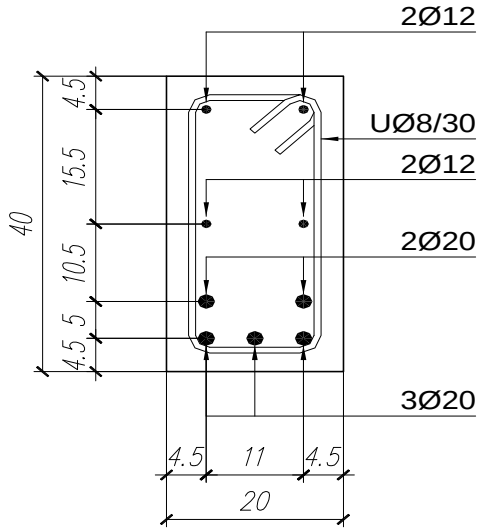
1. Odrediti napone u betonu i armaturi za presek prikazan na skici desno. Presek je opterećen momentima savijanja M_g = 29.6 kNm i M_p = 26 kNm. MB 30, GA 240/360.



$$a_1 = \frac{3\times 4.5 + 2\times 9.5}{5} = 6.5 \text{ cm} \quad ; \quad a_2 = 4.5 \text{ cm}$$

$$h = 40 - 6.5 = 33.5 \text{ cm} \quad ; \quad \alpha_2 = \frac{a_2}{h} = \frac{4.5}{33.5} = 0.134$$

$$A_{a1} = 15.71 \text{ cm}^2 \Rightarrow \mu_1 = \frac{A_{a1}}{b\times h} = \frac{15.71}{20\times 33.5} = 2.34\%$$

$$A_{a2} = 2.26 \text{ cm}^2 \Rightarrow \mu_2 = \frac{A_{a2}}{b\times h} = \frac{2.26}{20\times 33.5} = 0.34\%$$

$$\text{MB 30} \Rightarrow E_b = 31.5 \text{ GPa} \Rightarrow n = \frac{E_a}{E_b} = \frac{210}{31.5} = 6.67$$

$$s^2 + 2\times n\times(\mu_1 + \mu_2)\times s - 2\times n\times(\mu_1 + \mu_2\times\alpha_2) = 0$$

$$s^2 + 2\times 6.67\times(2.34 + 0.34)\times 10^{-2}\times s - 2\times 6.67\times(2.34 + 0.34\times 0.134)\times 10^{-2} = 0$$

$$s^2 + 0.358\times s - 0.319 = 0 \Rightarrow \mathbf{s = 0.414}$$

$$J_{IIb} = \frac{s^2}{2}\times\left(1 - \frac{s}{3}\right) = \frac{0.414^2}{2}\times\left(1 - \frac{0.414}{3}\right) = 0.074$$

$$M_a = M = M_g + M_p = 29.6 + 26.0 = 55.6 \text{ kNm}$$

$$\sigma_b = \frac{M_a}{b\times h^2}\times\frac{s}{J_{IIb} + n\times\mu_2\times(s-\alpha_2)\times(1-\alpha_2)}$$

$$\sigma_b = \frac{55.6\times 10^2}{20\times 33.5^2}\times\frac{0.414}{0.074 + 6.67\times 0.34\times 10^{-2}\times(0.414 - 0.134)\times(1 - 0.134)} = 1.29 \text{ kN/cm}^2$$

$$\sigma_{a1} = n\times\sigma_b\times\frac{1-s}{s} = 6.67\times 1.29\times\frac{1-0.414}{0.414} = 12.23 \text{ kN/cm}^2$$

$$\sigma_{a2} = n\times\sigma_b\times\frac{s-\alpha_2}{s} = 6.67\times 1.29\times\frac{0.414 - 0.134}{0.414} = 5.82 \text{ kN/cm}^2$$

$$\varepsilon_b = \frac{\sigma_b}{E_b} = \frac{1.29}{3.15\times 10^3} = 0.411‰$$

$$\varepsilon_{a1} = \frac{\sigma_{a1}}{E_a} = \frac{12.23}{21\times 10^3} = 0.583‰ \quad ; \quad \varepsilon_{a2} = \frac{\sigma_{a2}}{E_a} = \frac{5.82}{21\times 10^3} = 0.277‰$$

Napomene:

U praktičnim zadacima ***nije obavezno*** uvrstiti u proračun površinu pritisnute armature u preseku (ukoliko nije računski potrebna). Za slučaj $\mathbf{A_{a2} = 0}$, sledi **s = 0.424**, $\mathbf{s_b = 13.6}$ **MPa**, odnosno $\mathbf{s_{a1} = 123.0}$ **MPa**.

Potrebno je uočiti da su vrednosti sračunatih napona vrlo bliske vrednostima dopuštenih napona u betonu i armaturi. Naime, pored dimenzionisanja po teoriji granične nosivosti (Godišnji zadatak, list 2), isti primer je dimenzionisan i u Zadatku 1 - klasična teorija i u računskom smislu bio JEDNOSTRUKO ARMIRAN).

