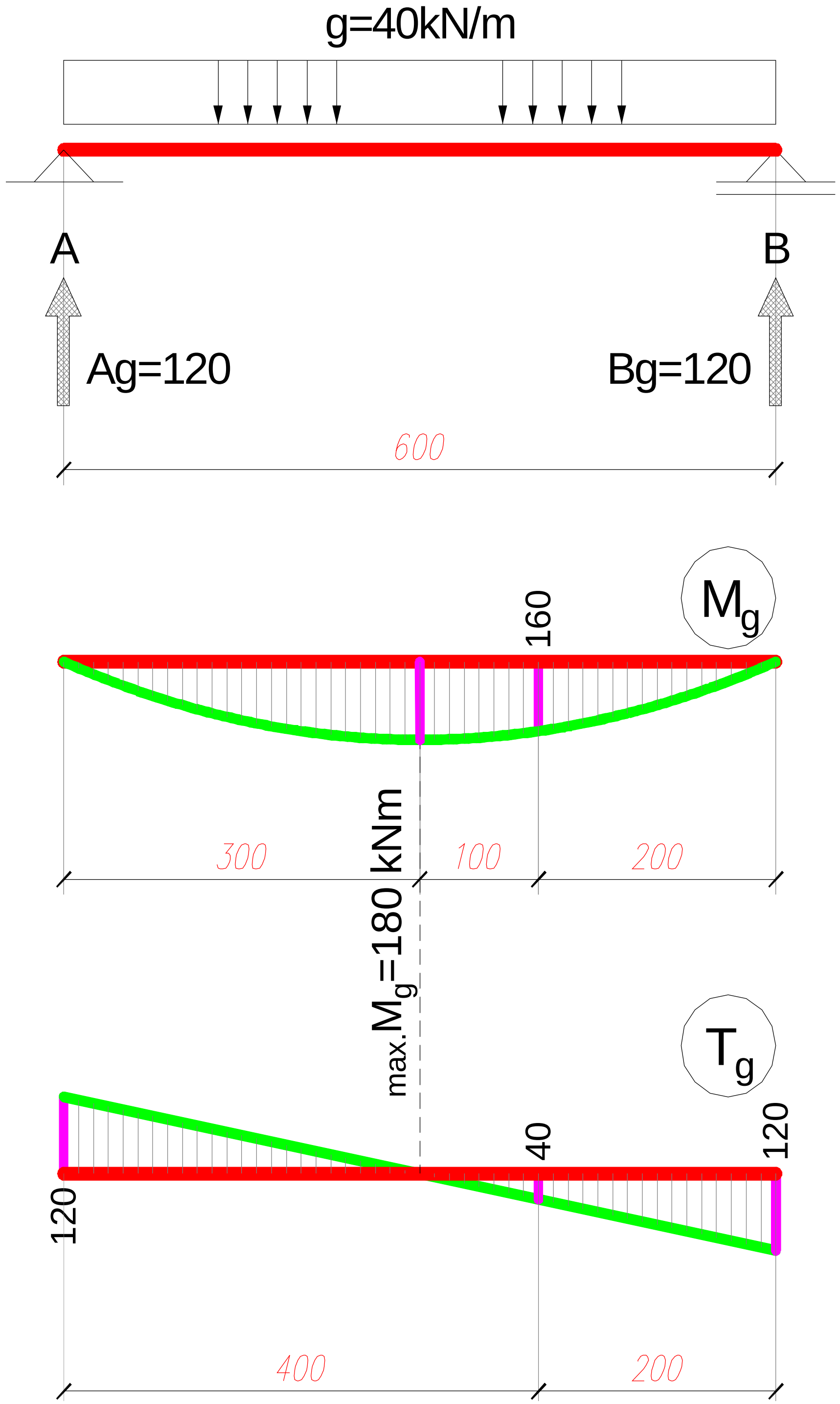
$$\Delta Z_{au} = Z_{au,r} - Z_{au} = -\frac{T_{mu}}{2} \cdot \left(ctg\theta - ctg\alpha \right)$$

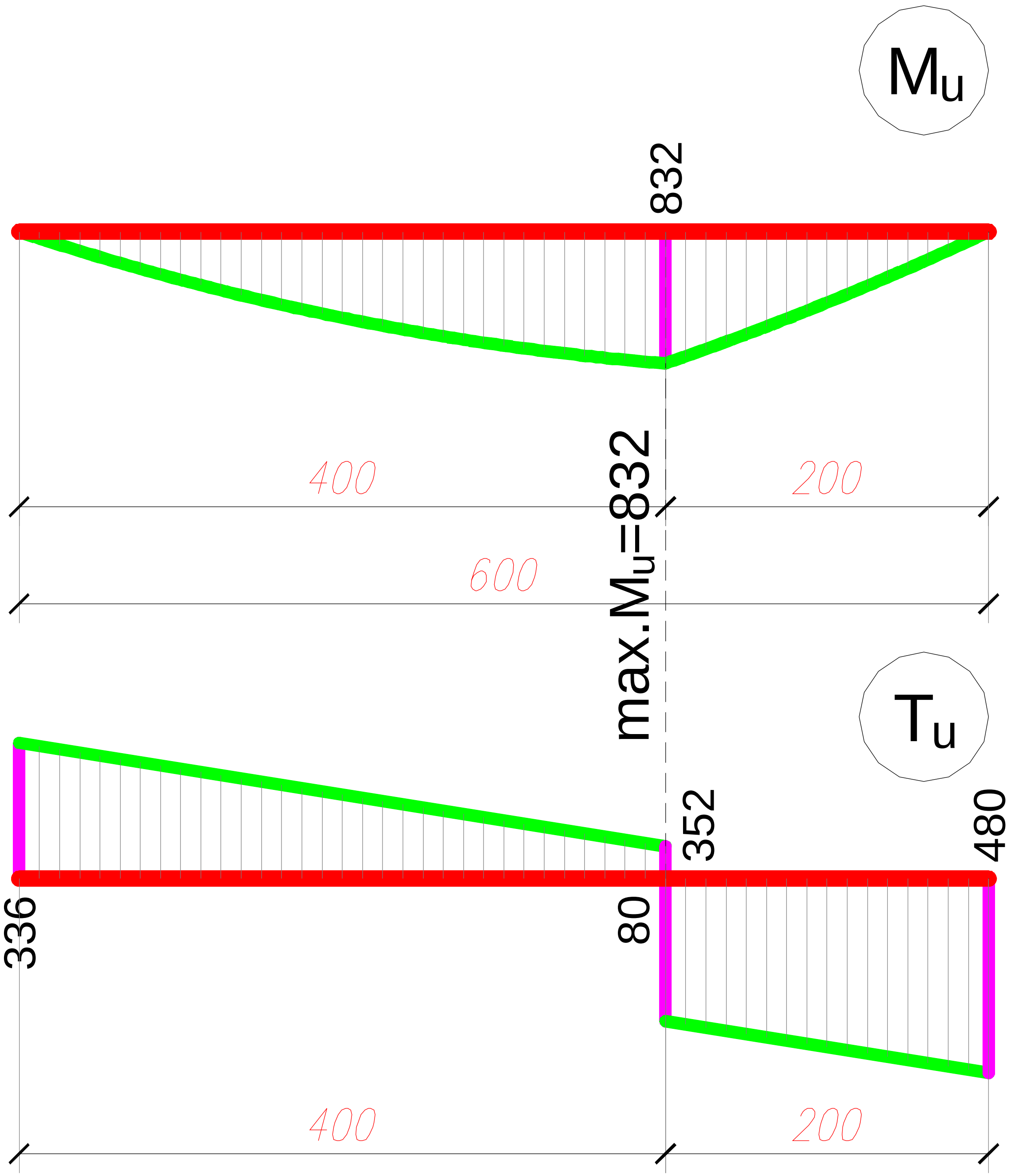
$$\Delta A_{a1} = \frac{\Delta Z_{au}}{\sigma_v} = \frac{T_{mu}}{2\sigma_v} \cdot \left(ctg\theta - ctg\alpha \right) \geq 0$$

Primer 1: (videti materijal "07" na http:imksus... !!!)

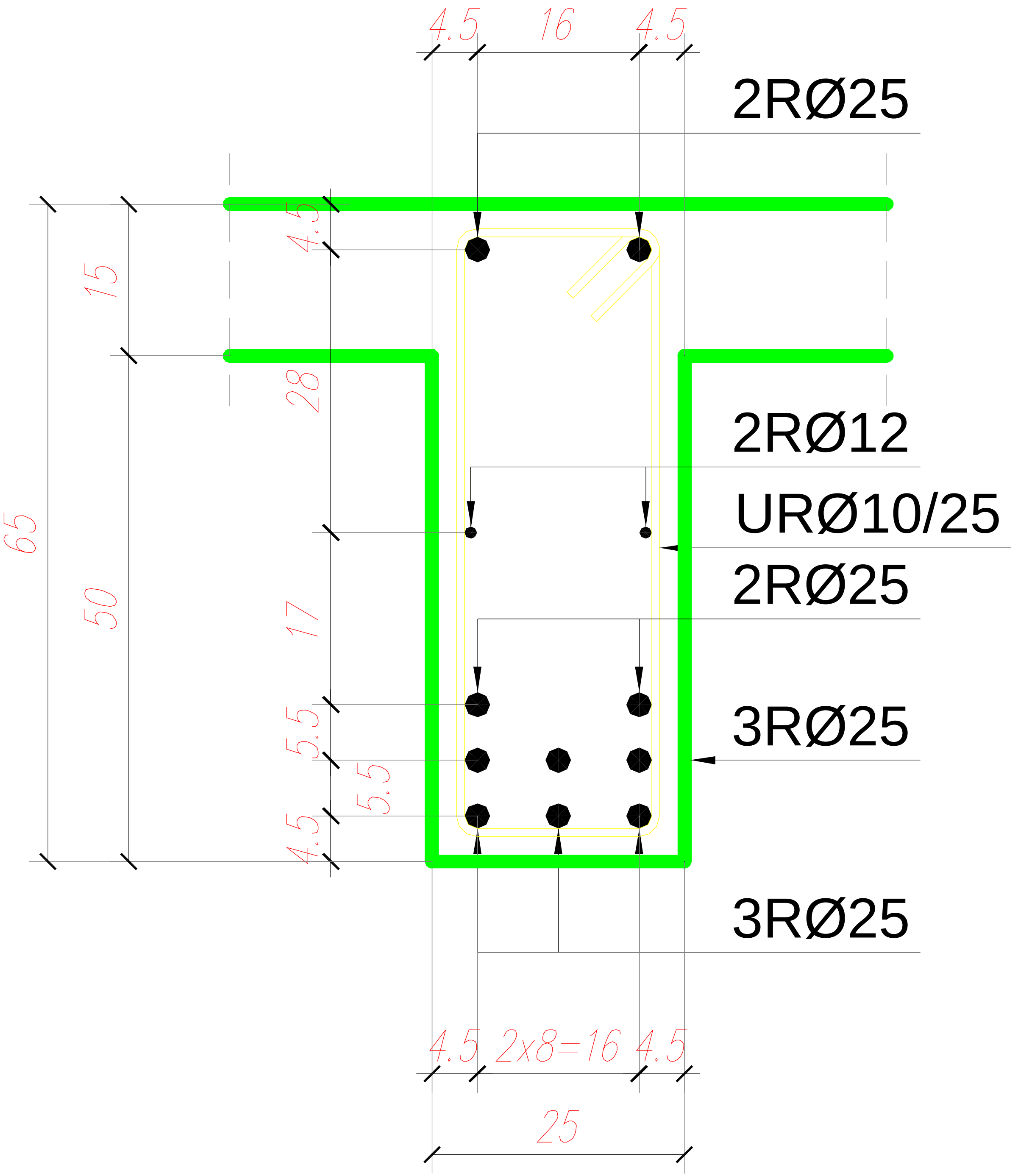
Dimenzionisati nosač sistema proste grede, čiji su opterećenje i poprečni presek prikazani na skici. MB 30, RA 400/500.



