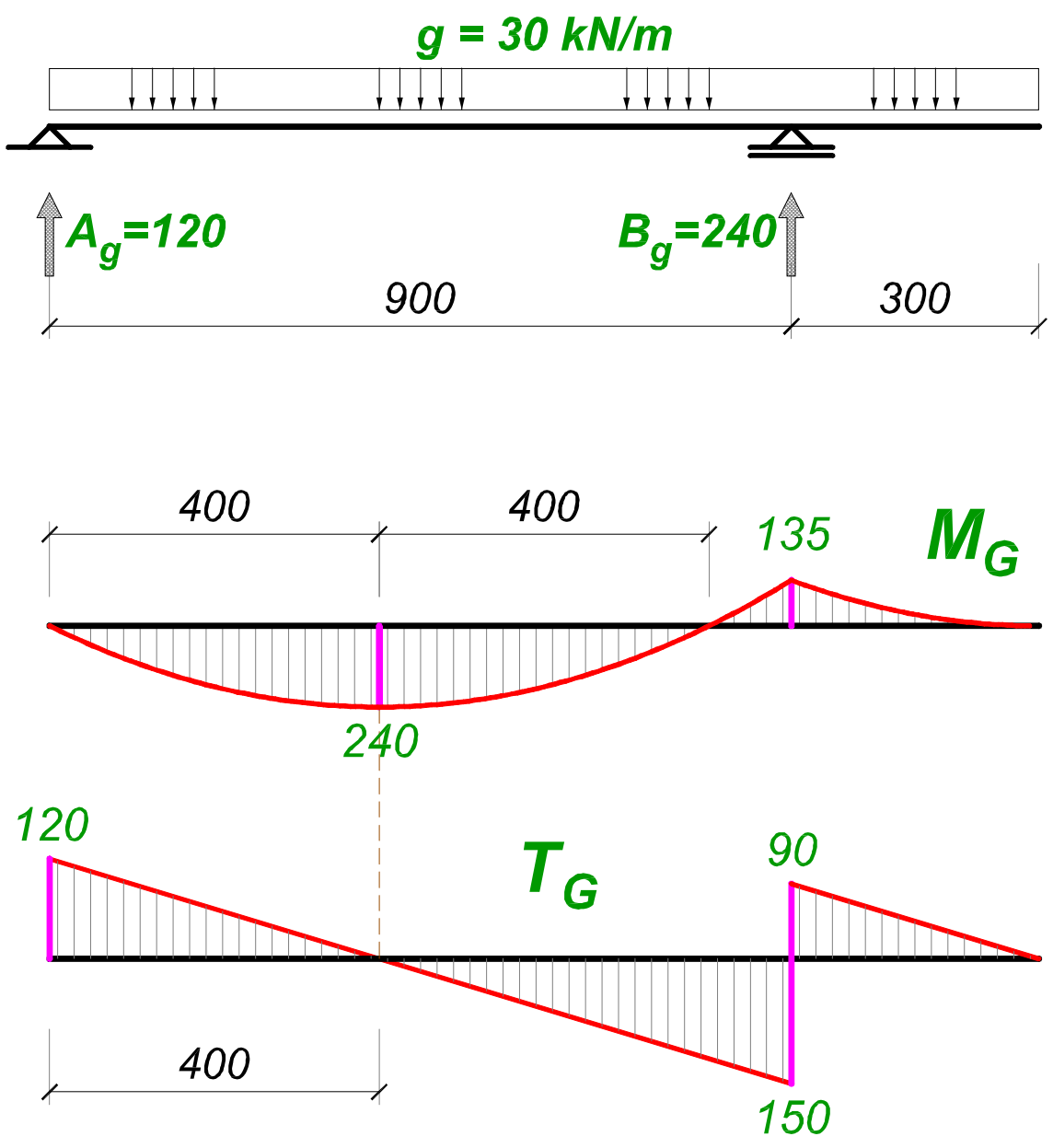


# Stalno opterećenje



$$B_g = (30 \times 12^2 / 2) / 9 = 240 \text{ kN}$$

$$A_g = 30 \times 12 - 240 = 120 \text{ kN}$$

$$M_g^{OSL} = 30 \times 3.0^2 / 2$$

$$M_g^{OSL} = 135 \text{ kNm}$$

maks. M u polju

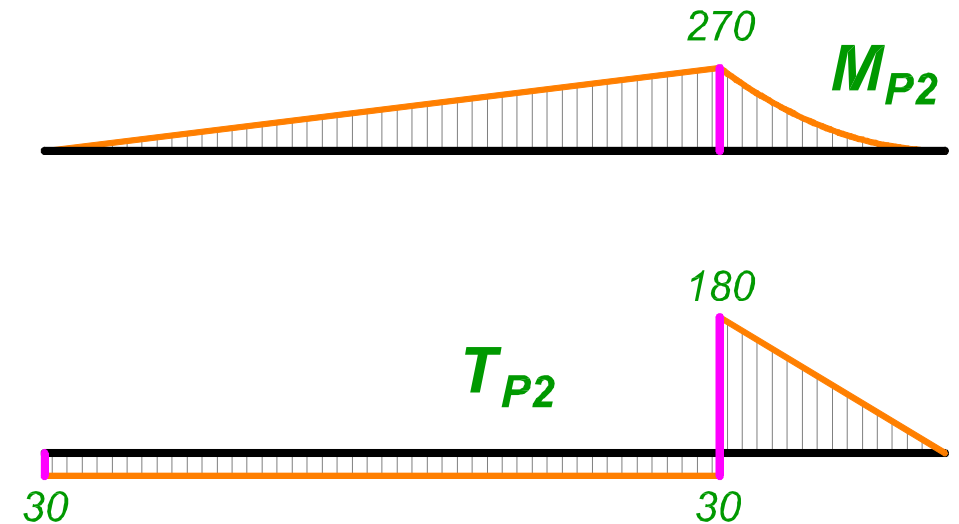
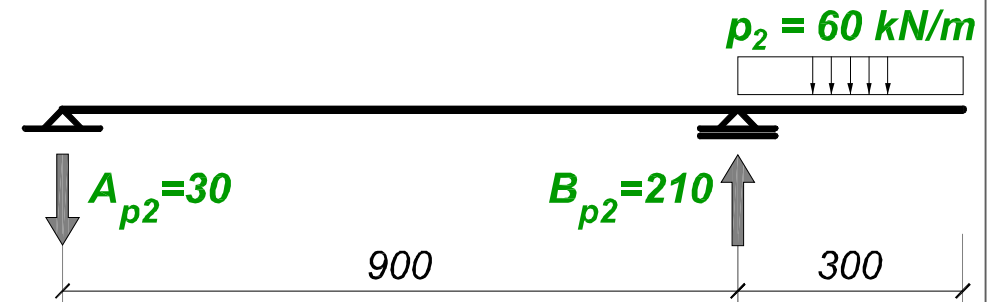
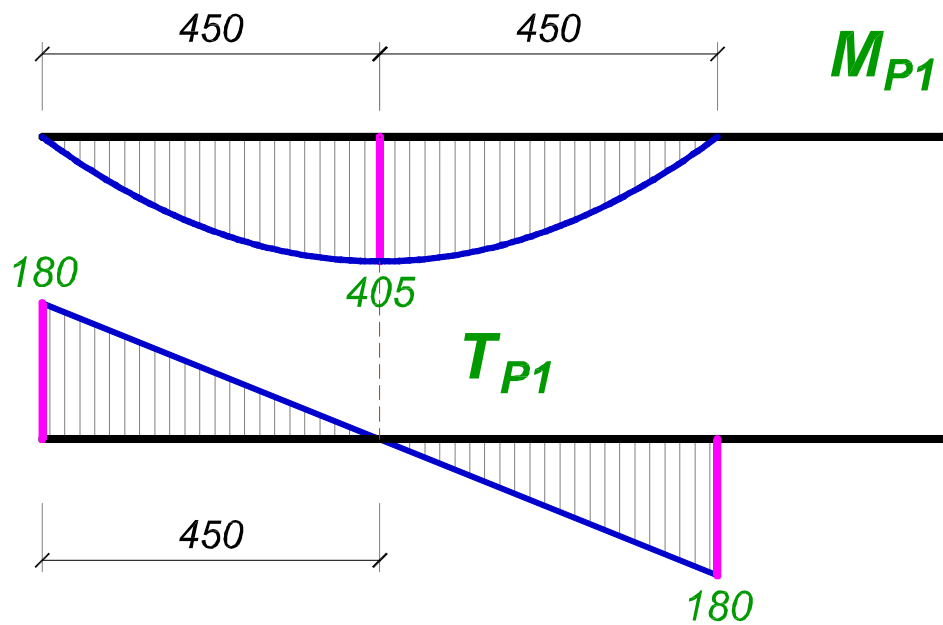
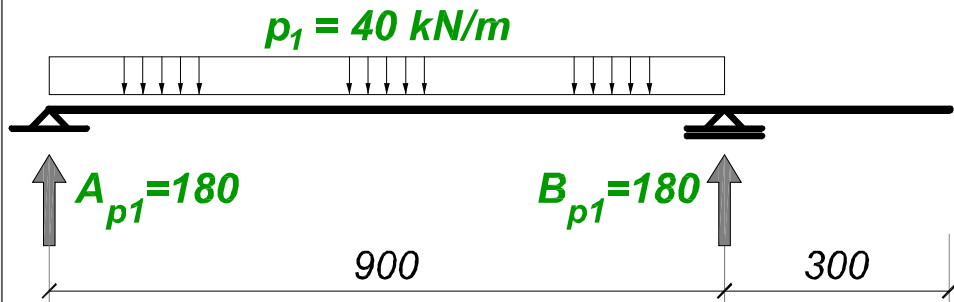
$$x_{max} = T_g^{A,desno} / g$$