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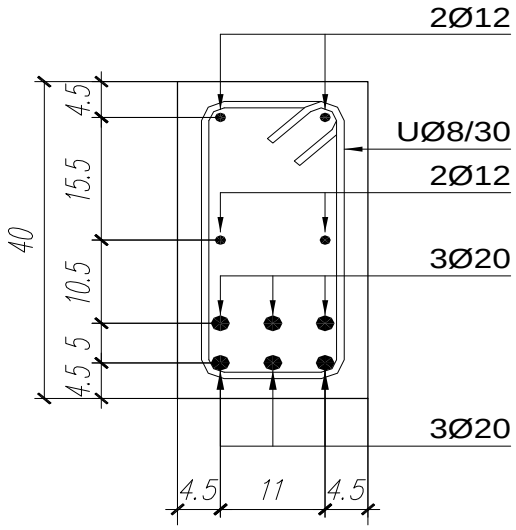
2. Odrediti napone u betonu i armaturi za presek na skici desno. Pored momenata savijanja M_g = 29.6 kNm i M_p = 26 kNm, presek je opterećen i silama zatezanja Z_g = 48.3 kN i Z_p = 42.6 kN. MB 30, GA 240/360.



$$a_1 = \frac{3\times 4.5 + 3\times 9.5}{6} = 7.0 \text{ cm} \quad ; \quad a_2 = 4.5 \text{ cm}$$

$$h = 40 - 7.0 = 33.0 \text{ cm} \quad ; \quad \alpha_2 = \frac{a_2}{h} = \frac{4.5}{33} = 0.136$$

$$A_{a1} = 18.85 \text{ cm}^2 \;\Rightarrow\; \mu_1 = \frac{A_{a1}}{b\times h} = \frac{18.85}{20\times 33} = 2.86\%$$

$$A_{a2} = 2.26 \text{ cm}^2 \;\Rightarrow\; \mu_2 = \frac{A_{a2}}{b\times h} = \frac{2.26}{20\times 33} = 0.34\%$$

$$\text{MB 30} \;\Rightarrow\; E_b = 31.5 \text{ GPa} \;\Rightarrow\; n = \frac{E_a}{E_b} = \frac{210}{31.5} = 6.67$$

$$s^3 + 3\left(\frac{e_{a1}}{h} - 1\right)s^2 + 6n\left(\frac{e_{a1}}{h}\mu_1 + \frac{e_{a2}}{h}\mu_2\right)s - 6n\left(\frac{e_{a1}}{h}\mu_1 + \frac{e_{a2}}{h}\mu_2\alpha_2\right) = 0$$

$$\left.\begin{aligned} M &= M_g + M_p = 29.6 + 26 = 55.6 \text{ kNm} \\ N &= Z_g + Z_p = -(48.3 + 42.6) = -90.9 \text{ kN} \end{aligned}\right\} \;\Rightarrow\; e = \frac{M}{N} = \frac{55.6\times 10^2}{-90.9} = -61.2 \text{ cm}$$

$$e_{a1} = e + y_{a1} = \frac{M}{N} + \left(\frac{d}{2} - a_1\right) = -61.2 + \frac{40}{2} - 7 = -48.2 \text{ cm}$$

$$e_{a2} = e - y_{a2} = \frac{M}{N} - \left(\frac{d}{2} - a_2\right) = -61.2 - \left(\frac{40}{2} - 4.5\right) = -76.7 \text{ cm}$$

$$A = 3\times\left(\frac{e_{a1}}{h} - 1\right) = 3\times\left(\frac{-48.2}{33} - 1\right) = -7.379$$

$$B = 6n\left(\frac{e_{a1}}{h}\mu_1 + \frac{e_{a2}}{h}\mu_2\right) = 6\times 6.67\times\left(\frac{-48.2}{33}\times 2.86 + \frac{-76.7}{33}\times 0.34\right)\times 10^{-2} = -1.986$$

$$C = -6n\left(\frac{e_{a1}}{h}\mu_1 + \frac{e_{a2}}{h}\mu_2\alpha_2\right) = -6\times 6.67\times\left(\frac{-48.2}{33}\times 2.86 + \frac{-76.7}{33}\times 0.34\times 0.136\right)\times 10^{-2} = 1.711$$

$$s^3 + As^2 + Bs + C = s^3 - 7.379\times s^2 - 1.986\times s + 1.711 = 0 \;\Rightarrow\; \mathbf{s = 0.372}$$

$$J_{IIb} = \frac{s^2}{2}\times\left(1 - \frac{s}{3}\right) = \frac{0.372^2}{2}\times\left(1 - \frac{0.372}{3}\right) = 0.061$$

$$M_a = M + N\times y_{a1} = M + N\times\left(\frac{d}{2} - a_1\right) = 55.2 - 90.9\times\left(\frac{0.40}{2} - 0.07\right) = 43.78 \text{ kNm}$$

$$\sigma_b = \frac{M_a}{b\times h^2}\times\frac{s}{J_{IIb} + n\times\mu_2\times(s - \alpha_2)\times(1 - \alpha_2)}$$

$$\sigma_b = \frac{43.78\times 10^2}{20\times 33^2}\times\frac{0.372}{0.061 + 6.67\times 0.34\times 10^{-2}\times(0.372 - 0.136)\times(1 - 0.136)} = 1.14 \text{ kN/cm}^2$$

$$\sigma_{a1} = n \times \sigma_b \times \frac{1-s}{s} = 6.67 \times 1.14 \times \frac{1-0.372}{0.372} = 12.87 \text{ kN/cm}^2$$

$$\sigma_{a2} = n \times \sigma_b \times \frac{s-\alpha_2}{s} = 6.67 \times 1.14 \times \frac{0.372-0.136}{0.372} = 4.84 \text{ kN/cm}^2$$

$$\varepsilon_b = \frac{\sigma_b}{E_b} = \frac{1.14}{3.15 \times 10^3} = 0.363‰$$

$$\varepsilon_{a1} = \frac{\sigma_{a1}}{E_a} = \frac{12.87}{21 \times 10^3} = 0.613‰ \quad ; \quad \varepsilon_{a2} = \frac{\sigma_{a2}}{E_a} = \frac{4.84}{21 \times 10^3} = 0.230‰$$

Napomena:
Za slučaj $\mathbf{A_{a2} = 0}$, sledi **s = 0.383**, s_b **= 12.0 MPa**, odnosno s_{a1} **= 128.9 MPa**.