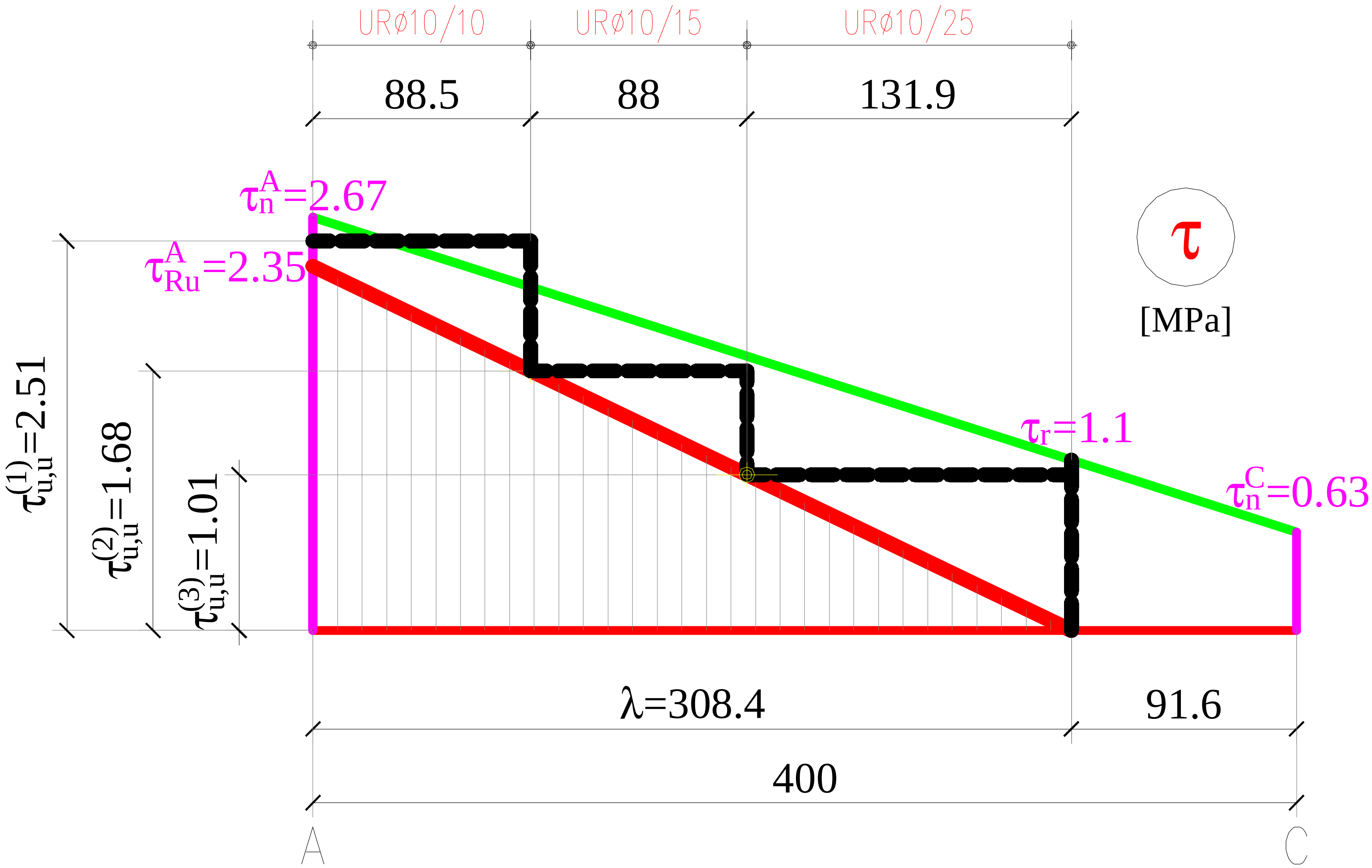

g=40kN/m
A
B
Ag=120
Bg=120
600
Mg
160
300
100
200
max.Mg=180 kNm
Tg
40
120
120
400
200



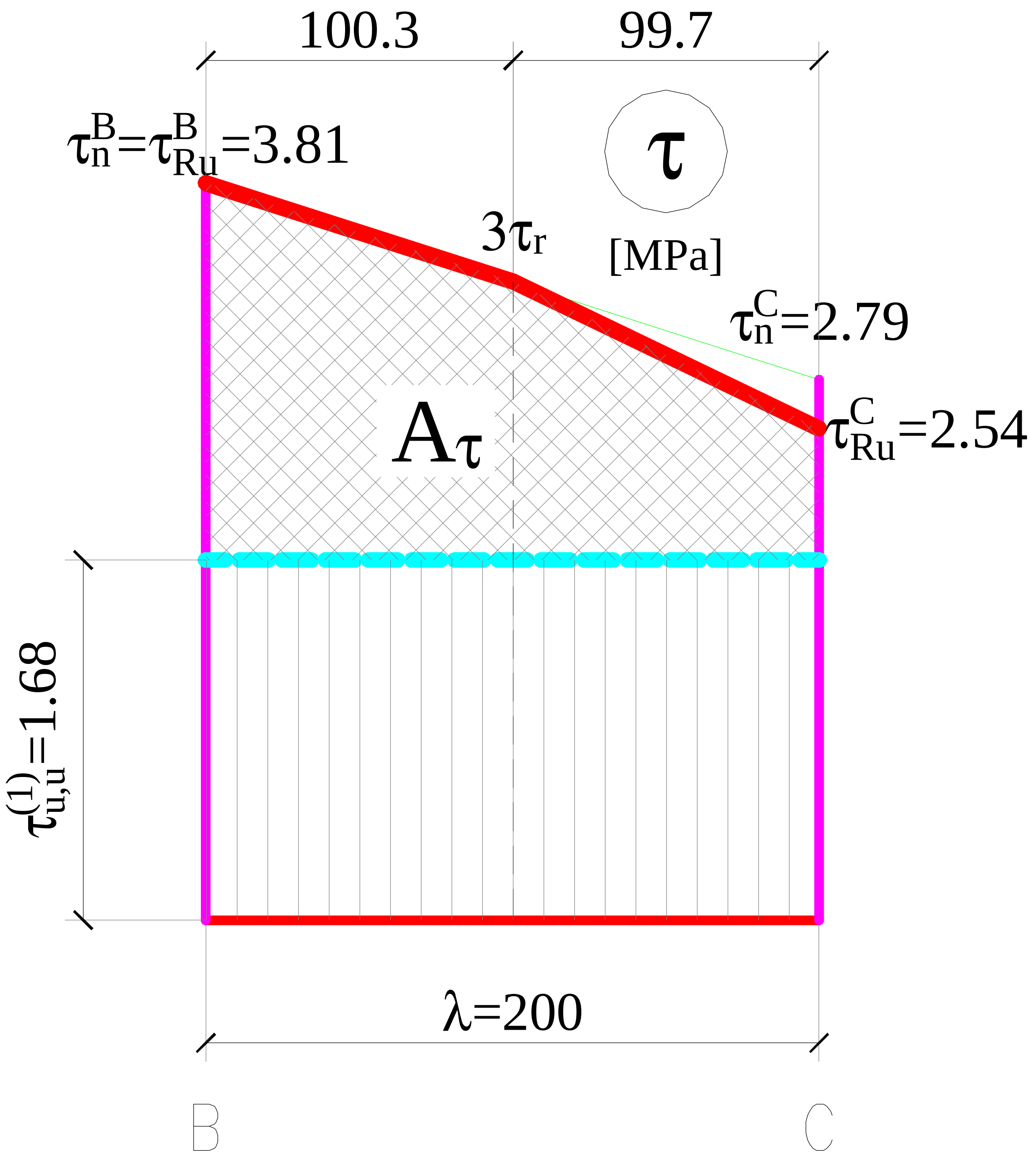
Mu
832
400
200
600
max.Mu=832
Tu
352
480
336
80
400
200