$$x_{max} = 120 / 30 = 4.0 \text{ m}$$

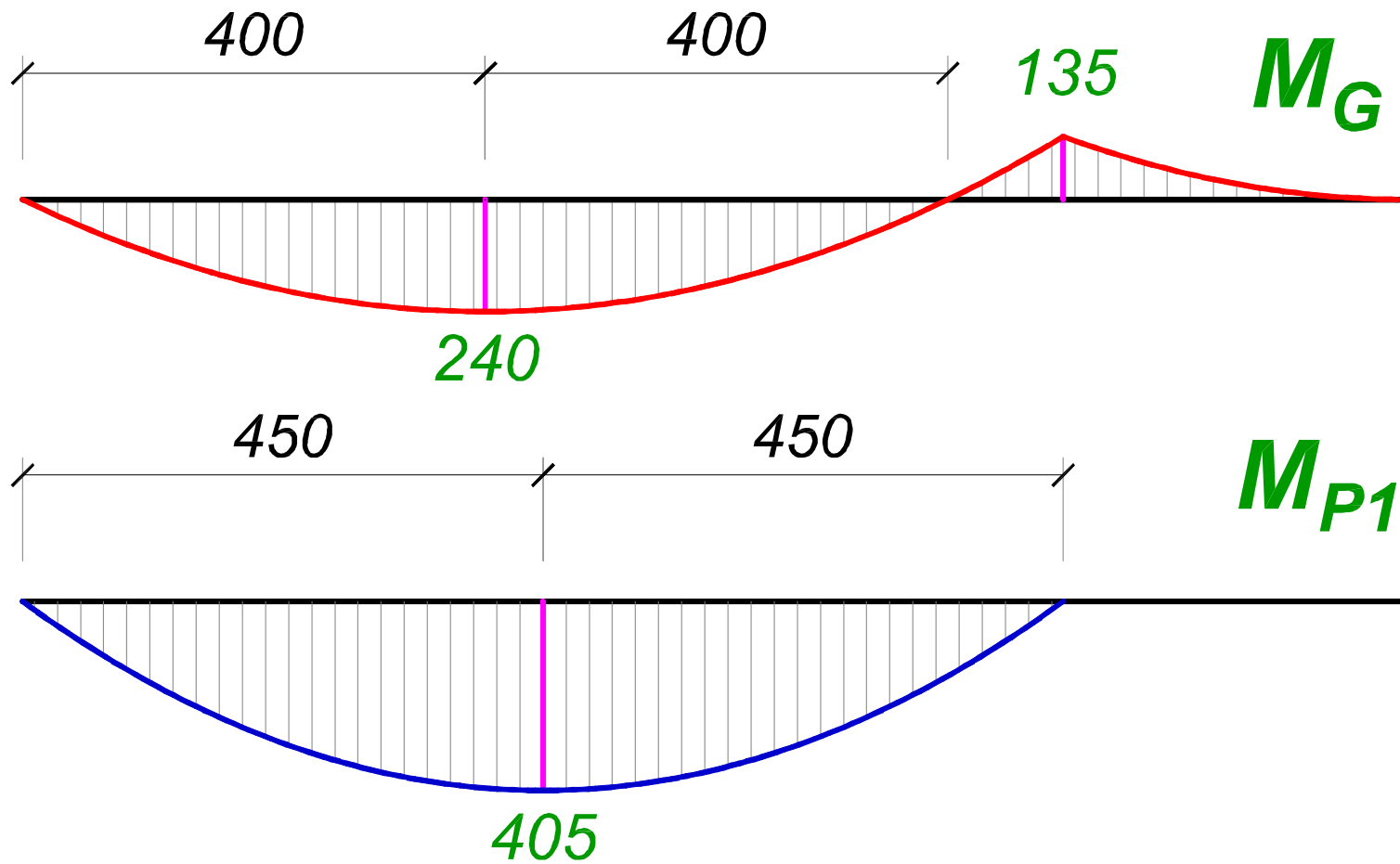
$$max M_g = 120 \times 4 - 30 \times 4^2 / 2$$