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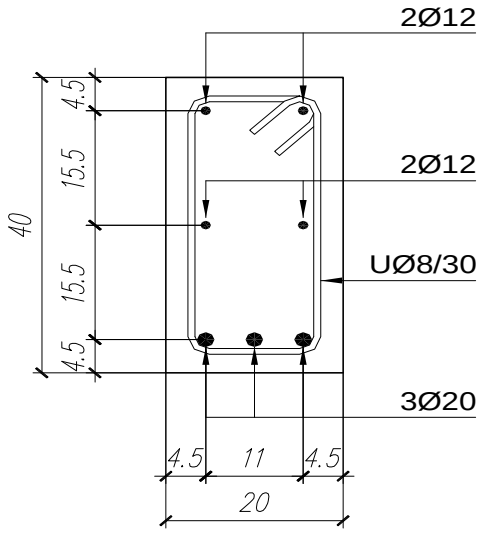
3. Odrediti napone u betonu i armaturi za presek na skici desno. Pored momenata savijanja M_g = 29.6 kNm i M_p = 26 kNm, presek je opterećen i silama pritiska N_g = 64.4 kN i N_p = 56.9 kN. MB 30, GA 240/360.



$a_1 = a_2 = 4.5$ cm

$h_. = 40 - 4.5 = 35.5$ cm ; $\alpha_2 = \frac{a_2}{h} = \frac{4.5}{35.5} = 0.127$

$A_{a1} = 9.42 \text{ cm}^2 \Rightarrow \mu_1 = \frac{A_{a1}}{b \times h} = \frac{9.42}{20 \times 35.5} = 1.33\%$

$A_{a2} = 2.26 \text{ cm}^2 \Rightarrow \mu_2 = \frac{A_{a2}}{b \times h} = \frac{2.26}{20 \times 35.5} = 0.32\%$

MB 30 $\Rightarrow E_b = 31.5$ GPa $\Rightarrow n = \frac{E_a}{E_b} = \frac{210}{31.5} = 6.67$

$$s^3 + 3\left(\frac{e_{a1}}{h} - 1\right)s^2 + 6n\left(\frac{e_{a1}}{h}\mu_1 + \frac{e_{a2}}{h}\mu_2\right)s - 6n\left(\frac{e_{a1}}{h}\mu_1 + \frac{e_{a2}}{h}\mu_2\alpha_2\right) = 0$$

$$\left.\begin{aligned} M &= M_g + M_p = 29.6 + 26 = 55.6 \text{ kNm} \\ N &= N_g + N_p = 64.4 + 56.9 = 121.3 \text{ kN} \end{aligned}\right\} \Rightarrow e = \frac{M}{N} = \frac{55.6 \times 10^2}{121.3} = 45.8 \text{ cm}$$

$$e_{a1} = e + y_{a1} = \frac{M}{N} + \left(\frac{d}{2} - a_1\right) = 45.8 + \frac{40}{2} - 4.5 = 61.3 \text{ cm}$$

$$e_{a2} = e - y_{a2} = \frac{M}{N} - \left(\frac{d}{2} - a_2\right) = 45.8 - \left(\frac{40}{2} - 4.5\right) = 30.3 \text{ cm}$$

$$A = 3 \times \left(\frac{e_{a1}}{h} - 1\right) = 3 \times \left(\frac{61.3}{35.5} - 1\right) = 2.183$$

$$B = 6n\left(\frac{e_{a1}}{h}\mu_1 + \frac{e_{a2}}{h}\mu_2\right) = 6 \times 6.67 \times \left(\frac{61.3}{35.5} \times 1.33 + \frac{30.3}{35.5} \times 0.32\right) \times 10^{-2} = 1.026$$

$$C = -6n\left(\frac{e_{a1}}{h}\mu_1 + \frac{e_{a2}}{h}\mu_2\alpha_2\right) = -6\times 6.67\times\left(\frac{61.3}{35.5}\times 1.33 + \frac{30.3}{35.5}\times 0.32\times 0.127\right)\times 10^{-2} = -0.931$$

$$s^3 + As^2 + Bs + C = s^3 + 2.183\times s^2 + 1.026\times s - 0.931 = 0 \quad \Rightarrow \quad \mathbf{s = 0.432}$$

$$J_{IIb} = \frac{s^2}{2}\times\left(1-\frac{s}{3}\right) = \frac{0.432^2}{2}\times\left(1-\frac{0.432}{3}\right) = 0.080$$

$$M_a = M + N\times y_{a1} = M + N\times\left(\frac{d}{2} - a_1\right) = 55.2 + 121.3\times\left(\frac{0.40}{2} - 0.045\right) = 74.4 \text{ kNm}$$

$$\sigma_b = \frac{M_a}{b\times h^2}\times\frac{s}{J_{IIb} + n\times\mu_2\times(s-\alpha_2)\times(1-\alpha_2)}$$

$$\sigma_b = \frac{74.4\times 10^2}{20\times 35.5^2}\times\frac{0.432}{0.080 + 6.67\times 0.32\times 10^{-2}\times(0.432-0.127)\times(1-0.127)} = 1.49 \text{ kN/cm}^2$$