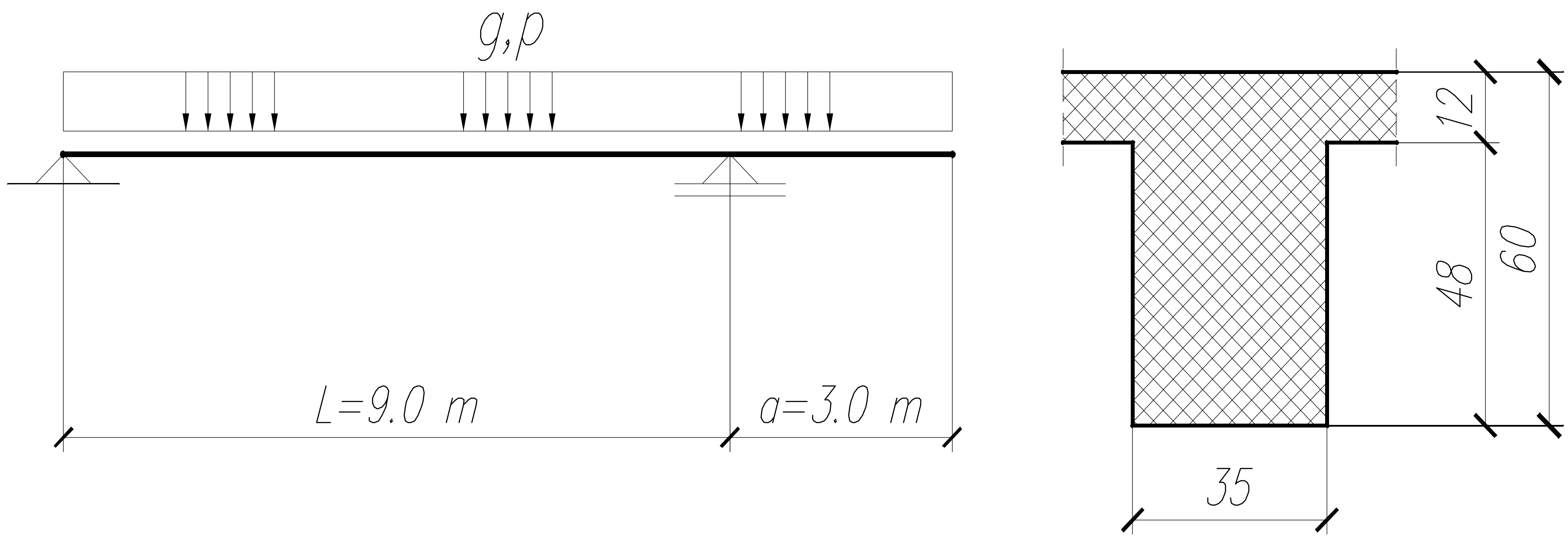


4.5
16
4.5
2RØ25
15
4.5
65
28
2RØ12
URØ10/25
50
2RØ25
17
5.5
3RØ25
5.5
4.5
3RØ25
4.5
2x8=16
4.5
25



URø10/10
URø10/15
URø10/25
88.5
88
131.9
τ
[MPa]
τ_n^A=2.67
τ_{Ru}^A=2.35
τ_r=1.1
τ_n^C=0.63
$\tau_{u,u}^{(1)}$=2.51
$\tau_{u,u}^{(2)}$=1.68
$\tau_{u,u}^{(3)}$=1.01
λ=308.4
91.6
400
A
C


100.3
99.7
τ
[MPa]
τnB=τRuB=3.81
3τr
τnC=2.79
τRuC=2.54
Aτ
τu,u(1)=1.68
λ=200
B
C