$$max M_g = 240 \text{ kNm}$$

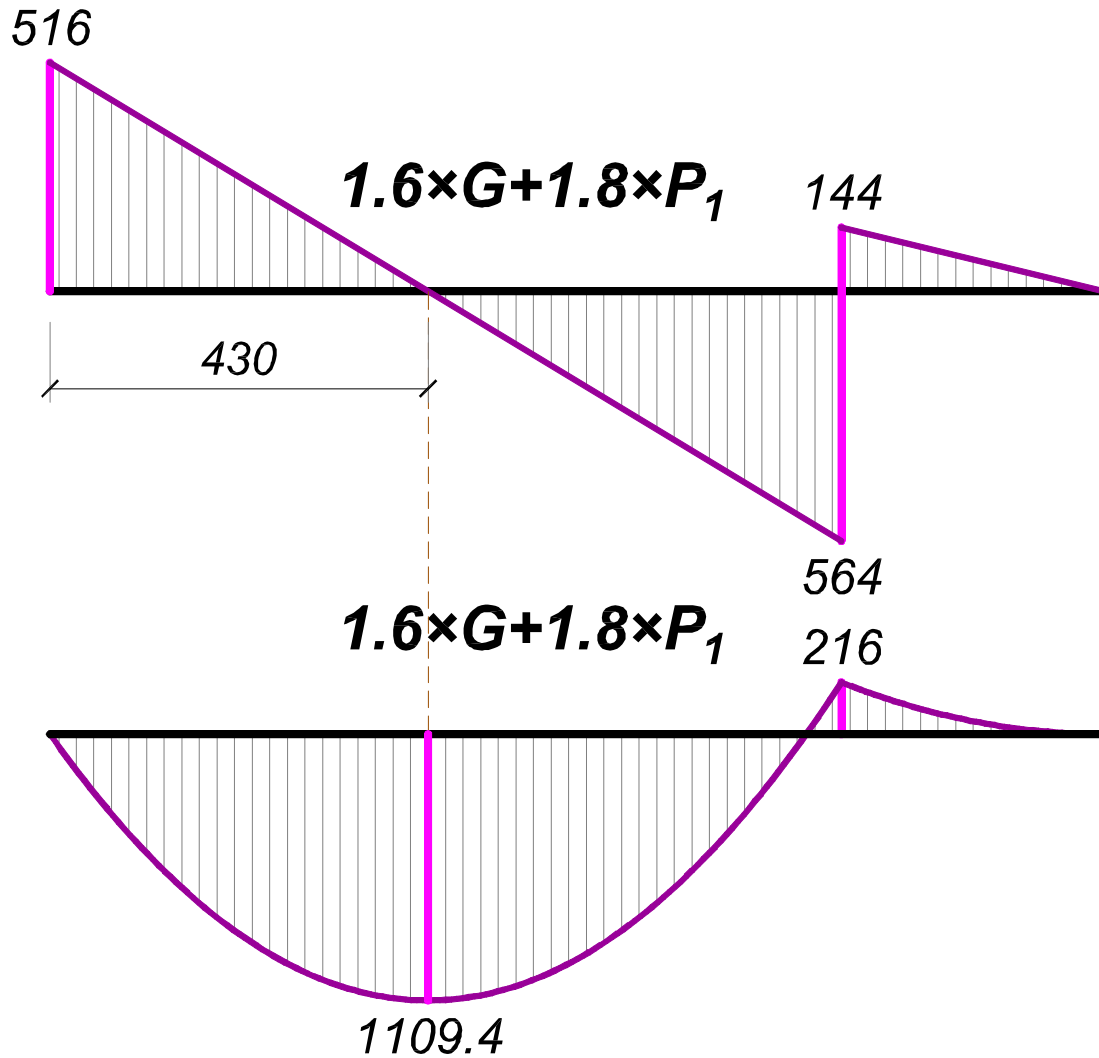
# Povremeno opterećenje



# Maksimalni moment u polju



# Maksimalni moment u polju



$$T_{u1} = 1.6 \times 120 + 1.8 \times 180 = 516 \text{ kN}$$

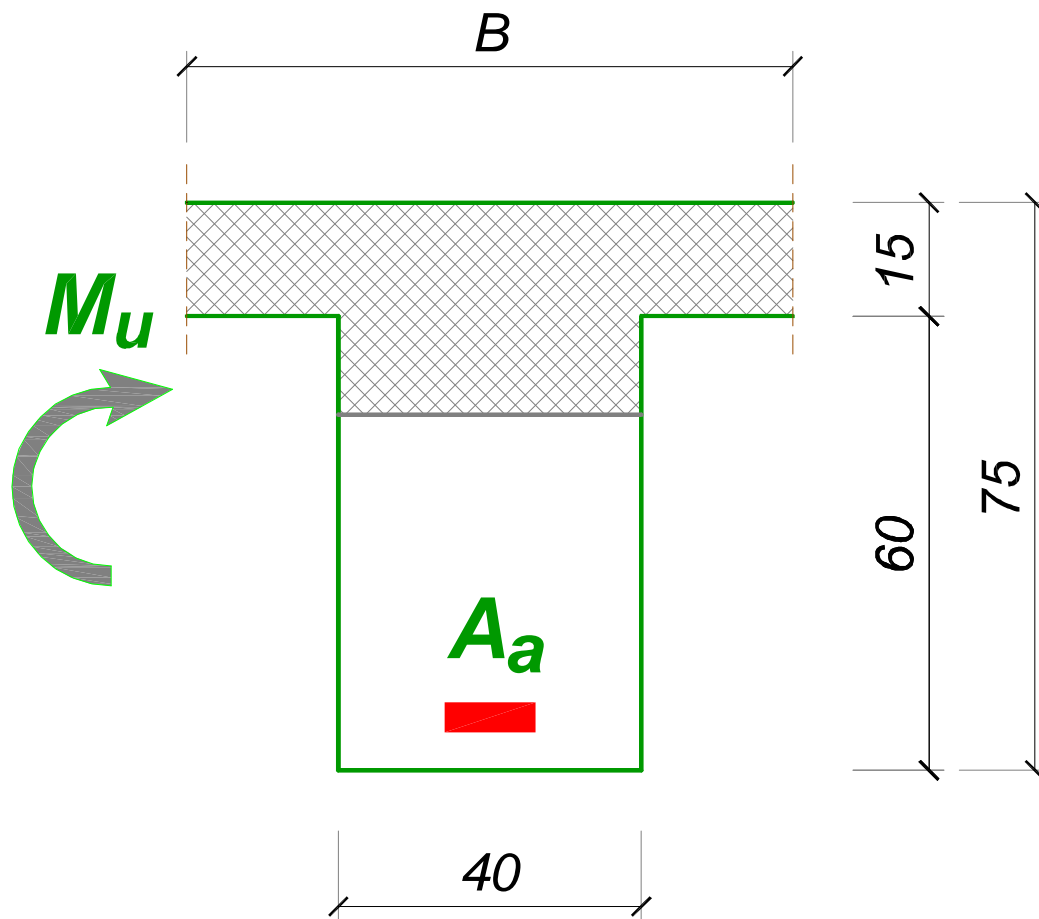
$$q_{u1} = 1.6 \times 30 + 1.8 \times 40 = 120 \text{ kN/m}$$

$$x_{max.} = 516 / 120 = 4.3 \text{ m}$$

$$M_{u,max.} = 516 \times 4.3 - 120 \times 4.3^2 / 2$$

$$M_{u,max.} = 1109.4 \text{ kNm}$$

# Dimenzionisanje - presek u polju



$$B = \min. \left\{ \begin{array}{l} b + 0.25 \times l_0 \\ b + 20 \times d_p \\ e \end{array} \right\}$$

$$l_0 = 8.60 \text{ m}$$

$$B = \min. \left\{ \begin{array}{l} 40 + 0.25 \times 860 = 255 \\ 40 + 20 \times 15 = 340 \end{array} \right\}$$

$$B = 255 \text{ cm}$$

## Dimenzionisanje - presek u polju

pretp.  $a_1 = 7 \text{ cm}$   $\text{P}$   $h = 75 - 7 = 68 \text{ cm}$

$$k = \frac{68}{\sqrt{\frac{1109.4 \times 10^2}{255 \times 2.05}}} = 4.668 \xrightarrow{\text{TABL.}} \begin{array}{l} \varepsilon_b / \varepsilon_a = 1.15 / 10\text{‰} \\ s = 0.103 \\ \bar{\mu} = 4.794\% \end{array}$$

$$x = 0.103 \times 68 = 7.0 \text{ cm} < d_p = 15 \text{ cm}$$

**Pretpostavka o položaju neutralne linije je dobra, pa se presek dimenzioniše kao pravougaoni, širine  $B = 255 \text{ cm}$ .**

## Dimenzionisanje - presek u polju

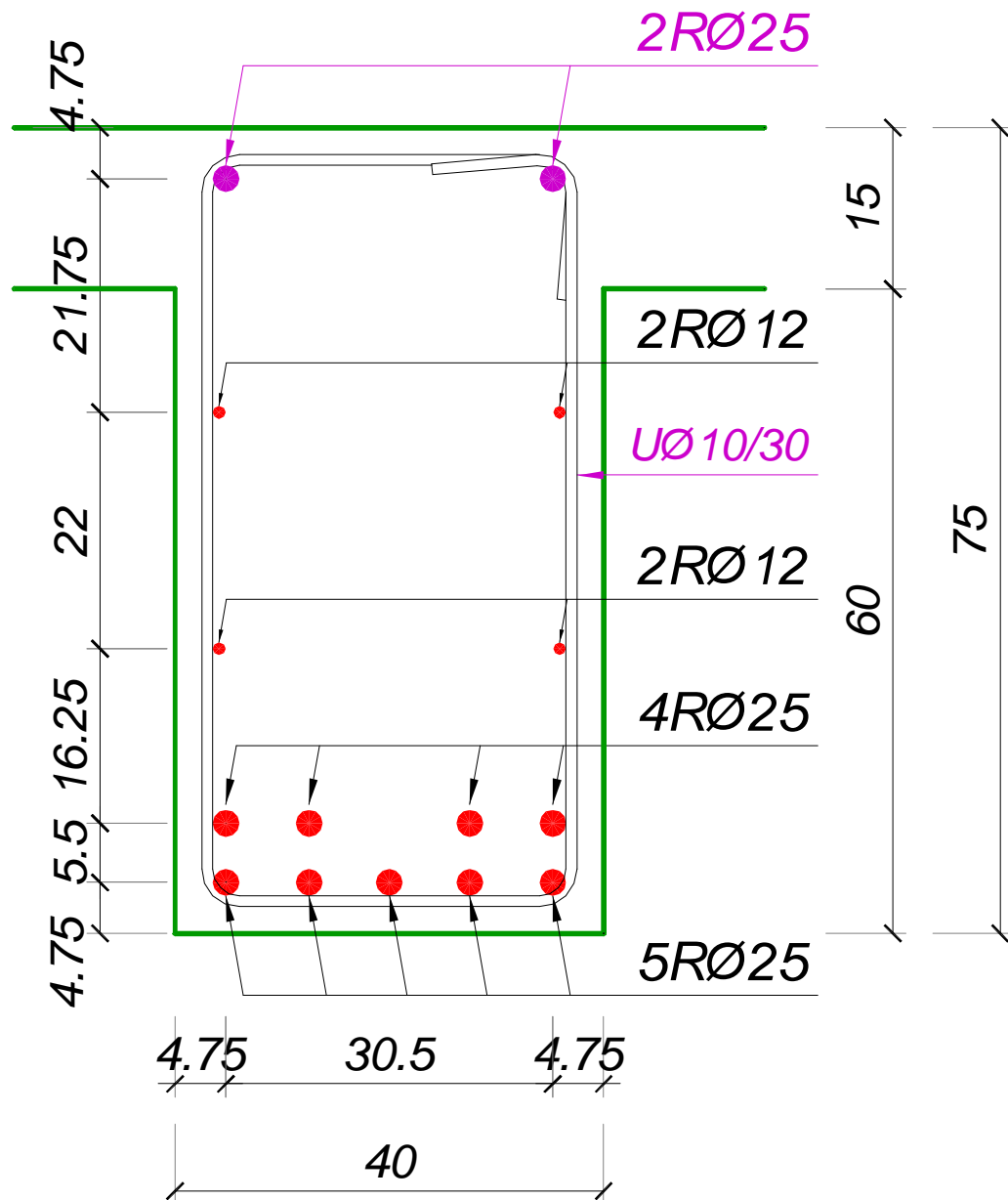
$\varepsilon_a$	$\varepsilon_b$	S	$\alpha_b$	$\eta$	$\zeta$	$\mu_{1M} \%$	k
10	1.15	0.103	0.465	0.353	0.964	4.794	4.653

$$A_a = 4.794 \times \frac{255 \times 68}{100} \times \frac{2.05}{40} = 42.60 \text{ cm}^2 \quad \text{ili:}$$

$$A_a = \frac{1109.4 \times 10^2}{0.964 \times 68 \times 40} = 42.33 \text{ cm}^2$$

usvojeno: **9RØ25** (44.18 cm<sup>2</sup>)