$$\sigma_{a1} = n\times\sigma_b\times\frac{1-s}{s} = 6.67\times 1.49\times\frac{1-0.432}{0.432} = 13.07 \text{ kN/cm}^2$$

$$\sigma_{a2} = n\times\sigma_b\times\frac{s-\alpha_2}{s} = 6.67\times 1.49\times\frac{0.432-0.127}{0.432} = 7.02 \text{ kN/cm}^2$$

$$\varepsilon_b = \frac{\sigma_b}{E_b} = \frac{1.49}{3.15\times 10^3} = 0.473‰$$

$$\varepsilon_{a1} = \frac{\sigma_{a1}}{E_a} = \frac{13.07}{21\times 10^3} = 0.623‰ \quad ; \quad \varepsilon_{a2} = \frac{\sigma_{a2}}{E_a} = \frac{7.02}{21\times 10^3} = 0.334‰$$

Napomena:
Za slučaj $\mathbf{A_{a2} = 0}$, sledi **s = 0.442**, s_b **= 15.7 MPa**, odnosno s_{a1} **= 132.1 MPa**.

I ovaj primer je dimenzionisan u Zadatku 1 - klasična teorija i u računskom smislu bio DVOSTRUKO ARMIRAN. To objašnjava znatnije odstupanje sračunatog napona pritiska u betonu od dopuštenog rubnog napona, koji za beton MB 30 iznosi σ_r = 12 MPa. Nasuprot tome, napon u zategnutoj armaturi je u granicama dopuštenog napona od σ_a = 140 MPa.

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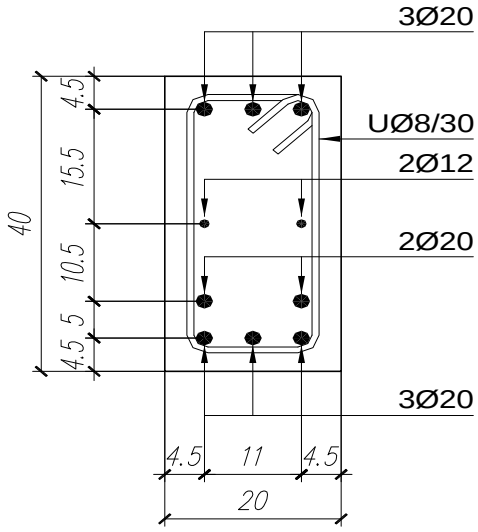
4. Odrediti napone u betonu i armaturi za presek na skici desno. Pored momenata savijanja M_g = 47.4 kNm i M_p = 41.9 kNm, presek je opterećen i silama pritiska N_g = 146.6 kN i N_p = 129.4 kN. MB 30, GA 240/360.



$$a_1 = \frac{3\times 4.5 + 2\times 9.5}{5} = 6.5 \text{ cm} \quad ; \quad a_2 = 4.5 \text{ cm}$$

$$h = 40 - 6.5 = 33.5 \text{ cm} \quad ; \quad \alpha_2 = \frac{a_2}{h} = \frac{4.5}{33.5} = 0.134$$

$$A_{a1} = 15.71 \text{ cm}^2 \quad \Rightarrow \quad \mu_1 = \frac{A_{a1}}{b\times h} = \frac{15.71}{20\times 33.5} = 2.34\%$$

$$A_{a2} = 9.42 \text{ cm}^2 \quad \Rightarrow \quad \mu_2 = \frac{A_{a2}}{b\times h} = \frac{9.42}{20\times 33.5} = 1.41\%$$

MB 30 $\Rightarrow$ $E_b = 31.5$ GPa $\Rightarrow$ $n = \dfrac{E_a}{E_b} = \dfrac{210}{31.5} = 6.67$

$$\left.\begin{aligned} M &= M_g + M_p = 47.4 + 41.9 = 89.3\ \text{kNm} \\ N &= N_g + N_p = 146.6 + 129.4 = 276\ \text{kN} \end{aligned}\right\} \Rightarrow\ e = \frac{M}{N} = \frac{89.3\times 10^2}{276} = 32.4\ \text{cm}$$

$$e_{a1} = e + y_{a1} = \frac{M}{N} + \left(\frac{d}{2} - a_1\right) = 32.4 + \frac{40}{2} - 6.5 = 45.9\ \text{cm}$$

$$e_{a2} = e - y_{a2} = \frac{M}{N} - \left(\frac{d}{2} - a_2\right) = 32.4 - \left(\frac{40}{2} - 4.5\right) = 16.9\ \text{cm}$$

$$A = 3\times\left(\frac{e_{a1}}{h} - 1\right) = 3\times\left(\frac{45.9}{33.5} - 1\right) = 1.106$$

$$B = 6n\left(\frac{e_{a1}}{h}\mu_1 + \frac{e_{a2}}{h}\mu_2\right) = 6\times 6.67\times\left(\frac{45.9}{33.5}\times 2.34 + \frac{16.9}{33.5}\times 1.41\right)\times 10^{-2} = 1.567$$

$$C = -6n\left(\frac{e_{a1}}{h}\mu_1 + \frac{e_{a2}}{h}\mu_2\alpha_2\right) = -6\times 6.67\times\left(\frac{45.9}{33.5}\times 2.34 + \frac{16.9}{33.5}\times 1.41\times 0.134\right)\times 10^{-2} = -1.322$$