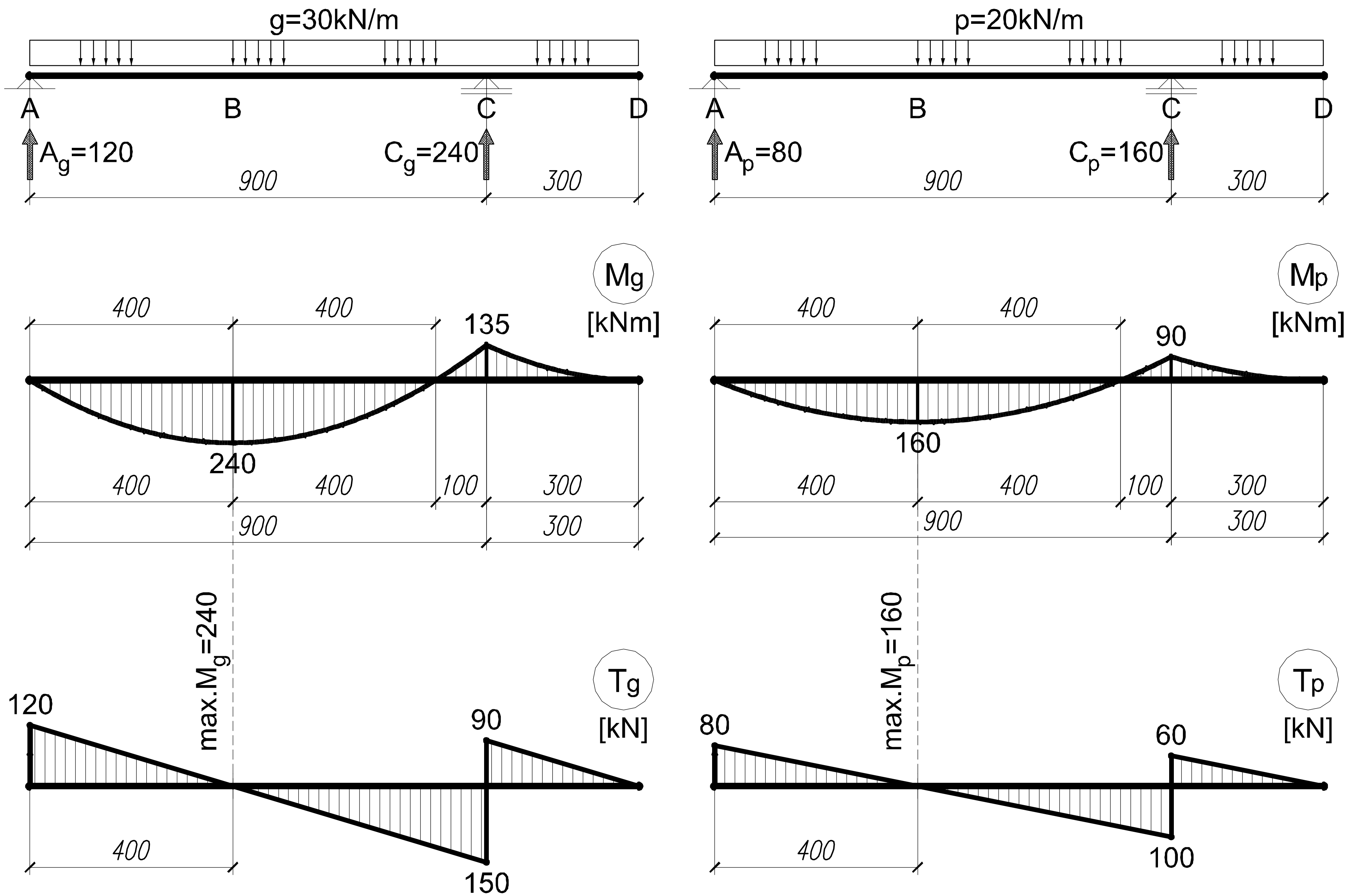


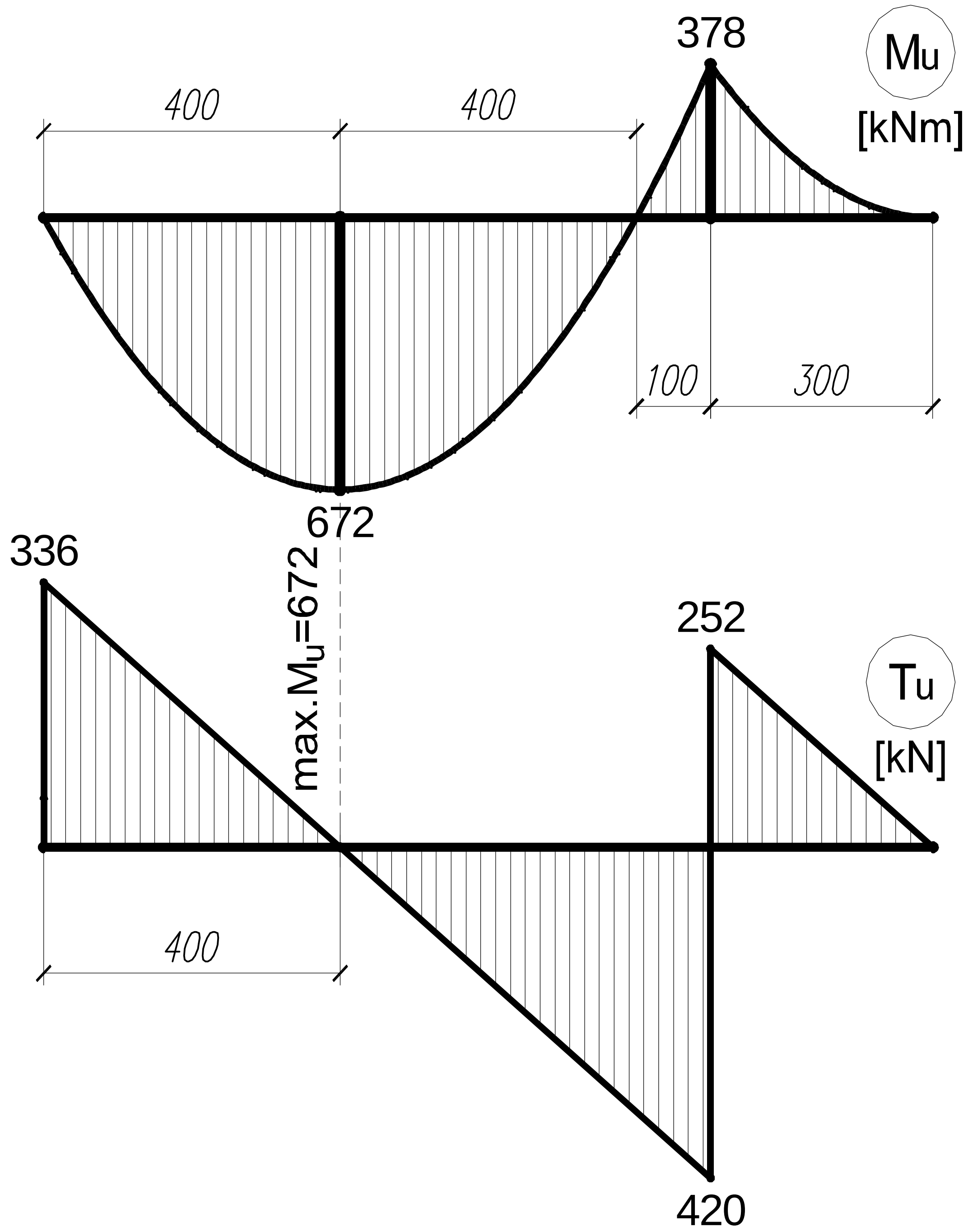
g = 30 kN/m

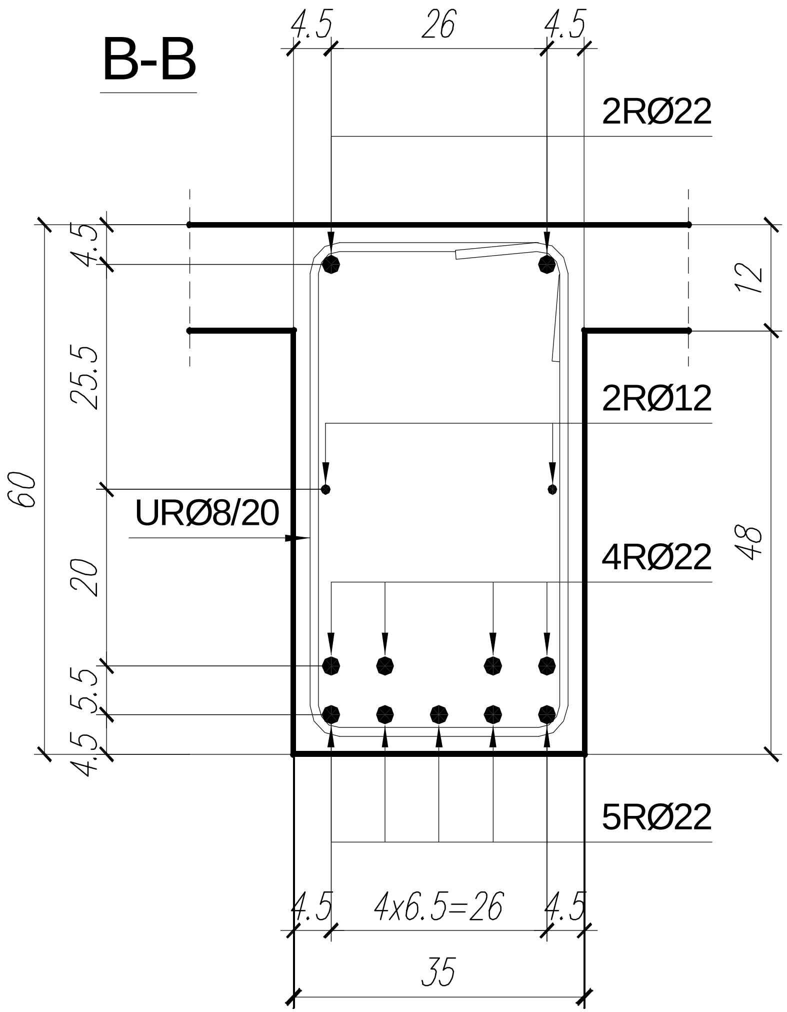
p = 20 kN/m

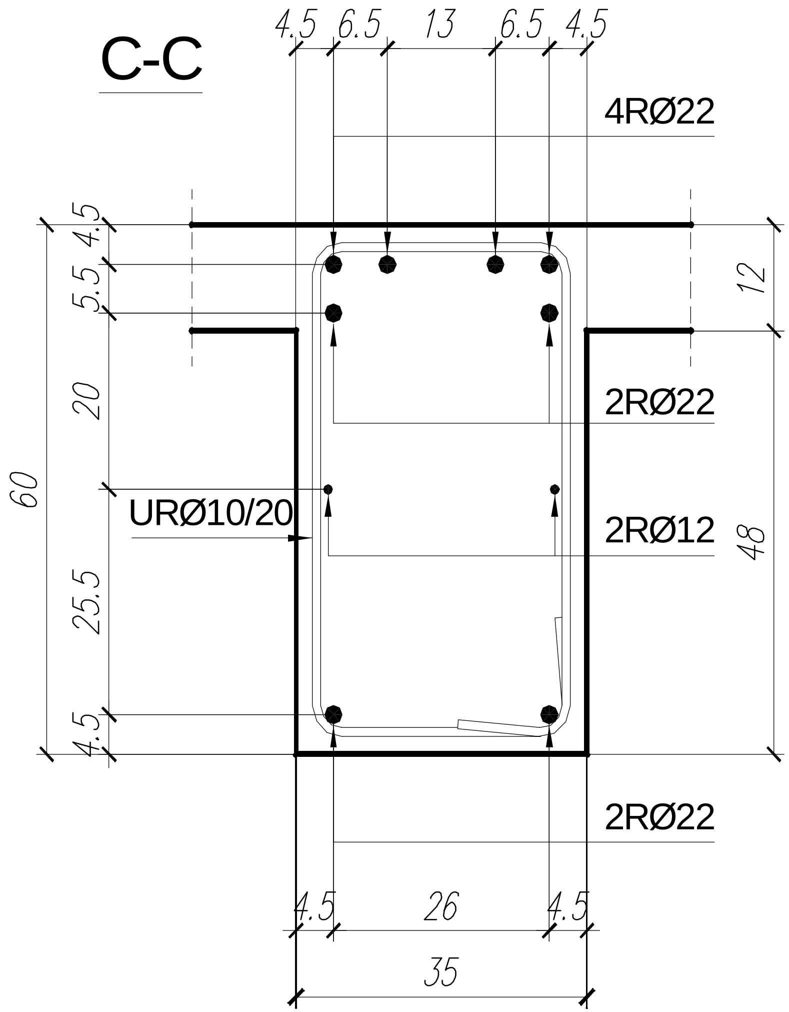
MB 30

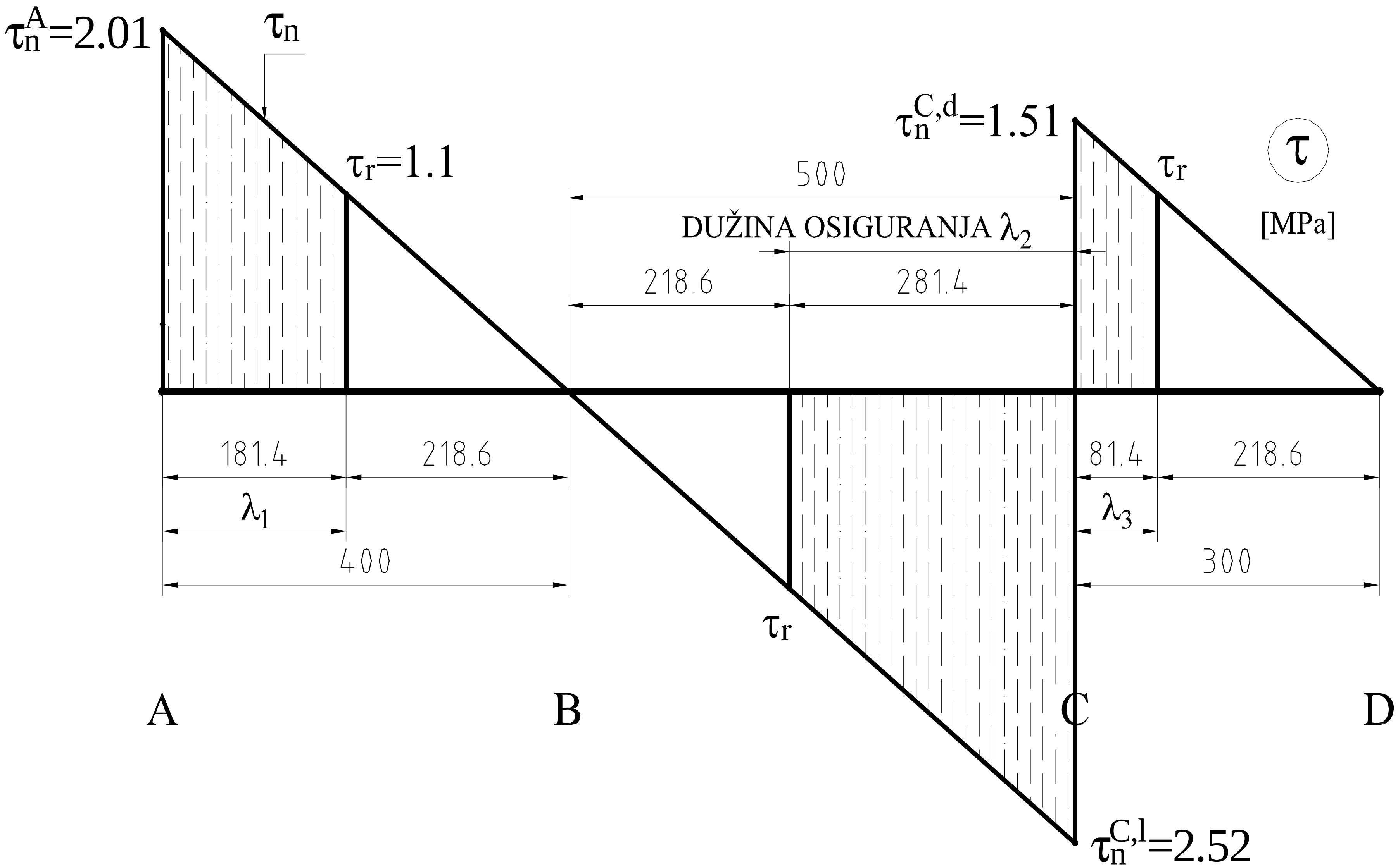
RA 400/500


g=30kN/m
A
B
C
D
A_g=120
C_g=240
900
300
M_g
[kNm]
400
400
135
240
400
400
100
300
900
300
max.M_g=240
T_g
[kN]
120
90
150
400
p=20kN/m
A
B
C
D
A_p=80
C_p=160
900
300
M_p
[kNm]
400
400
90
160
400
400
100
300
900
300
max.M_p=160
T_p
[kN]
80
60
100
400


378
Mu
[kNm]
400
400
100
300
672
336
max.Mu=672
252
Tu
[kN]
400
420


B-B
4.5
26
4.5
2RØ22
12
4.5
25.5
60
2RØ12
URØ8/20
48
4RØ22
20
5.5
4.5
5RØ22
4.5
4x6.5=26
4.5
35


C-C
4.5
6.5
13
6.5
4.5
4RØ22
4.5
5.5
12
20
2RØ22
60
URØ10/20
2RØ12
48
25.5
4.5
2RØ22
4.5
26
4.5
35



τnA=2.01
τn
τr=1.1
τnC,d=1.51
τr
τ
[MPa]
500
DUŽINA OSIGURANJA λ2
218.6
281.4
181.4
218.6
81.4
218.6
λ1
λ3
400
300
τr
A
B
C
D
τnC,l=2.52