# Dimenzionisanje - presek u polju



$$a' = 2.5 + 1.0 + 2.5/2 = 4.75 \text{ cm}$$

$$a'' = 4.75 + 3.0 + 2.5 = 10.25 \text{ cm}$$

$$a_1 = (5 \times 4.75 + 4 \times 10.25) / 9$$

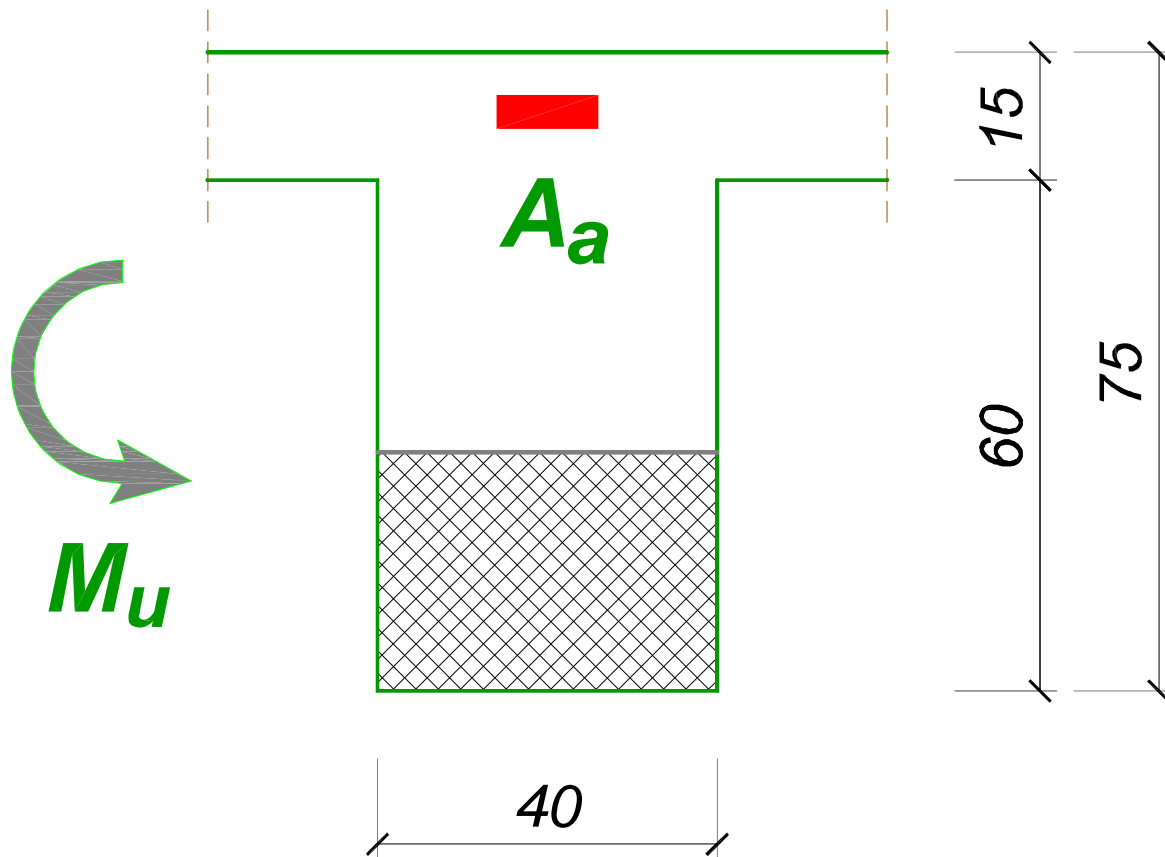
$$a_1 = 7.19 \text{ cm}$$

$$h = 75 - 7.19 = 67.81 \text{ cm}$$

$$h \approx 68 \text{ cm} = h_{\text{rač.}}$$



# Dimenzionisanje - oslonac



$$M_u = 1.6 \times M_g + 1.8 \times M_{p2}$$

$$M_u = 1.6 \times 135 + 1.8 \times 270$$

$$M_u = 702 \text{ kNm}$$

$$pretp. a_1 = 7 \text{ cm}$$

$$h = 75 - 7 = 68 \text{ cm}$$

## Dimenzionisanje - oslonac

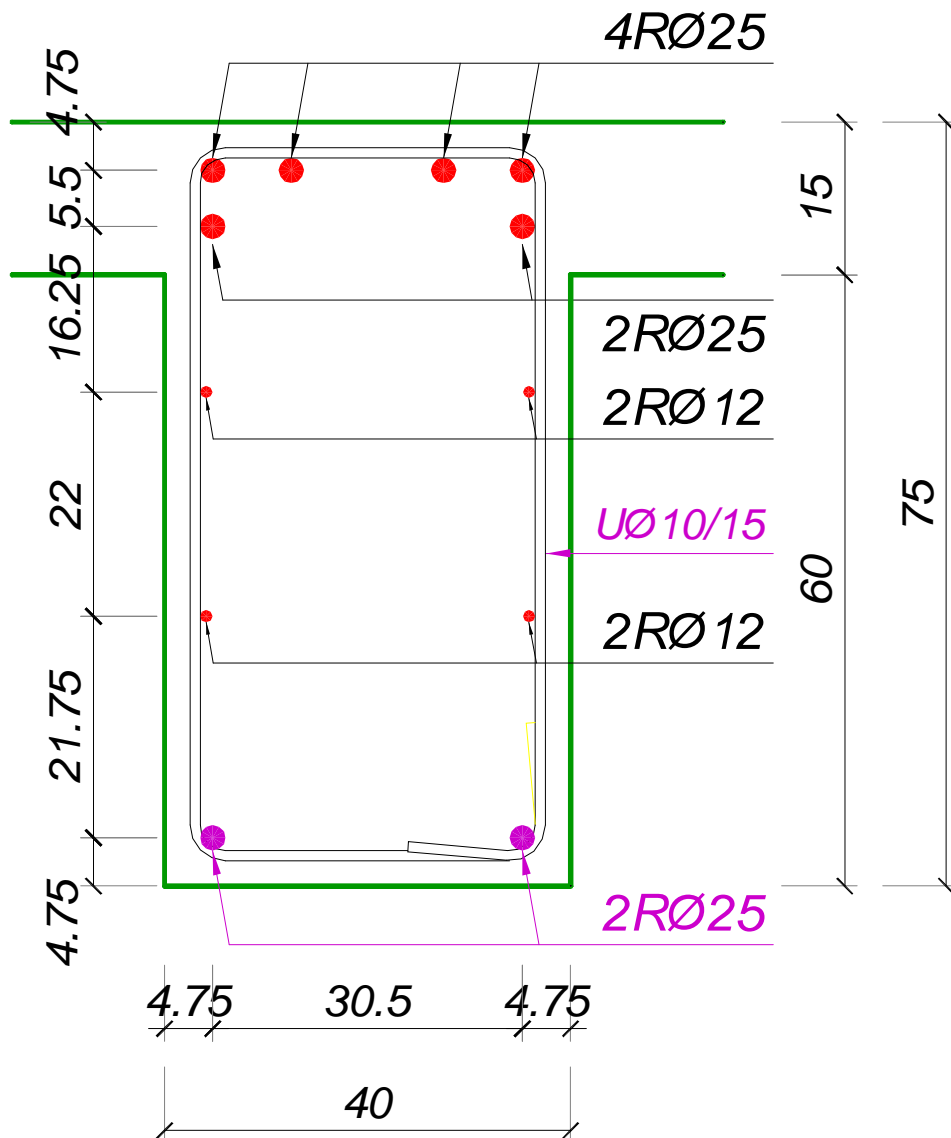
$$k = \frac{68}{\sqrt{\frac{702 \times 10^2}{40 \times 2.05}}} = 2.324 \xrightarrow{\text{TABL.}} \begin{matrix} \varepsilon_b / \varepsilon_a = 3.45 / 10\text{‰} \\ \bar{\mu} = 20.694\% \end{matrix}$$

$\varepsilon_a$	$\varepsilon_b$	$s$	$\alpha_b$	$\eta$	$\zeta$	$\mu_{1M} \%$	$k$
<b>10</b>	<b>3.45</b>	<b>0.257</b>	<b>0.807</b>	<b>0.415</b>	<b>0.894</b>	<b>20.694</b>	<b>2.325</b>

$$A_a = 20.694 \times \frac{40 \times 68}{100} \times \frac{2.05}{40} = 28.85 \text{ cm}^2$$

usvojeno: **6RØ25** (29.45 cm<sup>2</sup>)

# Dimenzionisanje - oslonac



$$a' = 2.5 + 1.0 + 2.5/2 = 4.75 \text{ cm}$$

$$a'' = 4.75 + 3.0 + 2.5 = 10.25 \text{ cm}$$

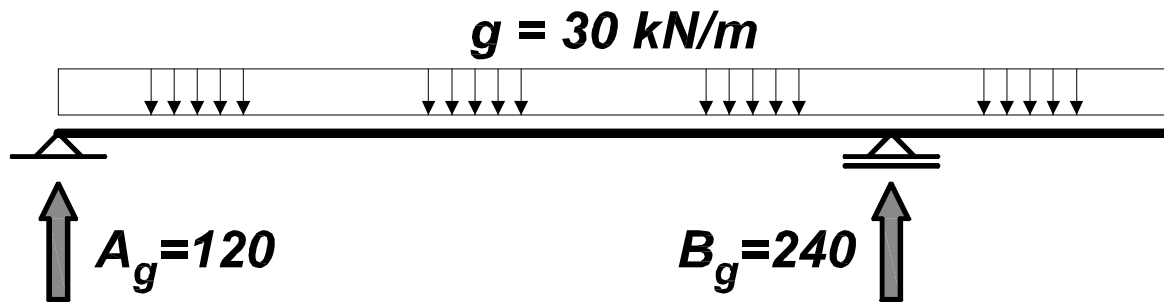
$$a_1 = (4 \times 4.75 + 2 \times 10.25) / 6$$

$$a_1 = 6.58 \text{ cm}$$

$$h = 75 - 6.58 = 68.42 \text{ cm}$$

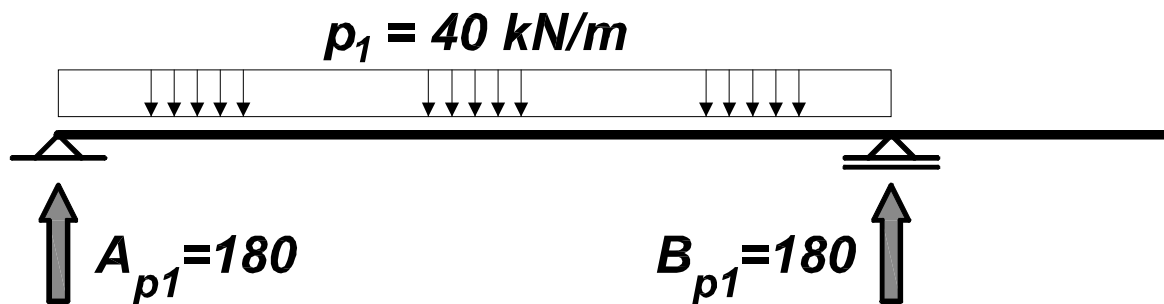
$$h \approx 68 \text{ cm} = h_{\text{rač.}}$$