$$s^3 + As^2 + Bs + C = s^3 + 1.106\times s^2 + 1.567\times s - 1.322 = 0 \quad \Rightarrow \quad \mathbf{s = 0.539}$$

$$J_{IIb} = \frac{s^2}{2}\times\left(1 - \frac{s}{3}\right) = \frac{0.539^2}{2}\times\left(1 - \frac{0.539}{3}\right) = 0.119$$

$$M_a = M + N\times y_{a1} = M + N\times\left(\frac{d}{2} - a_1\right) = 89.3 + 276\times\left(\frac{0.40}{2} - 0.065\right) = 126.6\ \text{kNm}$$

$$\sigma_b = \frac{M_a}{b\times h^2}\times\frac{s}{J_{IIb} + n\times\mu_2\times(s - \alpha_2)\times(1 - \alpha_2)}$$

$$\sigma_b = \frac{126.6\times 10^2}{20\times 33.5^2}\times\frac{0.539}{0.119 + 6.67\times 1.41\times 10^{-2}\times(0.539 - 0.134)\times(1 - 0.134)} = 2.00\ \text{kN/cm}^2$$

$$\sigma_{a1} = n\times\sigma_b\times\frac{1 - s}{s} = 6.67\times 2.00\times\frac{1 - 0.539}{0.539} = 11.41\ \text{kN/cm}^2$$

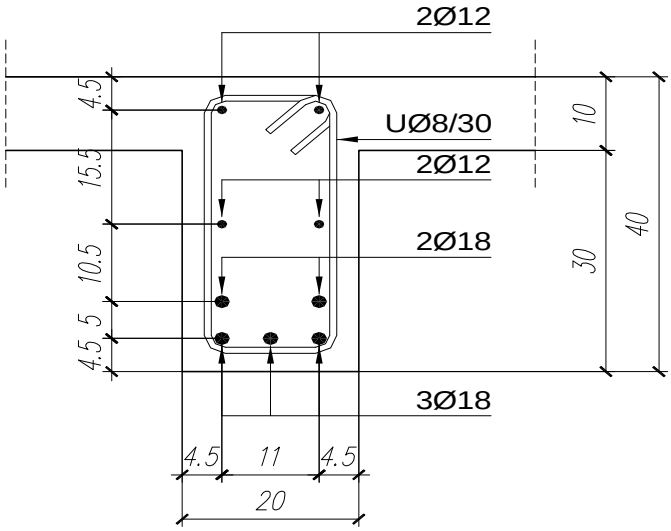
$$\sigma_{a2} = n\times\sigma_b\times\frac{s - \alpha_2}{s} = 6.67\times 2.00\times\frac{0.539 - 0.134}{0.539} = 10.01\ \text{kN/cm}^2$$

$$\varepsilon_b = \frac{\sigma_b}{E_b} = \frac{2.00}{3.15\times 10^3} = 0.635‰$$

$$\varepsilon_{a1} = \frac{\sigma_{a1}}{E_a} = \frac{11.41}{21\times 10^3} = 0.544‰ \quad ; \quad \varepsilon_{a2} = \frac{\sigma_{a2}}{E_a} = \frac{10.01}{21\times 10^3} = 0.477‰$$

Napomena:
Za slučaj $\mathbf{A_{a2} = 0}$, sledi **s = 0.572**, $\mathbf{s_b = 24.4\ MPa}$, odnosno $\mathbf{s_{a1} = 121.5\ MPa}$. Međutim, ovde je očito da u preseku postoji računska pritisnuta armatura, pa bi njeno zanemarivanje bilo nedopustivo. Sračunati napona pritiska u betonu znatno premašuje vrednost dopuštenog rubnog napona, dok je napon u zategnutoj armaturi ponovo u granicama dopuštenog. Nešto manja vrednost napona σ_{a1} se može objasniti usvajanjem nešto veće površine armature od računski potrebne (15.71 umesto 14.01 cm^2).

5. Odrediti napone u betonu i armaturi za presek prikazan na skici desno. Presek je opterećen momentima savijanja M_g = 29.6 kNm i M_p = 26 kNm. Računska aktivna širina ploče B = 105 cm. MB 30, GA 240/360.



$$a_1 = \frac{3\times 4.5 + 2\times 9.5}{5} = 6.5 \text{ cm}$$

$h_. = 40 - 6.5 = 33.5$ cm

$$a_2 = 4.5 \text{ cm} \;\Rightarrow\; \alpha_2 = \frac{a_2}{h} = \frac{4.5}{33.5} = 0.134$$

Iz praktičnih razloga se pretpostavlja da se neutralna linija nalazi u ploči, pa se ***s*** određuje za pravougaoni presek širine B = 105 cm:

$$A_{a1} = 12.72 \text{ cm}^2 \;\Rightarrow\; \mu_1 = \frac{A_{a1}}{B\times h} = \frac{12.72}{105\times 33.5} = 0.36\%$$

$$A_{a2} = 2.26 \text{ cm}^2 \;\Rightarrow\; \mu_2 = \frac{A_{a2}}{B\times h} = \frac{2.26}{105\times 33.5} = 0.06\%$$

$$\text{MB 30} \;\Rightarrow\; E_b = 31.5 \text{ GPa} \;\Rightarrow\; n = \frac{E_a}{E_b} = \frac{210}{31.5} = 6.67$$

$$s^2 + 2\times n\times(\mu_1 + \mu_2)\times s - 2\times n\times(\mu_1 + \mu_2\times\alpha_2) = 0$$

$$s^2 + 2\times 6.67\times(0.36 + 0.06)\times 10^{-2}\times s - 2\times 6.67\times(0.36 + 0.06\times 0.134)\times 10^{-2} = 0$$

$$s^2 + 0.057\times s - 0.049 = 0 \;\Rightarrow\; \mathbf{s = 0.196} < \delta = \frac{d_p}{h} = \frac{10}{33.5} = 0.299$$