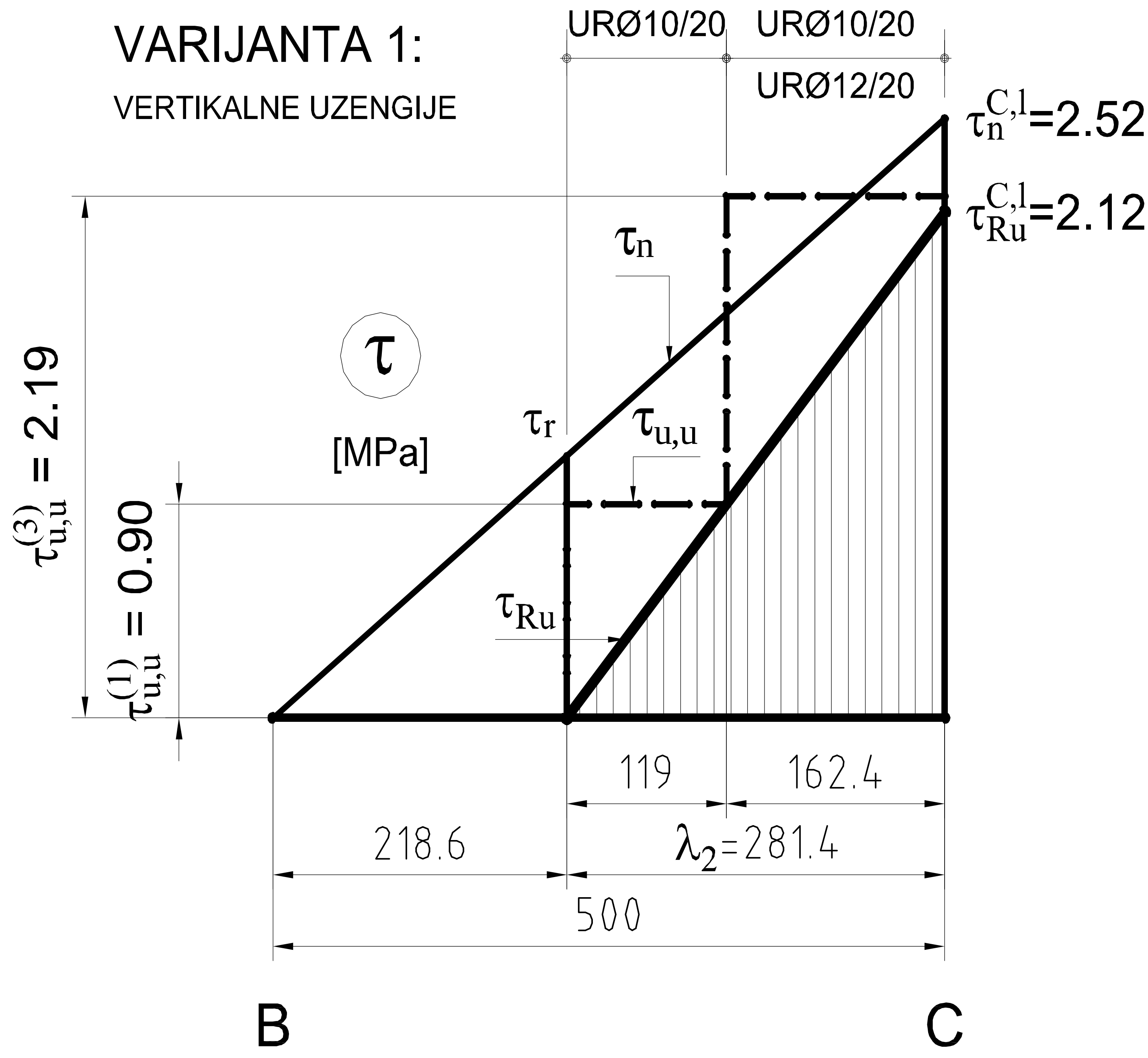
$$T_{mu}^{C,l} = 420\ \mathrm{kN} \Rightarrow \tau_n^{C,l} = \frac{420}{35 \times 47.7} = 0.252\ \mathrm{kN/cm^2}$$

$$T_{bu} = \frac{3 \times 0.11 - 0.252}{2} \times 35 \times 47.7 = 65.5\,\mathrm{kN}$$

$$T_{Ru} = T_{mu} - T_{bu} = 420 - 65.5 = 354.5\, kN$$

$$\tau_{Ru} = \frac{T_{Ru}}{b \times z} = \frac{354.5}{35 \times 47.7} = 0.212\ \mathrm{kN/cm^2}$$

$$e_u = \frac{2 \times 0.785}{35 \times 0.212} \times 40 \times (0 + 1 \times 1) = 10.76 \times a_u^{(1)} = 8.45\ \mathrm{cm}$$



VARIJANTA 1:
VERTIKALNE UZENGIJE
URØ10/20
URØ10/20
URØ12/20
$\tau_n^{C,l}$=2.52
$\tau_{Ru}^{C,l}$=2.12
τ_n
τ
[MPa]
τ_r
$\tau_{u,u}$
τ_{Ru}
$\tau_{u,u}^{(3)}$ = 2.19
$\tau_{u,u}^{(1)}$ = 0.90
119
162.4
218.6
λ_2=281.4
500
B
C

$$e_u \leq \frac{m \times a_u^{(1)}}{b \times \mu_{uz,min.}} = \frac{2 \times 0.785}{35 \times 0.2 \times 10^{-2}} = 22.4 \text{ cm}$$

$$e_u \leq \min.\left\{\begin{matrix} b = 35\text{cm} \\ h/2 = 26.5\text{cm} \\ 25\text{cm} \end{matrix}\right\} \Rightarrow \text{usv.}e_u = 20 \text{ cm}$$

$$\tau_{u,u}^{(1)} = \frac{m \times a_u^{(1)}}{b \times e_u} \times \sigma_v \times \left(\cos\alpha + \sin\alpha \times \text{ctg}\theta\right)$$

$$\tau_{u,u}^{(1)} = \frac{2 \times 0.785}{35 \times 20} \times 40 \times \left(0 + 1 \times 1\right) = 0.090 \text{ kN/cm}^2$$

$$\lambda_2 \times \frac{\tau_{u,u}^{(1)}}{\tau_{Ru}^{C,l}} = 281.4 \times \frac{0.90}{2.12} = 119\ \text{cm}$$

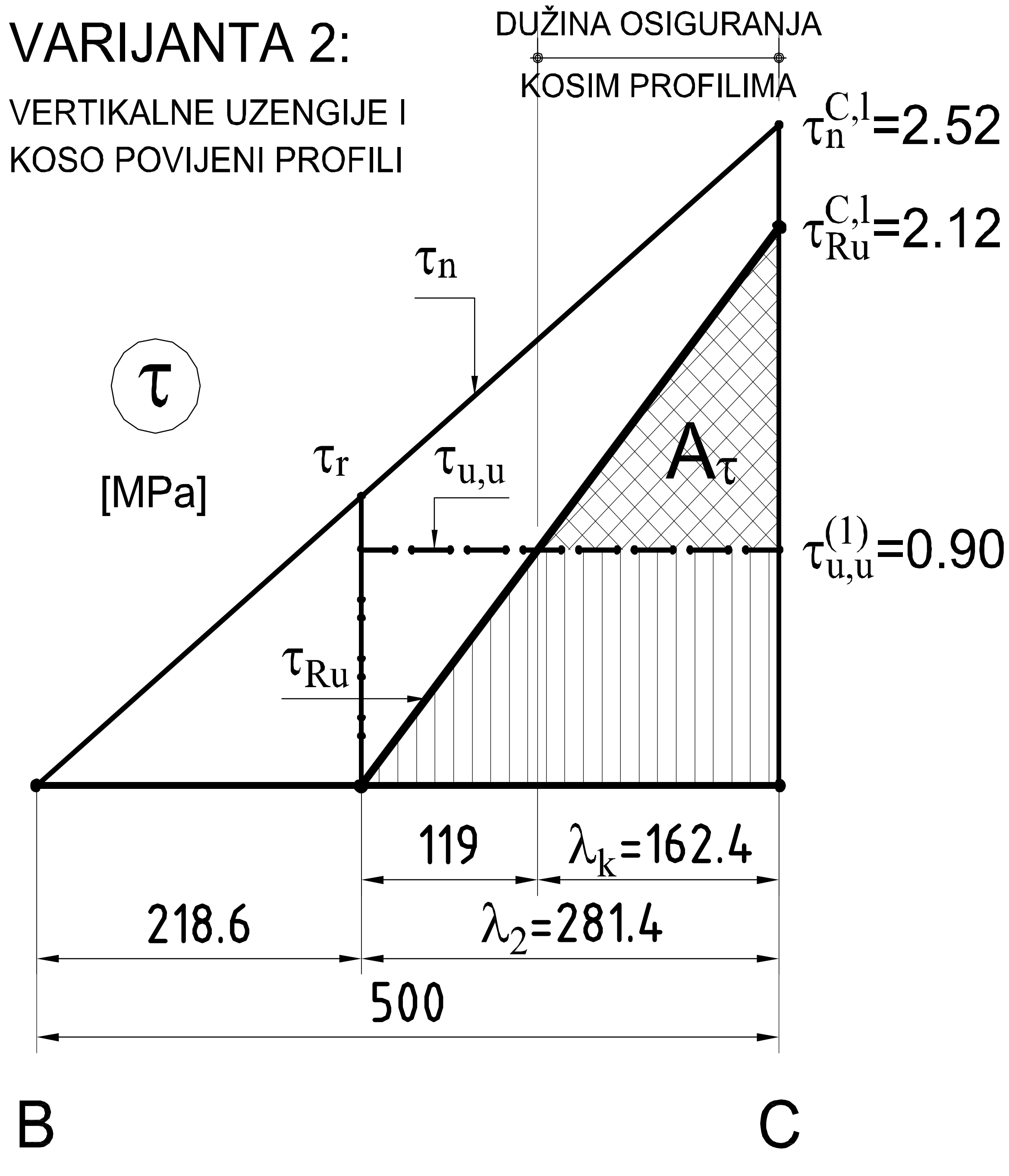
$$\Delta\lambda_2 = 281.4 - 119 = 162.4\ \text{cm}$$

$$\Delta\tau_{Ru}^{C,l} = \tau_{Ru,max}^{C,l} - \tau_{u,u}^{(1)} = 2.12 - 0.90 = 1.22\ \text{MPa}$$

$$a_u^{(1)} = \frac{b \times \Delta\tau_{Ru}^{C,l}}{m \times \sigma_v} \times \frac{1}{(\cos\alpha + \sin\alpha \times \text{ctg}\theta)} \times e_u$$

$$a_u^{(1)} = \frac{35 \times 0.122}{2 \times 40} \times \frac{1}{(0 + 1 \times 1)} \times 20 = 1.07\ \text{cm}^2$$

$$\tau_{u,u}^{(3)} = \frac{2 \times (0.785 + 1.131)}{35 \times 20} \times 40 \times (0 + 1 \times 1) = 0.219\ \text{kN/cm}^2$$