# Reakcije oslonaca

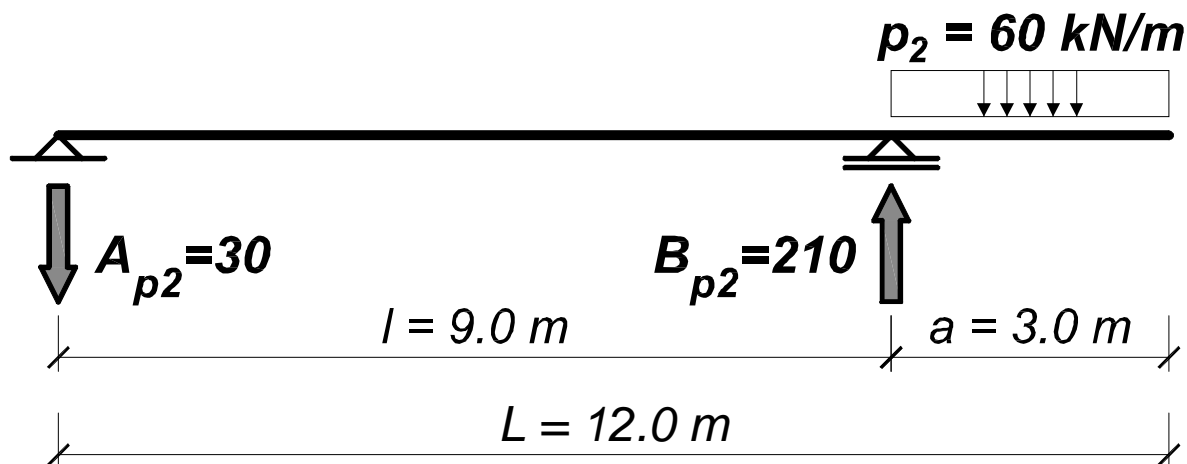


$$B_g = (30 \times 12^2 / 2) / 9 = 240 \text{ kN}$$

$$A_g = 30 \times 12 - 240 = 120 \text{ kN}$$



$$A_{p1} = B_{p1} = 40 \times 9 / 2 = 180 \text{ kN}$$

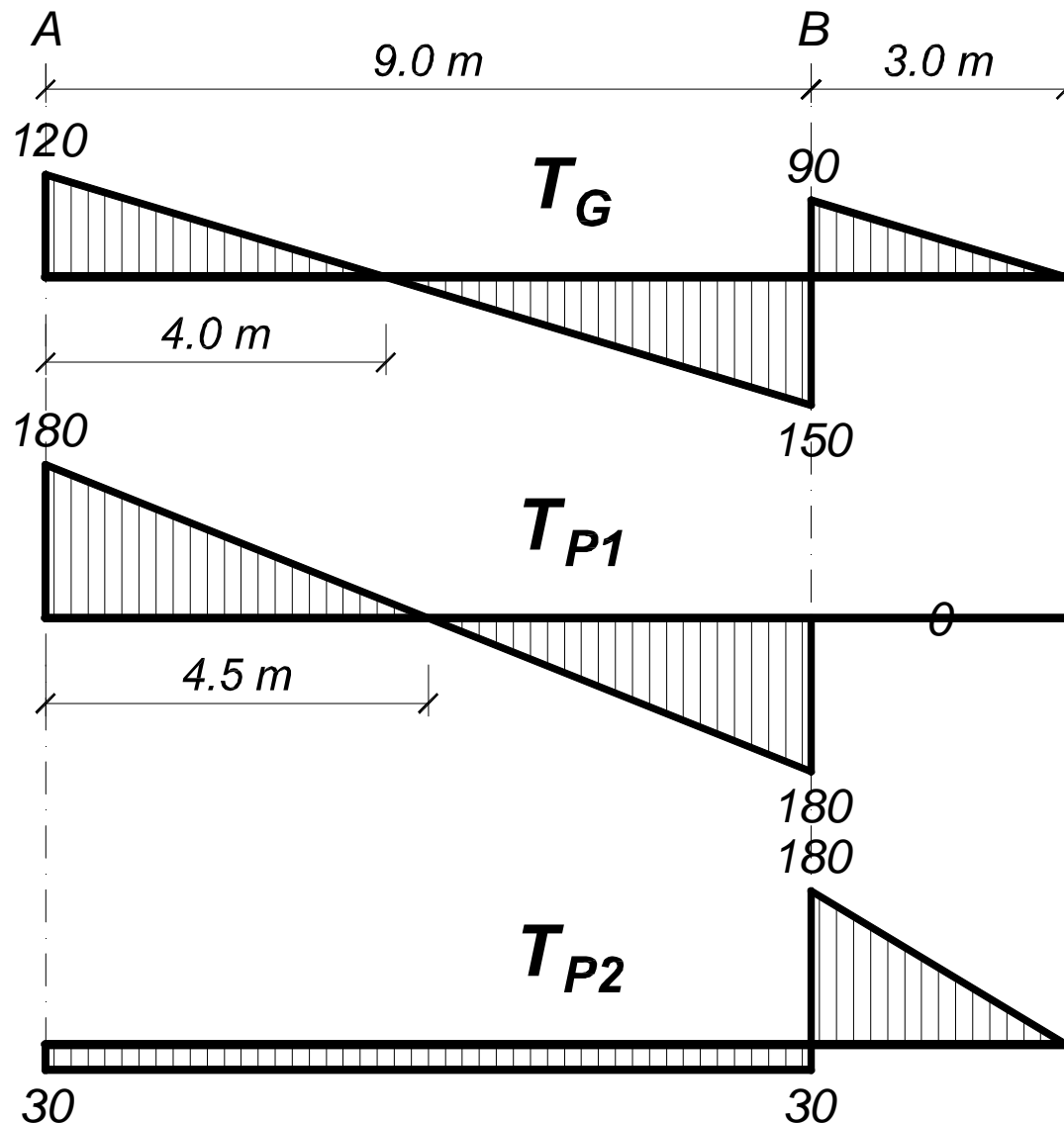


$$B_{p2} = [(60 \times 3 \times (9 + 3/2))] / 9$$

$$B_{p2} = 210 \text{ kN}$$

$$A_{p2} = 60 \times 3 - 210 = -30 \text{ kN}$$

# Dijagrami transverzalnih sila



$$T_g^A = A_g = 120 \text{ kN}$$

B desno:

$$T_g^{B,d} = g \times a = 30 \times 3 = 90 \text{ kN}$$

B levo:

$$T_g^{B,l} = T_g^{B,d} - B_g$$

$$T_g^{B,l} = 90 - 240 = -150 \text{ kN}$$

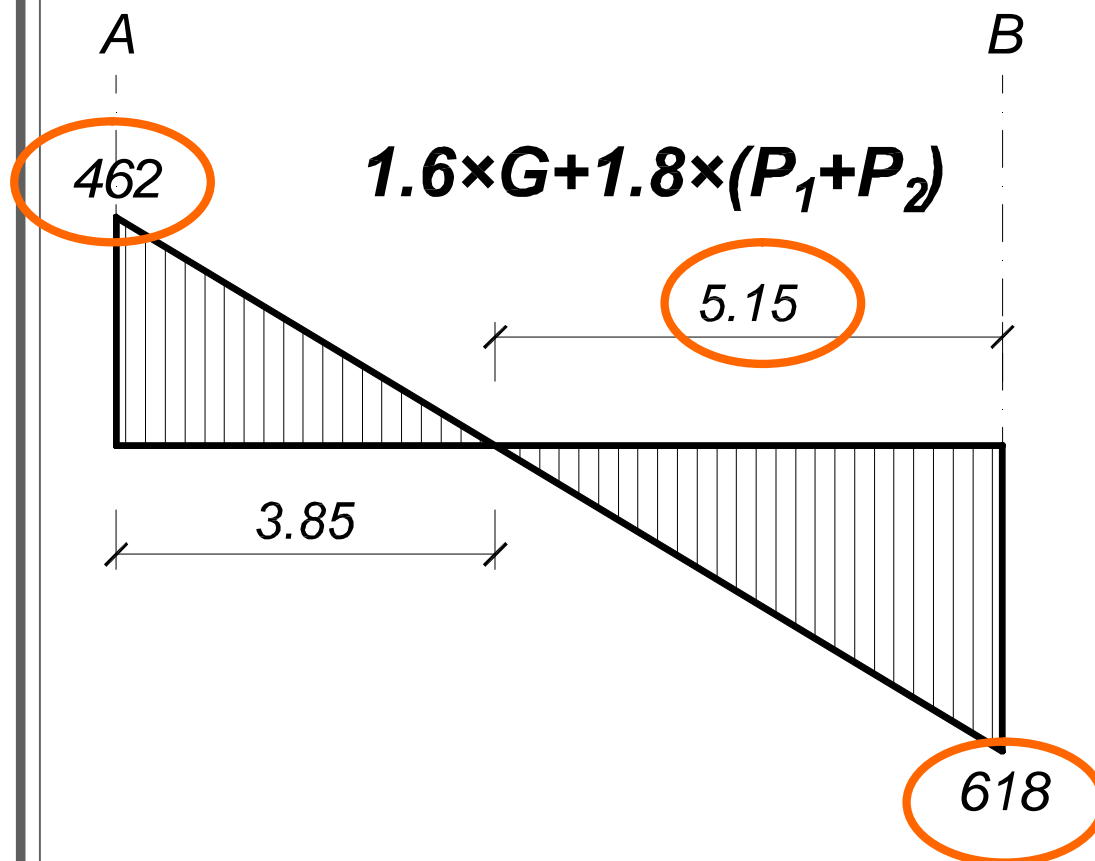
$$T_{p1}^A = -T_{p1}^{B,l} = 180 \text{ kN}$$