Pretpostavka o položaju neutralne linije je tačna, pa dalje sledi:

$$J_{IIb} = \frac{s^2}{2}\times\left(1 - \frac{s}{3}\right) = \frac{0.196^2}{2}\times\left(1 - \frac{0.196}{3}\right) = 0.018$$

$M_a = M = M_g + M_p = 29.6 + 26.0 = 55.6$ kNm

$$\sigma_b = \frac{M_a}{B\times h^2}\times\frac{s}{J_{IIb} + n\times\mu_2\times(s - \alpha_2)\times(1 - \alpha_2)}$$

$$\sigma_b = \frac{55.6\times 10^2}{105\times 33.5^2}\times\frac{0.196}{0.018 + 6.67\times 0.06\times 10^{-2}\times(0.196 - 0.134)\times(1 - 0.134)} = 0.51 \text{ kN/cm}^2$$

$$\sigma_{a1} = n\times\sigma_b\times\frac{1 - s}{s} = 6.67\times 0.51\times\frac{1 - 0.196}{0.196} = 13.97 \text{ kN/cm}^2$$

$$\sigma_{a2} = n\times\sigma_b\times\frac{s - \alpha_2}{s} = 6.67\times 0.51\times\frac{0.196 - 0.134}{0.196} = 1.06 \text{ kN/cm}^2$$

$$\varepsilon_b = \frac{\sigma_b}{E_b} = \frac{0.51}{3.15\times 10^3} = 0.162‰$$

$$\varepsilon_{a1} = \frac{\sigma_{a1}}{E_a} = \frac{13.97}{21\times10^3} = 0.665‰ \quad ; \quad \varepsilon_{a2} = \frac{\sigma_{a2}}{E_a} = \frac{1.06}{21\times10^3} = 0.051‰$$

Napomene:
Za slučaj $\mathbf{A_{a2} = 0}$, sledi $\mathbf{s = 0.197}$, $\mathbf{s_b = 5.1\ MPa}$, odnosno $\mathbf{s_{a1} = 139.6\ MPa}$.

Jasno je da kod preseka sa ovako velikom pritisnutom površinom betona doprinos armature smeštene uz pritisnutu ivicu preseka treba zanemariti. Očekivano, sračunati napon u betonu je znatno ispod dopuštenog rubnog napona, dok je napon u zategnutoj armaturi ponovo vrlo blizak dopuštenom.

Ponovo se ukazuje da iz pretpostavljenog položaja neutralne linije sledi proračun za presek ***pravougaonog oblika***, koga karakteriše SAMO JEDNA širina (u konkretnom slučaju, B=105 cm), bez obzira da li je u opštim izrazima za proračun napona kod pravougaonog preseka označena malim ili velikim slovom. Za slučaj da se neutralna linija nađe u rebru, proračun se sprovodi za odgovarajući presek oblika **T**.

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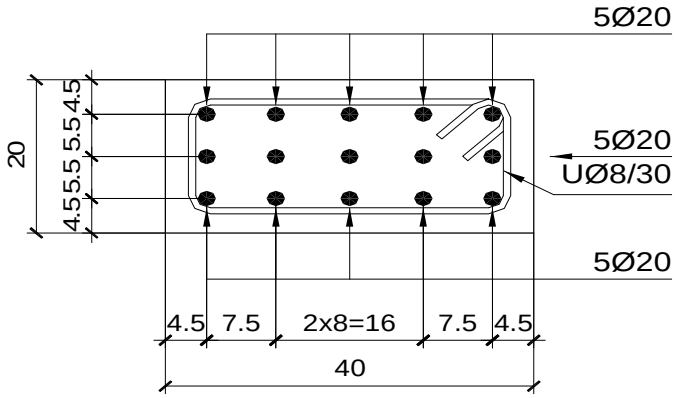
6. Odrediti napon u armaturi za presek prikazan na skici desno. Presek je opterećen silama zatezanja Z_g = 305 kN i Z_p = 337 kN. MB 25, GA 240/360.



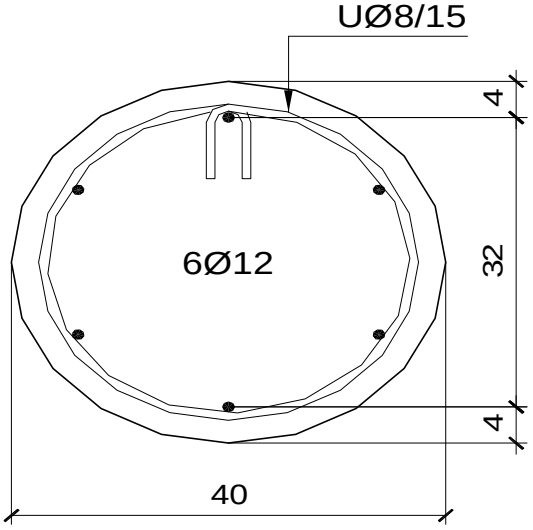
A_a = 47.12 cm^2 (15Ø20)

$Z = Z_g + Z_p = 305 + 337 = 642$ kN

$$\sigma_a = \frac{Z}{A_a} = \frac{642}{47.12} = 13.62\ \text{kN/cm}^2$$

$$\varepsilon_a = \frac{\sigma_a}{E_a} = \frac{13.62}{21\times10^3} = 0.649‰$$

7. Odrediti napone u betonu i armaturi za presek prikazan na skici desno. Presek je opterećen silama pritiska N_g = 630 kN i N_p = 398 kN. MB 25, GA 240/360.