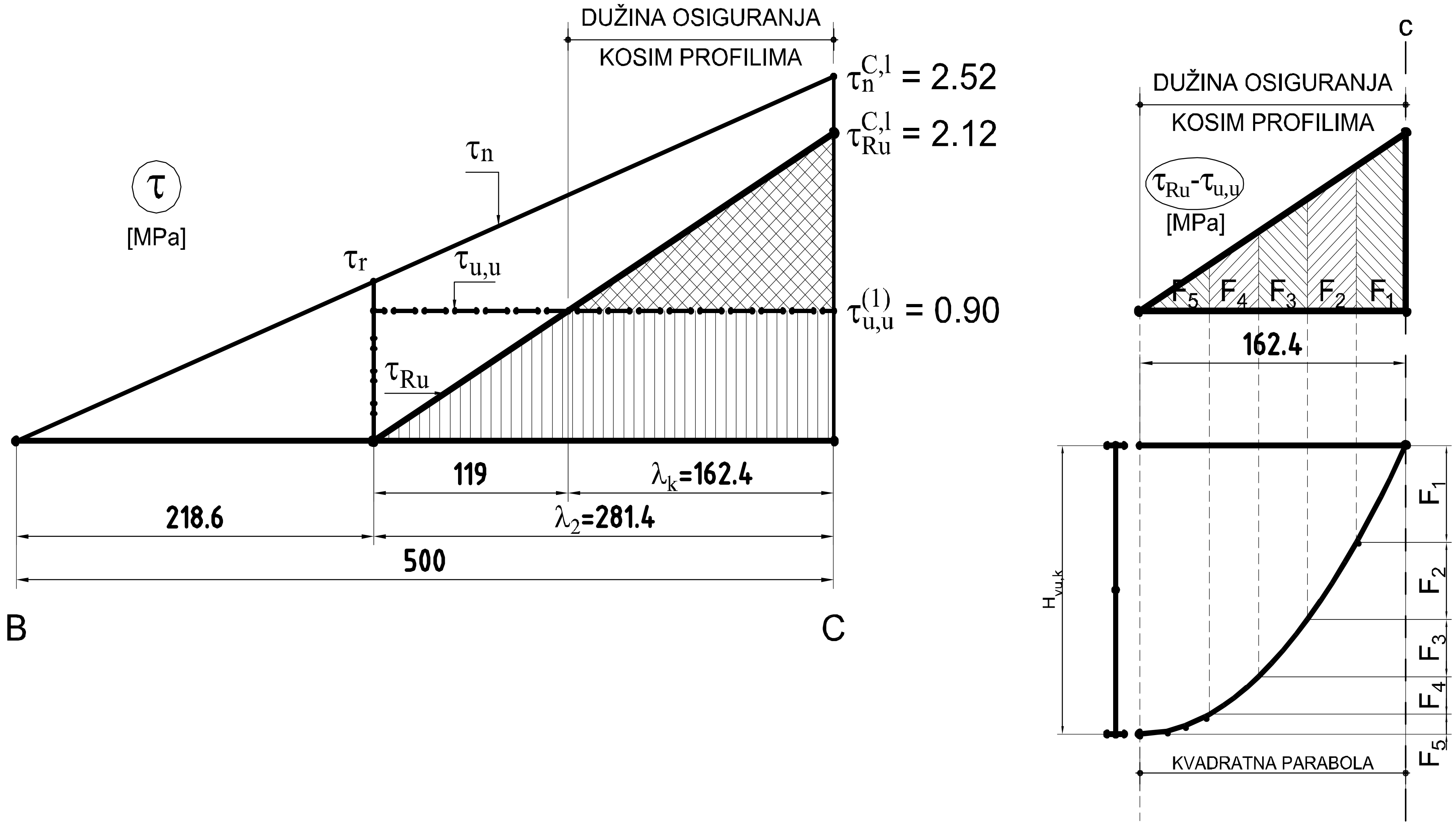
VARIJANTA 2:
VERTIKALNE UZENGIJE I
KOSO POVIJENI PROFILI
DUŽINA OSIGURANJA
KOSIM PROFILIMA
$\tau_n^{C,l}=2.52$
$\tau_{Ru}^{C,l}=2.12$
τ_n
τ
[MPa]
τ_r
$\tau_{u,u}$
A_τ
$\tau_{u,u}^{(1)}=0.90$
τ_{Ru}
119
$\lambda_k=162.4$
218.6
$\lambda_2=281.4$
500
B
C

$$H_{vu,k} = A_{\tau} \times b$$

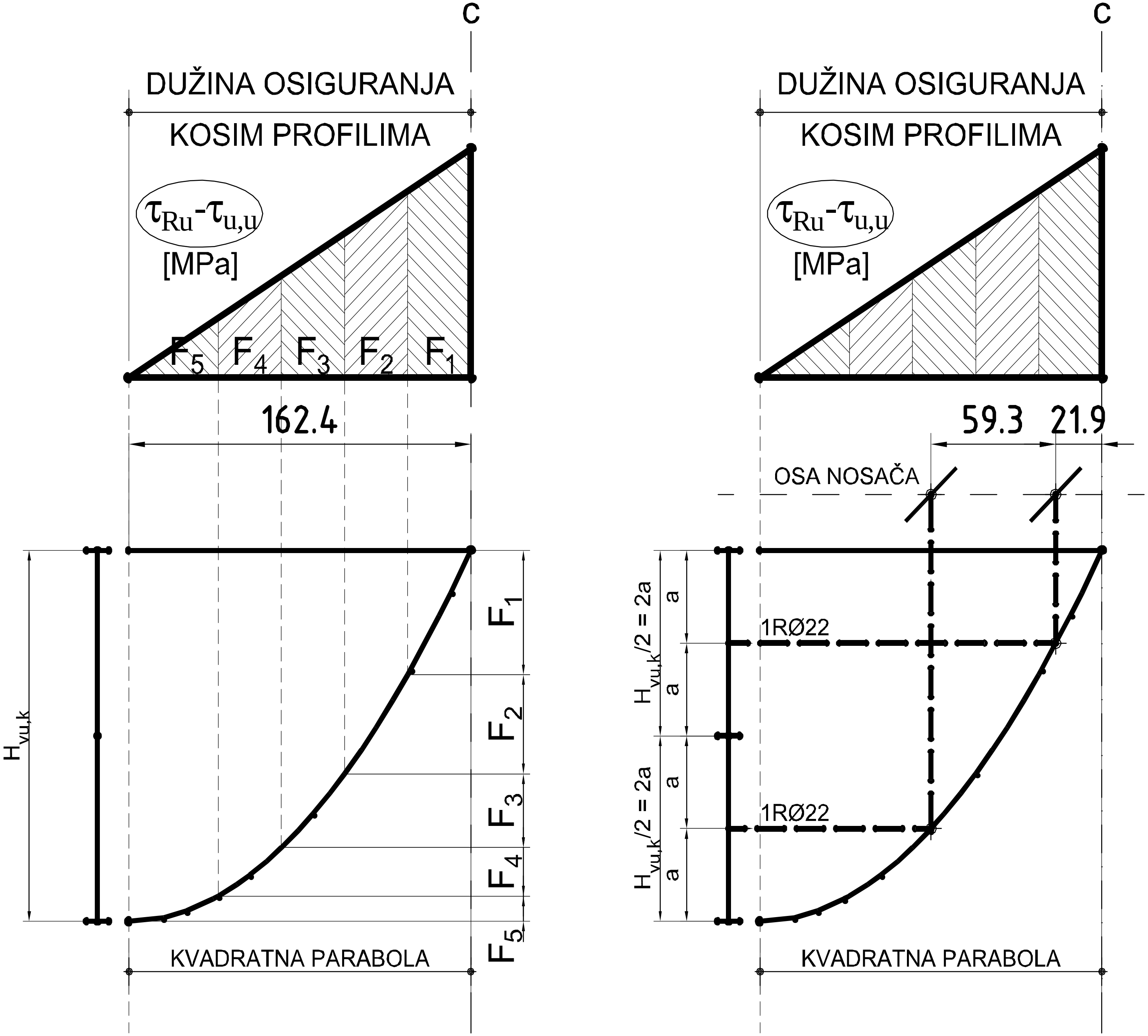
$$H_{vu,k} = \left(\frac{0.212 - 0.090}{2} \times 162.4 \right) \times 35 = 348.5 \text{ kN}$$

$$A_{a,k} = \frac{H_{vu,k}}{\sigma_v \times (\cos\alpha_k + \sin\alpha_k \times \text{ctg}\theta)}$$

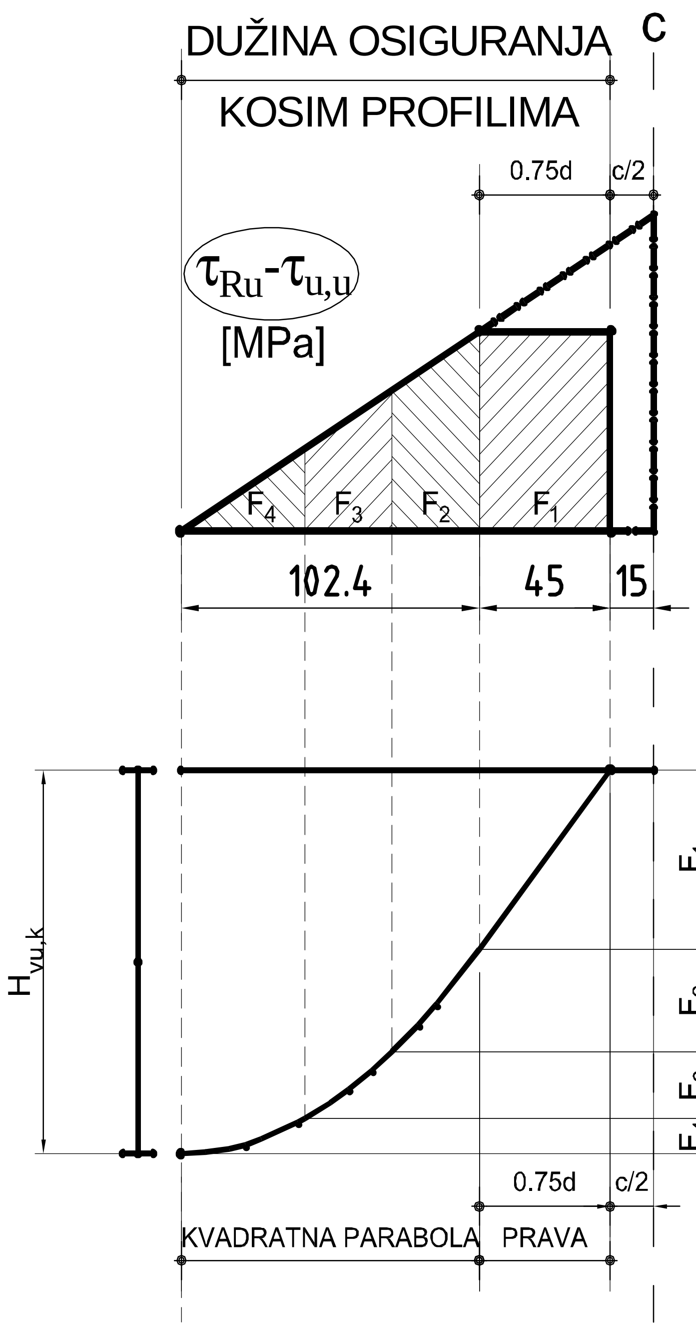
$$A_{a,k} = \frac{348.5}{40 \times (0.707 + 0.707 \times 1.0)} = 6.16 \text{ cm}^2$$