$$T_{p2}^A = A_{p2} = -30 \text{ kN} = T_{p2}^{B,l}$$

$$T_{p2}^{B,d} = 60 \times 3 = 180 \text{ kN}$$

## Presek B levo: određivanje sile $T_u$

Maksimalna sila  $T_u$  se javlja u preseku B levo, pri istovremenom delovanju svih opterećenja.



presek B levo:

$$\max. T_u^{B,l} = 1.6 \times 150 + 1.8 \times (180 + 30)$$

$$\max. T_u^{B,l} = 618 \text{ kN}$$

presek A:

$$\text{odg. } T_u^A = 1.6 \times 120 + 1.8 \times (180 - 30)$$

$$\text{odg. } T_u^A = 462 \text{ kN}$$

$$I_{0,T} = 618 / (618 + 462) \times 9.0$$

$$I_{0,T} = 5.15 \text{ m}$$

# Proračun nominalnog napona smicanja

$$\tau_n = \frac{T_{mu}}{b \times z}$$

usvojeno za sve preseke:

$$z \approx 0.9 \times h = 0.9 \times 68 = 61.2 \text{ cm}$$

$$\tau_n = \frac{618}{40 \times 61.2} = 0.252 \frac{\text{kN}}{\text{cm}^2} > \tau_r = 0.11 \frac{\text{kN}}{\text{cm}^2}$$

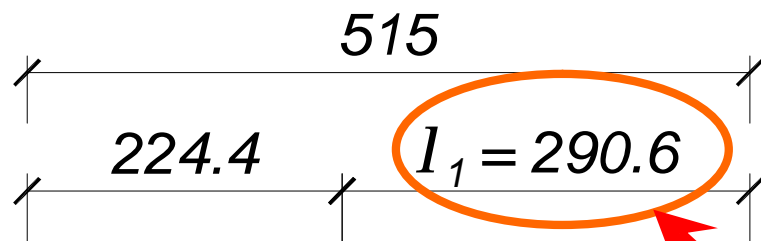
<b>MB</b>	<b>15</b>	<b>20</b>	<b>30</b>	<b>40</b>	<b>50</b>	<b>60</b>	<b>[MPa]</b>
<b><math>t_r</math></b>	<b>0.6</b>	<b>0.8</b>	<b>1.1</b>	<b>1.3</b>	<b>1.5</b>	<b>1.6</b>	<b>[MPa]</b>

(član 89 Pravilnika BAB 87)

# Određivanje dužine osiguranja

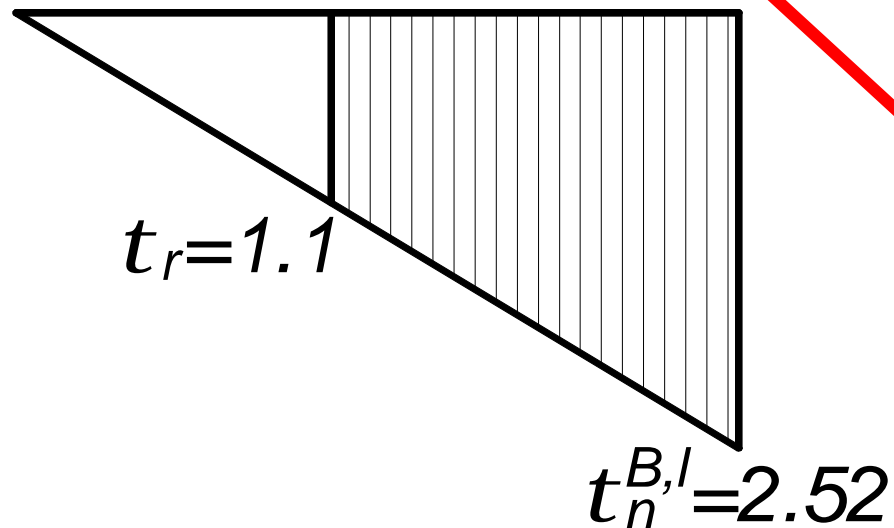
Dužina na kojoj je prekoračen nominalni napon smicanja.

Na ovom delu nosača je potrebno proračunati **POPREČNU** i **PODUŽNU** armaturu za prihvatanje uticaja od dejstva  $T$  sila



$$\lambda_1 = l_{0,T} \times \left( 1 - \frac{\tau_r}{\tau_n^{B,levo}} \right)$$

$$\lambda_1 = 515 \times \left( 1 - \frac{1.1}{2.52} \right)$$



$$I_1 = 290.6 \text{ cm}$$



## Određivanje redukovane $T$ sile

Kako je  $t_n < 3t_r$ , proračunava se REDUKOVANA TRANSVERZALNA SILA (član 91 PBAB 87):

$$T_{Ru} = T_{mu} - T_{bu}$$

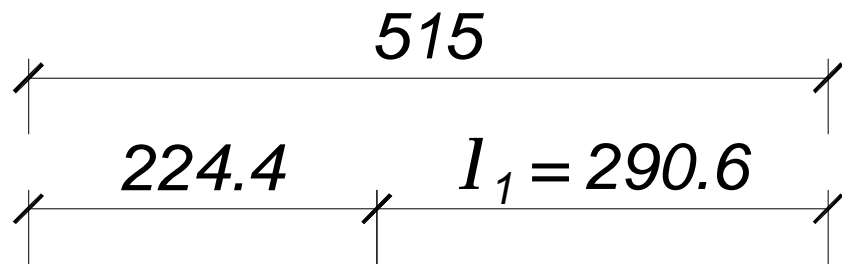
$$T_{bu} = \frac{1}{2} \times (3\tau_r - \tau_n) \times b \times z$$

$$T_{bu} = \frac{1}{2} \times (3 \times 0.11 - 0.252) \times 40 \times 61.2 = 94.9 \text{ kN}$$

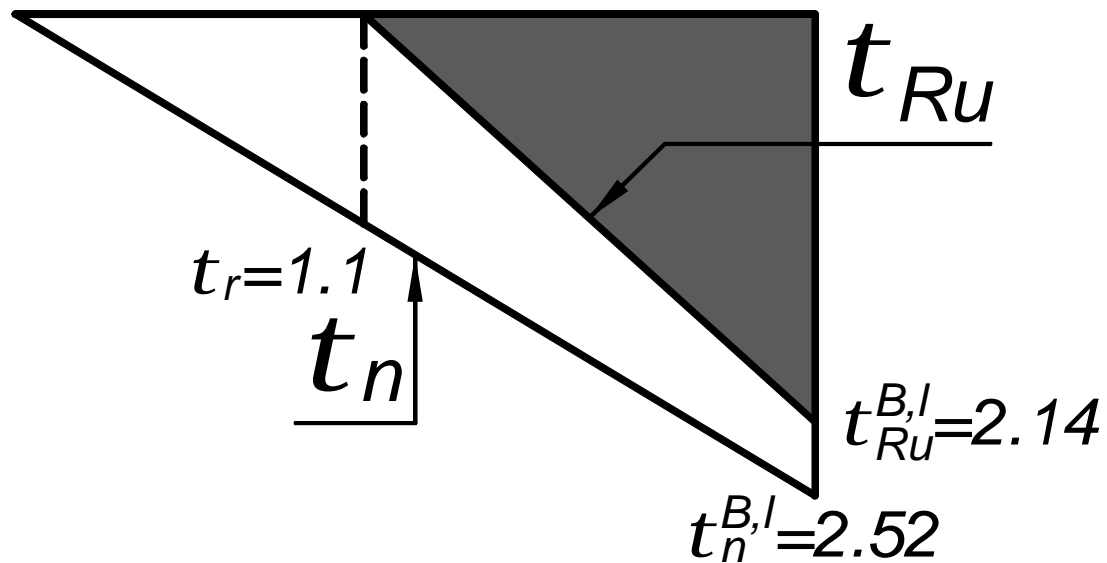
$$T_{Ru} = T_{mu} - T_{bu} = 618 - 94.9 = 523.1 \text{ kN}$$

$$\tau_{Ru} = \frac{T_{Ru}}{b \times z} = \frac{523.1}{40 \times 61.2} = 0.214 \frac{\text{kN}}{\text{cm}^2}$$