MB 25 $\Rightarrow$ E_b = 30 GPa $\Rightarrow$ $n = \frac{E_a}{E_b} = \frac{210}{30} = 7$

A_a = 6.79 cm^2 (6Ø12)

$$A_b = \frac{D^2\times\pi}{4} = \frac{40^2\times\pi}{4} = 1257\ \text{cm}^2$$

Površina idealizovanog preseka:

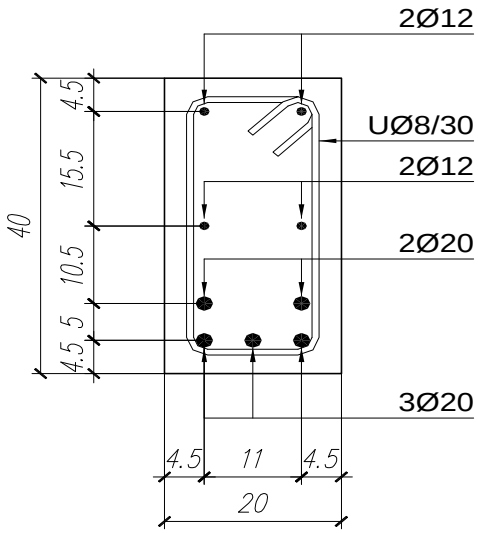
$A_i = A_b + n\times A_a = 1257 + 7\times6.79 = 1304$ cm^2

$N = N_g + N_p = 630 + 398 = 1028$ kN

$$\sigma_b = \frac{N}{A_i} = \frac{1028}{1304} = 0.79\ \text{kN/cm}^2 \quad \Rightarrow \quad \varepsilon_b = \frac{\sigma_b}{E_b} = \frac{0.79}{3\times10^3} = 0.263‰ = \varepsilon_a$$

$$\sigma_a = \varepsilon_a \times E_a = n\times\sigma_b = 0.263\times10^{-3}\times21\times10^3 = 5.52\ \text{kN/cm}^2$$

8. Za presek iz Primera 1 odrediti napone u betonu i armaturi, vodeći računa o dugotrajnom dejstvu stalnog opterećenja. Usvojeno: $\varphi_\infty = 2.5$; $\chi_\infty = 2.5$ $\varepsilon_{s\infty} = 0.4‰$.



Pored geometrijskih karakteristika preseka, u Primeru 1 sračunato, za UKUPNO G+P opterećenje (trenutak t=0):

$s_0 = 0.414$

$$\sigma_{b0}^{G+P} = 12.9\,\text{MPa} \quad \Rightarrow \quad \varepsilon_{b0}^{G+P} = 0.411‰$$
$$\sigma_{a1,0}^{G+P} = 122.3\,\text{MPa} \quad \Rightarrow \quad \varepsilon_{a1,0}^{G+P} = 0.583‰$$
$$\sigma_{a2,0}^{G+P} = 58.2\,\text{MPa} \quad \Rightarrow \quad \varepsilon_{a2,0}^{G+P} = 0.277‰$$

Od toga, usled dugotrajnog (stalnog) opterećenja:

$$\sigma_{b0}^{G} = \frac{29.6}{55.6}\times 12.9 = 6.9\,\text{MPa} \quad \Rightarrow \quad \varepsilon_{b0}^{G} = 0.219‰$$
$$\sigma_{a1,0}^{G} = \frac{29.6}{55.6}\times 122.3 = 65.2\,\text{MPa} \quad \Rightarrow \quad \varepsilon_{a1,0}^{G} = 0.311‰$$
$$\sigma_{a2,0}^{G} = \frac{29.6}{55.6}\times 58.2 = 31.0\,\text{MPa} \quad \Rightarrow \quad \varepsilon_{a2,0}^{G} = 0.148‰$$

Položaj neutralne linije u trenutku $t\to\infty$ određuje se iz jednačine oblika:

$$\boxed{A\cdot s_t^3 + B\cdot s_t^2 + C\cdot s_t + D = 0}$$

pri čemu je:

$$a_t = n\times(1+\chi\times\varphi) = 6.67\times(1+0.8\times 2.5) = 20$$

$$b_t = n\times(1-\chi)\times\varphi = 6.67\times(1-0.8)\times 2.5 = 3.33$$

$$A = -(\mu_1+\mu_2)\cdot(b_t\sigma_{b0} + E_a\varepsilon_{st})\cdot\frac{bh^2}{M} \quad ; \quad B = 3+4\cdot(\mu_1+\mu_2\alpha_2)\cdot(b_t\sigma_{b0} + E_a\varepsilon_{st})\cdot\frac{bh^2}{M}$$

$$C = 6a_t\cdot(\mu_1+\mu_2\alpha_2) - 3\cdot(\mu_1+\mu_2\alpha_2^2)\cdot(b_t\sigma_{b0} + E_a\varepsilon_{st})\cdot\frac{bh^2}{M} \quad ; \quad D = -6a_t\cdot(\mu_1+\mu_2\alpha_2)$$