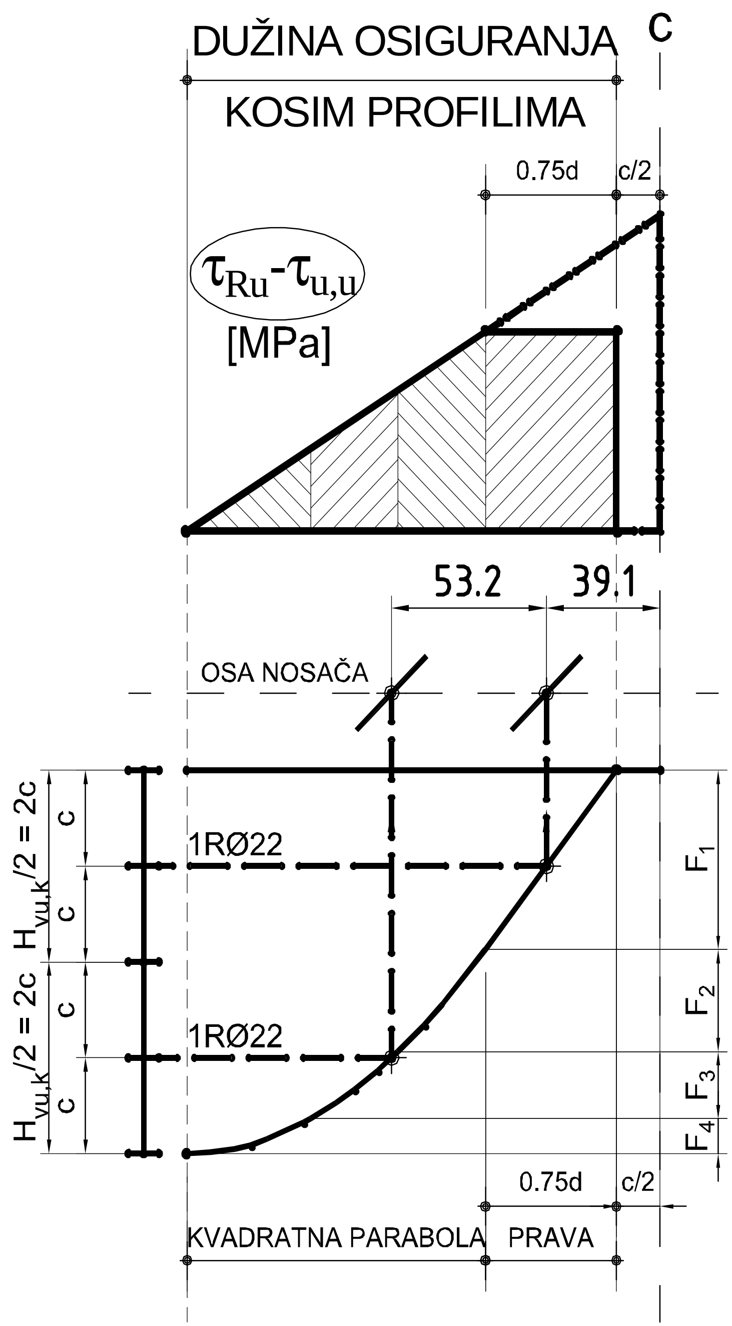
DUŽINA OSIGURANJA
KOSIM PROFILIMA
$\tau_n^{C,l} = 2.52$
$\tau_{Ru}^{C,l} = 2.12$
τ_n
τ
[MPa]
τ_r
$\tau_{u,u}$
$\tau_{u,u}^{(1)} = 0.90$
τ_{Ru}
119
λ_k=162.4
218.6
λ_2=281.4
500
B
C
c
DUŽINA OSIGURANJA
KOSIM PROFILIMA
τ_{Ru}-$\tau_{u,u}$
[MPa]
F_5 F_4 F_3 F_2 F_1
162.4
$H_{vu,k}$
F_1
F_2
F_3
F_4
F_5
KVADRATNA PARABOLA



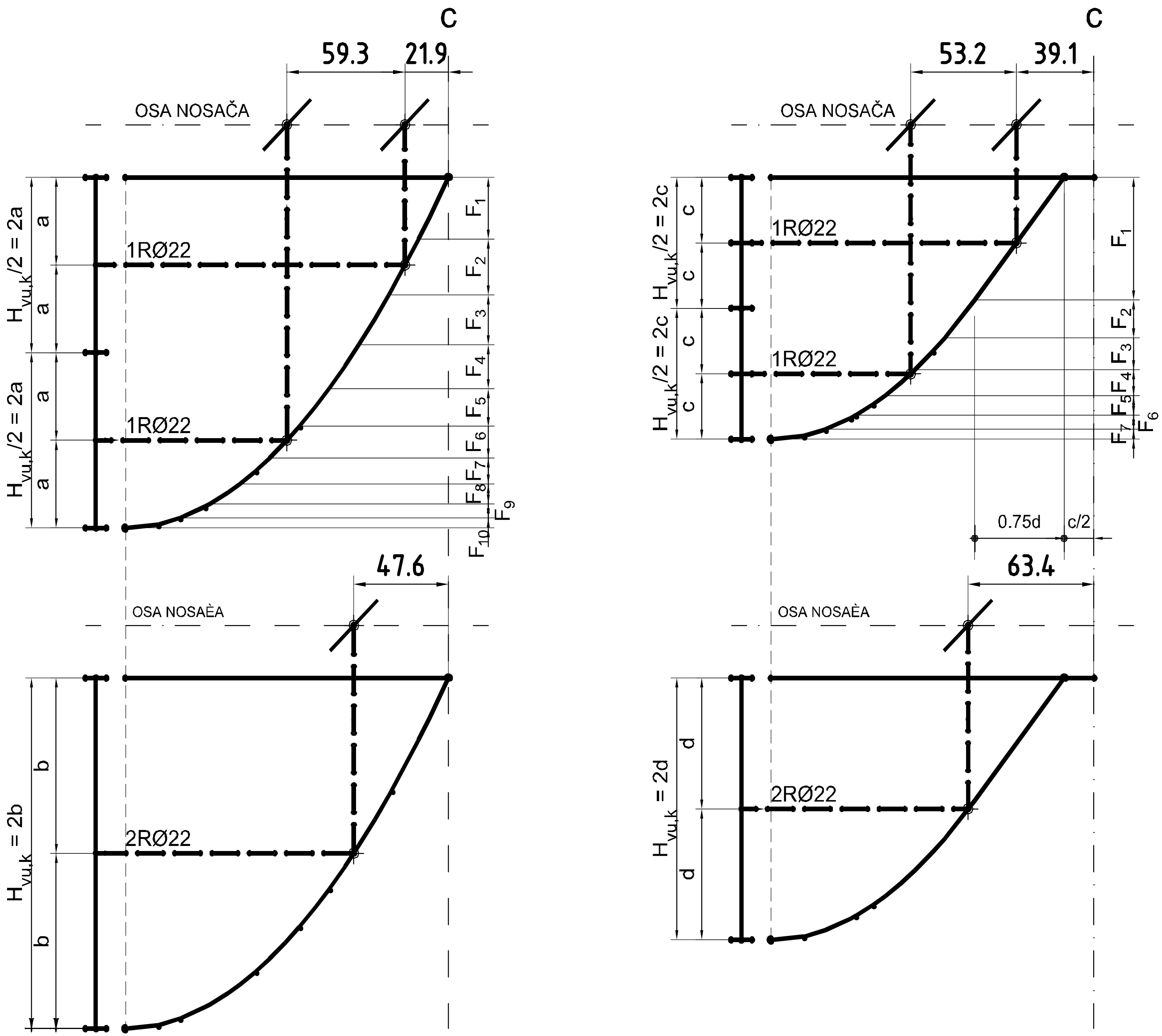
c
DUŽINA OSIGURANJA
KOSIM PROFILIMA
τRu-τu,u
[MPa]
F5 F4 F3 F2 F1
162.4
Hvu,k
F1
F2
F3
F4
F5
KVADRATNA PARABOLA
c
DUŽINA OSIGURANJA
KOSIM PROFILIMA
τRu-τu,u
[MPa]
59.3
21.9
OSA NOSAČA
1RØ22
1RØ22
Hvu,k/2 = 2a
Hvu,k/2 = 2a
a
a
a
a
KVADRATNA PARABOLA



DUŽINA OSIGURANJA
c
KOSIM PROFILIMA
0.75d
c/2
τRu-τu,u
[MPa]
F4
F3
F2
F1
102.4
45
15
Hvu,k
F1
F2
F3
F4
0.75d
c/2
KVADRATNA PARABOLA
PRAVA



DUŽINA OSIGURANJA
c
KOSIM PROFILIMA
0.75d
c/2
τRu-τu,u
[MPa]
53.2
39.1
OSA NOSAČA
Hvu,k/2 = 2c
c
c
1RØ22
Hvu,k/2 = 2c
c
c
1RØ22
F1
F2
F3
F4
0.75d
c/2
KVADRATNA PARABOLA
PRAVA


C
59.3
21.9
OSA NOSAČA
1RØ22
1RØ22
H_vu,k/2 = 2a
a
F1
F2
F3
F4
F5
F6
F7
F8
F9
F10
C
53.2
39.1
OSA NOSAČA
1RØ22
1RØ22
H_vu,k/2 = 2c
c
0.75d
c/2
47.6
OSA NOSAÈA
2RØ22
H_vu,k = 2b
b
63.4
OSA NOSAÈA
2RØ22
H_vu,k = 2d
d