# Određivanje redukovanog napona $t_{Ru}$



$$\tau_{Ru} = \frac{3}{2} \times (\tau_n - \tau_r)$$



$$\tau_{Ru}^{B,I} = \frac{3}{2} \times (2.52 - 1.1)$$

$$\tau_{Ru}^{B,I} = 2.14 \text{ MPa}$$

# Određivanje poprečne armature

## OSIGURANJE UZENGIJAMA

$$\tau_{u,u} = \frac{m \times a_u^{(1)}}{b \times e_u} \times \sigma_v \times (\cos \alpha + \sin \alpha \times \operatorname{ctg} \theta)$$

**usvojeno:**             $m = 2$             *(dvosečne uzengije)*  
                                $q = 45^\circ$   
                                $a = 90^\circ$             *(vertikalne uzengije)*

**Izjednačavanjem napona koji mogu prihvatiti uzengije i maksimalnog REDUKOVANOG napona smicanja  $t_{Ru}$ , sledi:**

## Određivanje poprečne armature

$$e_u = \frac{m \times a_u^{(1)}}{b \times \tau_{Ru,MAX.}} \times \sigma_v \times (\cos \alpha + \sin \alpha \times \text{ctg} \theta)$$

$$e_u = \frac{2 \times a_u^{(1)}}{40 \times 0.214} \times 40 \times (\cos 90^\circ + \sin 90^\circ \times \text{ctg} 45^\circ)$$

$$e_u = 9.36 \times a_u^{(1)}$$

$$\emptyset 10 \text{ P } a_u^{(1)} = 0.785 \text{ cm}^2$$

$$e_u = 9.36 \times 0.785 = 7.35 \text{ cm}$$

**potrebno:**

**URØ 10/7**

# Određivanje poprečne armature

Kako je rastojanje uzengija vrlo malo, na raspolaganju stoje sledeće mogućnosti:

- **povećanje prečnika** (max. RØ 12, Ø 16, čl. 140 BAB 87)

Ø 12  $\text{P } a_u^{(1)} = 1.13 \text{ cm}^2$ :  $e_u = 9.36 \times 1.13 = 10.6 \text{ cm}$   
 potrebno: URØ 12/10 ( $m=2$ )

- **povećanje sečnosti** ( $m=4$ )

$$e_u = \frac{m \times a_u^{(1)}}{b \times \tau_{Ru,MAX.}} \times \sigma_v \times (\cos \alpha + \sin \alpha \times \text{ctg} \theta)$$

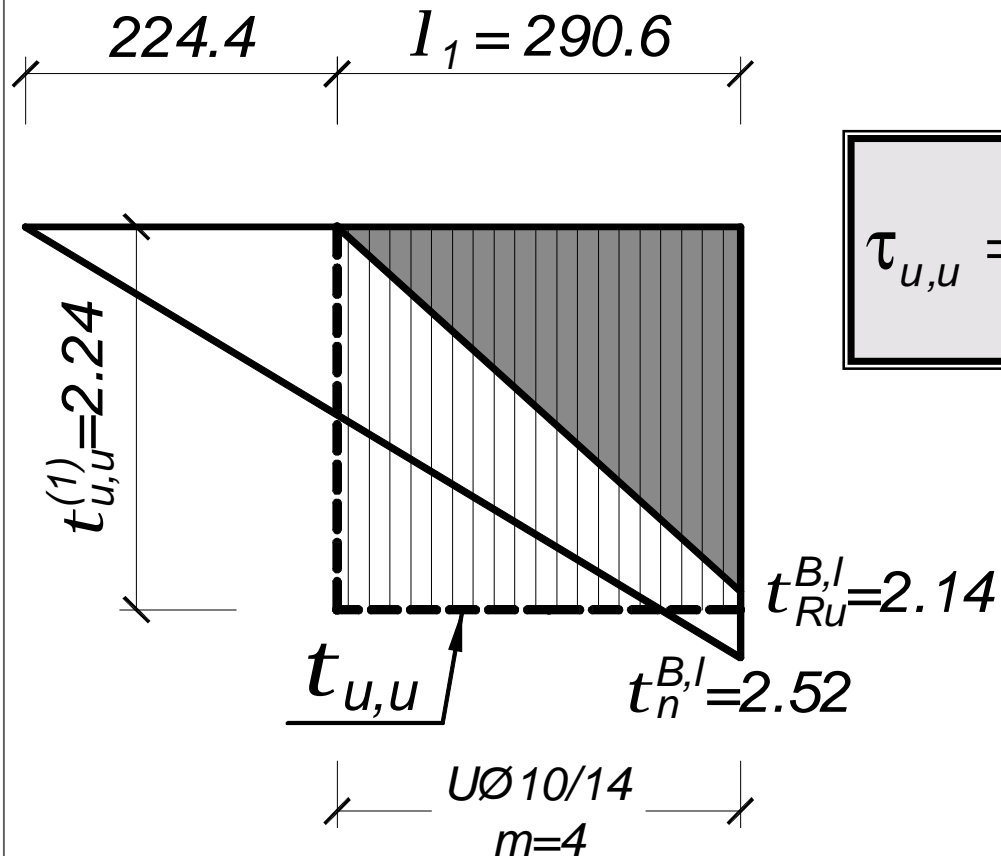
$e_u = 2 \times 7.35 = 14.7 \text{ cm}$

potrebno: URØ 10/14 ( $m=4$ )

- primena **KOSO POVIJENIH** profila

# Određivanje poprečne armature

Usvajanjem URØ 10/14 ( $m=4$ ) na čitavoj dužini osiguranja  $l_1$  proračun bi mogao biti završen. Nosivost usvojenih uzengija prikazana je dijagramom  $t_{u,u}$



$$\tau_{u,u} = \frac{m \cdot a_u^{(1)}}{b \cdot e_u} \cdot \sigma_v \cdot (\cos \alpha + \sin \alpha \cdot \operatorname{ctg} \theta)$$

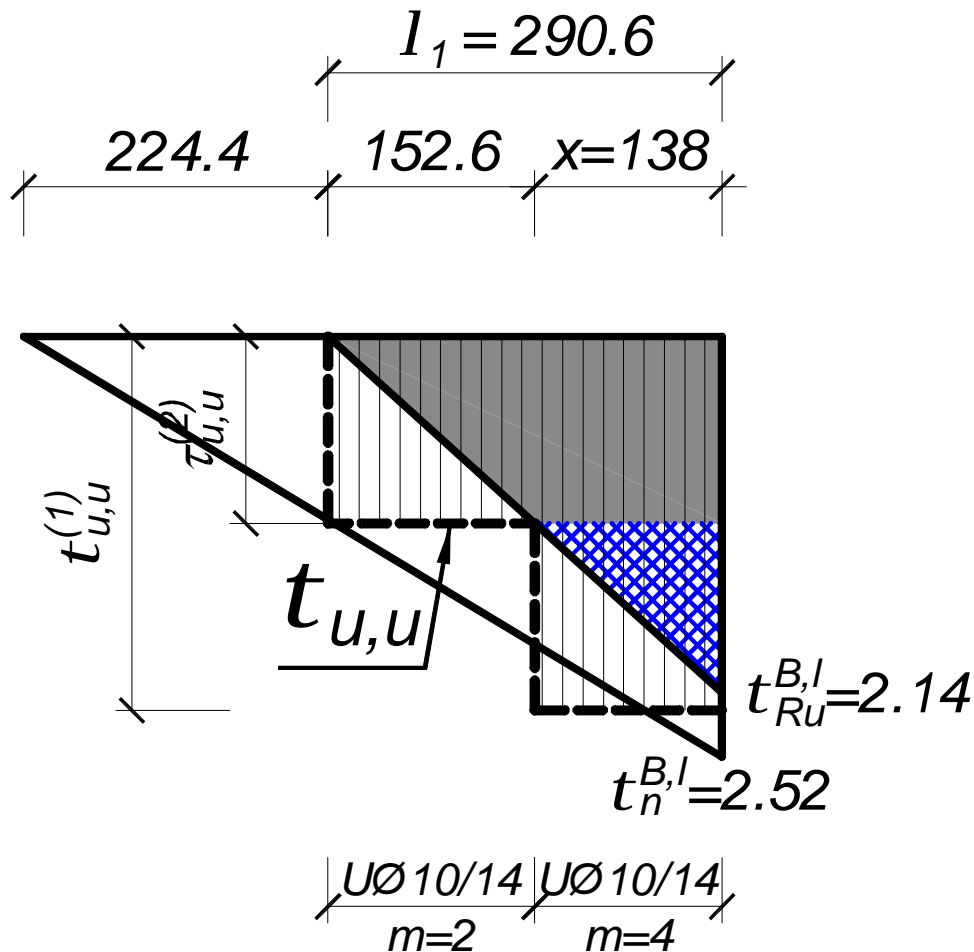
$$\alpha = 90^\circ ; \theta = 45^\circ :$$

$$\cos \alpha + \sin \alpha \cdot \operatorname{ctg} \theta = 1$$

$$\tau_{u,u}^{(1)} = \frac{4 \times 0.785}{40 \times 14} \times 40 = 0.224 \frac{\text{kN}}{\text{cm}^2}$$