Zamenom sračunatih vrednosti sledi:

$$A = -(2.34+0.34)\times 10^{-2}\times(3.33\times 0.69 + 210\times 0.40)\times\frac{20\times 33.5^2}{29.6\times 10^2} = -2.176$$

$$B = 3+4\times(2.34+0.34\times 0.134)\times 10^{-2}\times(3.33\times 0.69 + 210\times 0.40)\times\frac{20\times 33.5^2}{29.6\times 10^2} = 10.754$$

$$C = 6\times 20\times(2.34+0.34\times 0.134)\times 10^{-2} -$$
$$-3\times(2.34+0.34\times 0.134^2)\times 10^{-2}\times(3.33\times 0.69 + 210\times 0.40)\times\frac{20\times 33.5^2}{29.6\times 10^2} = -2.502$$

$$D = -6\times 20\times(2.34+0.34\times 0.134)\times 10^{-2} = -2.868$$

$$\boxed{-2.176\cdot s_t^3 + 10.754\cdot s_t^2 - 2.502\cdot s_t - 2.868 = 0} \quad \Rightarrow \quad \mathbf{s_t = 0.710}$$

Naponi u betonu i armaturi:

$$\sigma_{bt}=\frac{M}{bh^2}\cdot\frac{s_t}{J_{IIb,t}+a_t\cdot\mu_2\cdot(s_t-\alpha_2)\cdot(1-\alpha_2)}-\frac{(b_t\sigma_{b0}+E_a\varepsilon_{st})\cdot\mu_2\cdot(s_t-\alpha_2)\cdot(1-\alpha_2)}{J_{IIb,t}+a_t\cdot\mu_2\cdot(s_t-\alpha_2)\cdot(1-\alpha_2)}$$

$$J_{IIb,t}=\frac{s_t^2}{2}\times\left(1-\frac{s_t}{3}\right)=\frac{0.710^2}{2}\times\left(1-\frac{0.710}{3}\right)=0.192$$

$$\sigma_{bt}=\frac{29.6\times10^2}{20\times33.5^2}\times\frac{0.710}{0.192+20\times0.34\times10^2\times(0.710-0.134)\times(1-0.134)}-$$
$$-\frac{(3.33\times0.69+210\times0.40)\times0.34\times10^2\times(0.710-0.134)\times(1-0.134)}{0.192+20\times0.34\times10^2\times(0.710-0.134)\times(1-0.134)}=0.33\,\text{kN/cm}^2$$

$$\varepsilon_{bt}=\frac{1}{E_a}\times(a_t\times\sigma_{bt}+b_t\times\sigma_{b0})+\varepsilon_{st}=\frac{(20\times3.3+3.33\times6.9)}{210\times10^3}+0.4\times10^{-3}=0.828‰$$

$$\sigma_{a1t}=(a_t\sigma_{bt}+b_t\sigma_{b0}+E_a\varepsilon_{st})\cdot\frac{1-s_t}{s_t}=(20\times3.3+3.33\times6.9+210\times0.40)\cdot\frac{1-0.710}{0.710}=71.0\,\text{MPa}$$

$$\sigma_{a2t}=(a_t\sigma_{bt}+b_t\sigma_{b0}+E_a\varepsilon_{st})\cdot\frac{s_t-\alpha_2}{s_t}=(20\times3.3+3.33\times6.9+210\times0.40)\cdot\frac{0.710-0.134}{0.710}=141.0\,\text{MPa}$$

$$\varepsilon_{a1t}=\frac{\sigma_{a1t}}{E_a}=\frac{71.0}{210\times10^3}=0.338‰$$

$$\varepsilon_{a2t}=\frac{\sigma_{a2t}}{E_a}=\frac{141.0}{210\times10^3}=0.671‰$$

Ukupno, od opterećenja G+P, u trenutku t→∞:

$$\sigma_{bt}^{G+P}=12.9+(3.3-6.9)=9.4\,\text{MPa}\quad\Rightarrow\quad\varepsilon_{bt}^{G+P}=0.411+(0.828-0.219)=1.020‰$$
$$\sigma_{a1,t}^{G+P}=122.3+(71.0-65.2)=128.3\,\text{MPa}\quad\Rightarrow\quad\varepsilon_{a1,t}^{G+P}=0.611‰$$
$$\sigma_{a2,t}^{G+P}=58.2+(141.0-31.0)=168.2\,\text{MPa}\quad\Rightarrow\quad\varepsilon_{a2,t}^{G+P}=0.801‰$$

U donjoj tabeli su prikazane vrednosti napona i dilatacija po fazama, usled dugotrajnog, odnosno ukupnog dugotrajnog i kratkotrajnog opterećenja, sa i bez udela skupljanja betona:

		t = 0		**t→∞ sa skupljanjem**		**t→∞ bez skupljanja**	
		G_0	**$(G+P)_0$**	$G_{\infty,s}$	**$(G+P)_{\infty,s}$**	G_∞	**$(G+P)_\infty$**
σ_b	[MPa]	6.9	**12.9**	3.3	**9.4**	4.4	**10.4**
σ_{a1}	[MPa]	65.2	**122.5**	71.0	**128.3**	69.6	**126.9**
σ_{a2}	[MPa]	31.0	**58.3**	141.0	**168.2**	86.2	**113.5**
ε_b	[‰]	0.219	**0.411**	0.828	**1.020**	0.526	**0.718**
ε_{a1}	[‰]	0.311	**0.583**	0.338	**0.611**	0.331	**0.604**
ε_{a2}	[‰]	0.148	**0.277**	0.671	**0.801**	0.411	**0.540**