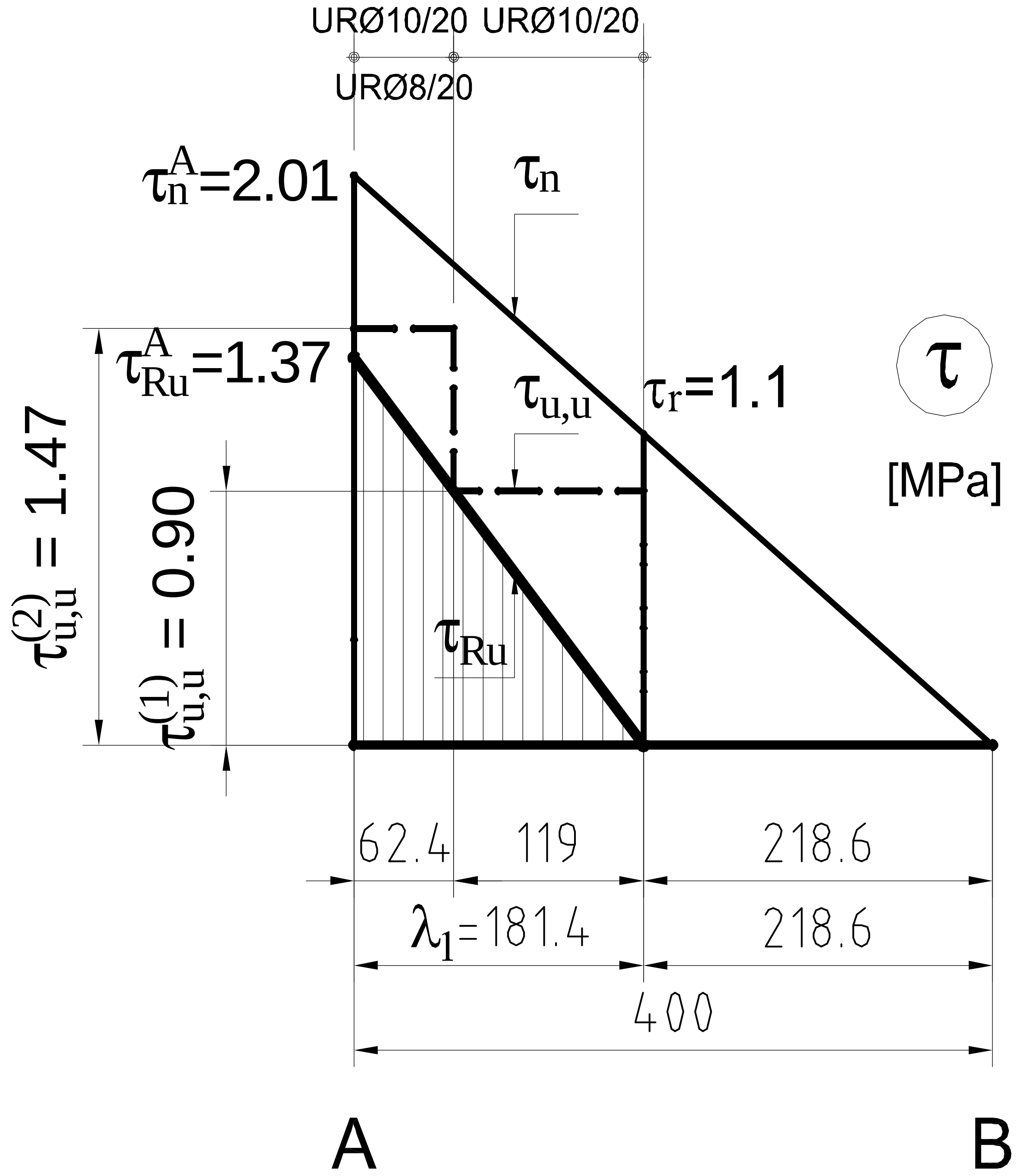
$$T_{mu}^{A} = 336\,kN \Rightarrow \tau_{n}^{A} = \frac{336}{35 \times 47.7} = 0.201\,kN/cm^{2}$$

$$T_{bu}^{A} = \frac{3 \times 0.11 - 0.201}{2} \times 35 \times 47.7 = 107.5\ kN$$

$$T_{Ru} = T_{mu} - T_{bu} = 336 - 107.5 = 228.5\,kN$$

$$\tau_{Ru} = \frac{228.5}{35 \times 47.7} = 0.137\ kN/cm^{2}$$

$$e_{u} = \frac{2 \times 0.785}{35 \times 0.137} \times 40 \times \left(0 + 1 \times 1\right) = 16.7 \times a_{u}^{(1)} = 13.1\ cm$$



URØ10/20 URØ10/20
URØ8/20
τnA=2.01
τn
τRuA=1.37
τu,u
τr=1.1
τ
[MPa]
τu,u(2) = 1.47
τu,u(1) = 0.90
τRu
62.4
119
218.6
λ1=181.4
218.6
400
A
B

$$\lambda_1 \times \frac{\tau_{u,u}^{(1)}}{\tau_{Ru}^{A}} = 181.4 \times \frac{0.90}{1.37} = 119\,\text{cm}$$

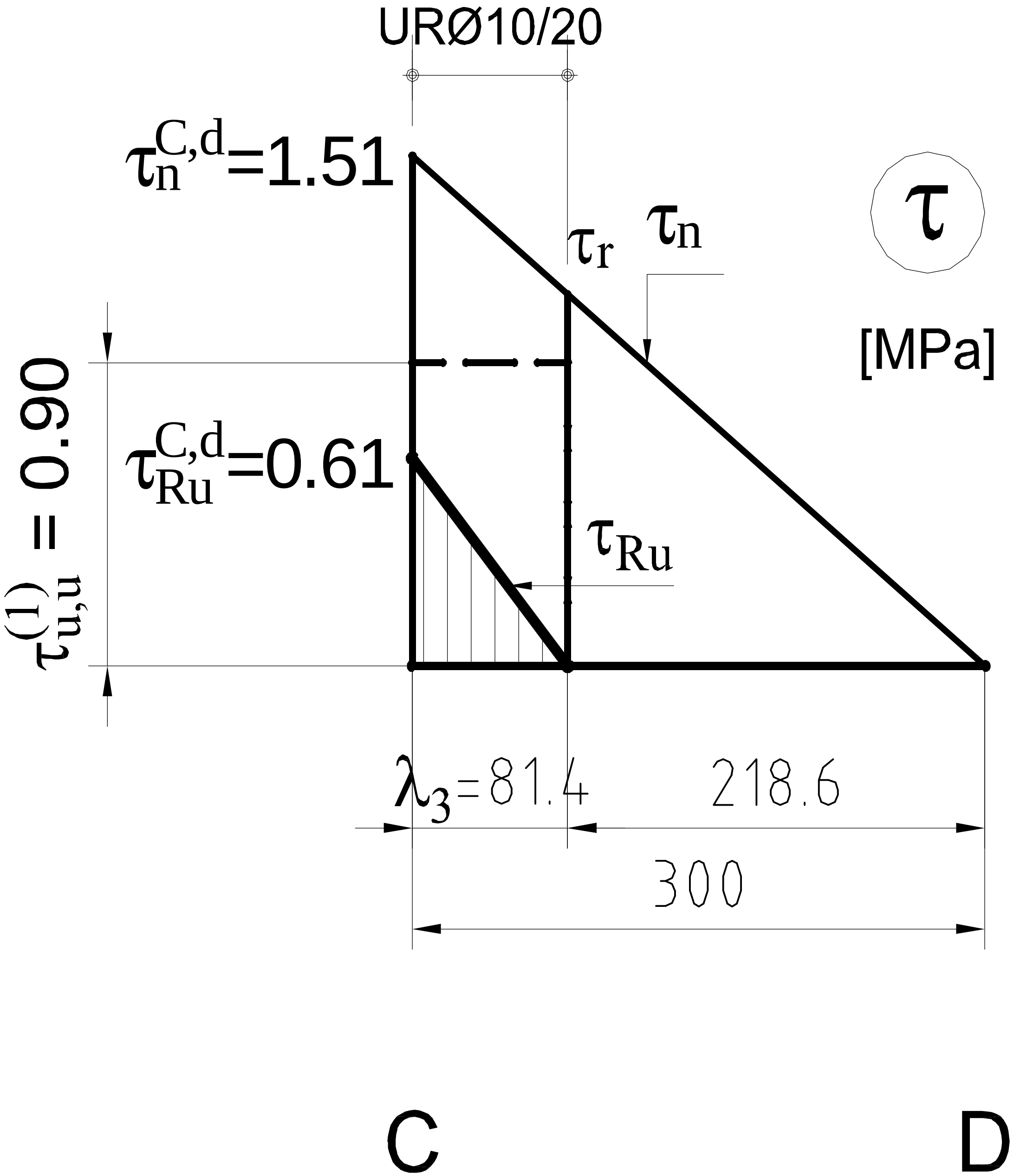
$$\Delta\lambda_1 = 181.4 - 119 = 62.4\ \text{cm}$$

$$\Delta\tau_{Ru}^{A} = \tau_{Ru,max}^{A} - \tau_{u,u}^{(1)} = 1.37 - 0.90 = 0.047\ \text{MPa}$$

$$a_u^{(1)} = \frac{b \times \Delta\tau_{Ru}^{A}}{m \times \sigma_v} \times \frac{1}{(\cos\alpha + \sin\alpha \times \text{ctg}\theta)} \times e_u$$

$$a_u^{(1)} = \frac{35 \times 0.047}{2 \times 40} \times \frac{1}{(0 + 1 \times 1)} \times 20 = 0.412\ \text{cm}^2$$

$$\tau_{u,u}^{(2)} = \frac{2 \times (0.785 + 0.503)}{35 \times 20} \times 40 \times (0 + 1 \times 1) = 0.147\ \text{kN/cm}^2$$



URØ10/20
τn C,d=1.51
τr
τn
τ
[MPa]
τRu C,d=0.61
τRu
τu,u n,(1) = 0.90
λ3=81.4
218.6
300
C
D

$$T_{mu}^{C,d} = 252\,\text{kN} \Rightarrow \tau_n^A = \frac{252}{35\times 47.7} = 0.151\,\text{kN/cm}^2$$

$$T_{bu}^{C,d} = \frac{3\times 0.11 - 0.151}{2}\times 35\times 47.7 = 149.5\ \text{kN}$$

$$T_{Ru} = T_{mu} - T_{bu} = 252 - 149.5 = 102.5\,\text{kN}$$

$$\tau_{Ru} = \frac{102.5}{35\times 47.7} = 0.061\,\text{kN/cm}^2$$

$$e_u = \frac{2\times 0.785}{35\times 0.061}\times 40\times\left(0+1\times 1\right) = 37.2\times a_u^{(1)} = 29.2\ \text{cm}$$