# Određivanje poprečne armature

S obzirom na veliku količinu armature za osiguranje i oblik dijagrama  $t_{Ru}$ , racionalno je odrediti dužinu na kojoj su dovoljne dvosečne uzengije URØ 10/14:



$$\tau_{u,u}^{(2)} = \frac{2 \times 0.785}{40 \times 14} \times 40 = 0.112 \frac{\text{kN}}{\text{cm}^2}$$

$$x = \lambda_1 \times \left( 1 - \frac{\tau_{u,u}^{(2)}}{\tau_{Ru}} \right)$$

$$x = 290.6 \times \left( 1 - \frac{1.12}{2.14} \right) = 138 \text{ cm}$$

Na delu nosača dužine  $x=138$  cm potrebno je usvojiti **čtvorosečne uzengije ili koso povijene profile**

# Određivanje poprečne armature

## PRIMENA KOSO POVIJENIH PROFILA

Potrebno je **na dužini osiguranja** obezbediti makar **MINIMALNE** vertikalne uzengije:

$$\mu_{uz} = \frac{m \cdot a_u^{(1)}}{b \cdot e_u} \geq 0.2\%$$

Pored toga, **na dužini osiguranja** **MAKSIMALNO RASTOJANJE** uzengija mora zadovoljiti sledeće uslove:

$$e_u \leq \left\{ \begin{array}{l} 25 \text{ cm} \\ b = 40 \\ h/2 = 68/2 = 34 \end{array} \right\} = 25 \text{ cm}$$



# Određivanje poprečne armature

**Za presek širine  $b=40$  cm, armiran DVOSEČNIM uzengijama, sledi:**

$$e_u = \frac{m \cdot a_u^{(1)}}{b \cdot \mu_{uz,min.}} = \frac{2 \times a_u^{(1)}}{40 \times 0.2 \times 10^{-2}} = 25 \times a_u^{(1)}$$

**$U\emptyset 8 \text{ } \vdash \text{ } e_{u,max} = 25 \times 0.503 = 12.6 \text{ cm} \text{ } \vdash \text{ } R\emptyset 8/12.5$**

**$U\emptyset 10 \text{ } \vdash \text{ } e_{u,max} = 25 \times 0.785 = 19.6 \text{ cm} \text{ } \vdash \text{ } R\emptyset 10/15$**

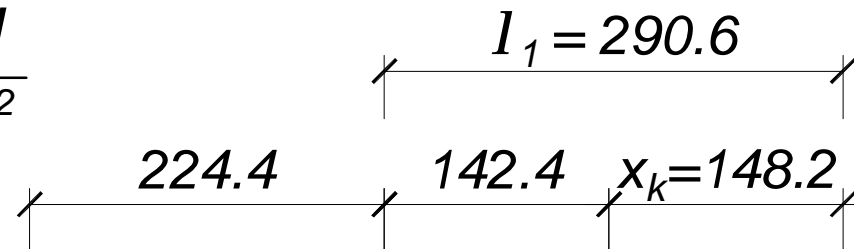
**$U\emptyset 12 \text{ } \vdash \text{ } e_{u,max} = 25 \times 1.13 = 28.3 \text{ cm} \text{ } \vdash \text{ } R\emptyset 12/25$**

**usvojeno:  $UR\emptyset 10/15$  ( $m=2$ )**

# Određivanje poprečne armature

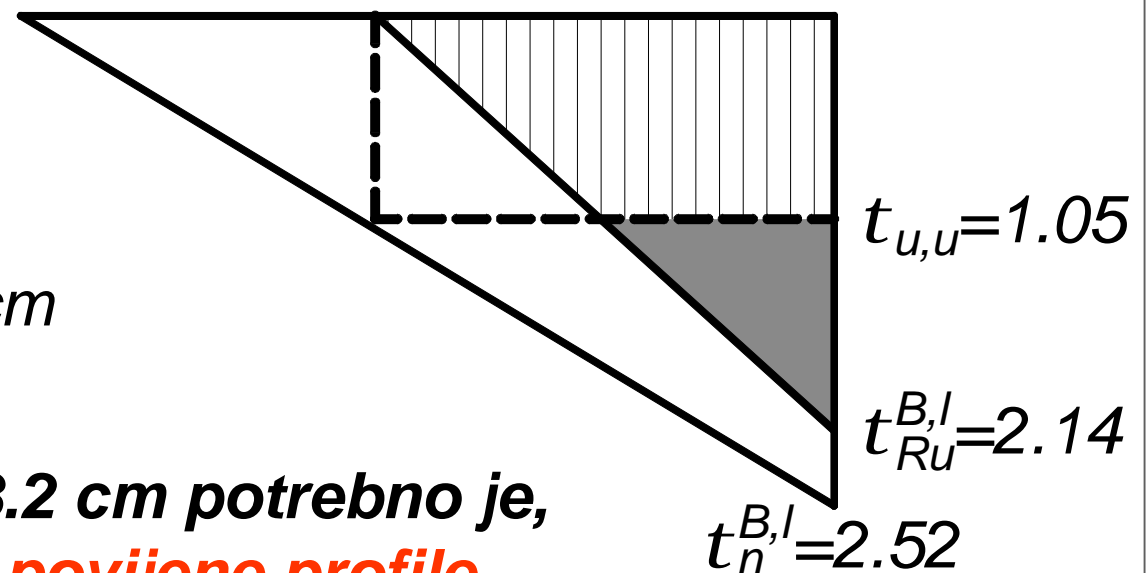
usvojeno: **URØ 10/15** ( $m=2$ )

$$\tau_{u,u} = \frac{2 \times 0.785}{40 \times 15} \times 40 = 0.105 \frac{\text{kN}}{\text{cm}^2}$$



$$x_k = \lambda_1 \times \left( 1 - \frac{\tau_{u,u}}{\tau_{Ru}} \right)$$

$$x_k = 290.6 \times \left( 1 - \frac{1.05}{2.14} \right) = 148.2 \text{ cm}$$



**Na delu nosača dužine  $x_k = 148.2$  cm potrebno je, pored uzengija, usvojiti koso povijene profile**

# Određivanje poprečne armature

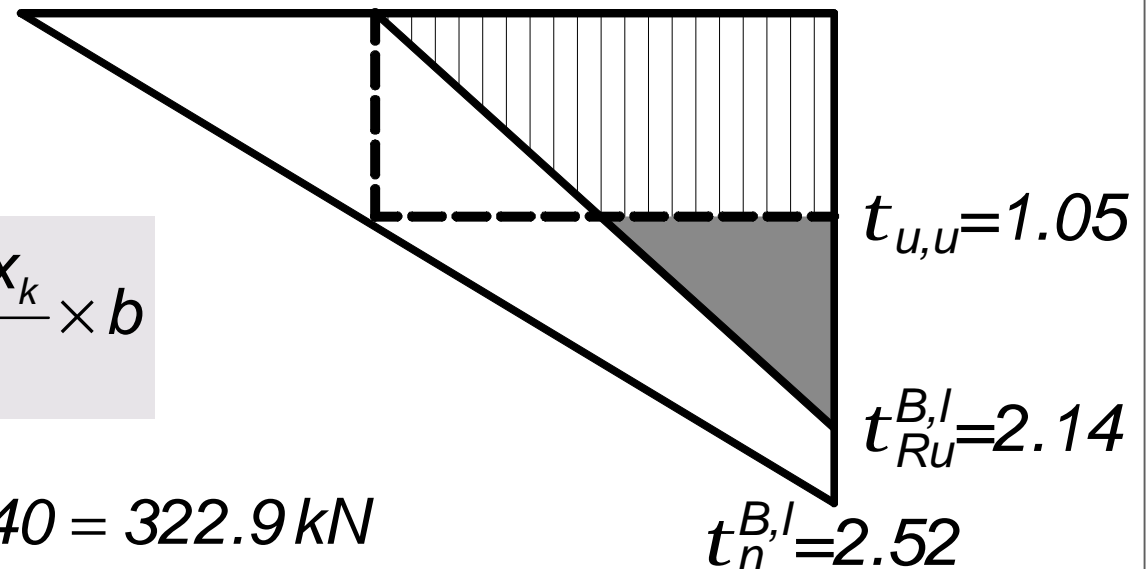
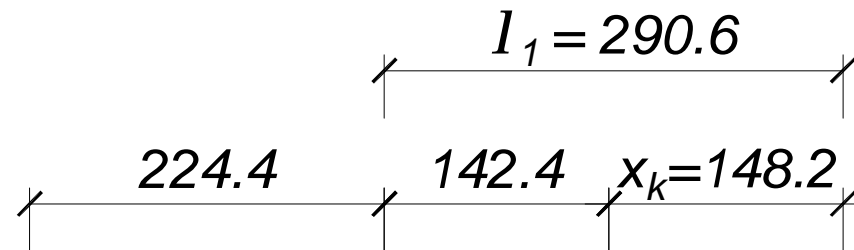
Ukupna sila smicanja, tj. horizontalna sila veze na dužini osiguranja nosača  $l = b - a$ :

$$H_{vu} = \int_{x=a}^{x=b} \frac{T_{Ru}}{z} dx$$

$$H_{vu} = \int_{x=a}^{x=b} \frac{b \cdot z \cdot \tau_{Ru}}{z} dx$$

$$H_{vuk} = b \cdot \int_{x=a}^{x=b} \tau_{Ru} dx = \frac{(\tau_{Ru} - \tau_{u,u}) \times x_k}{2} \times b$$

$$H_{vuk} = \frac{(0.214 - 0.105) \times 148.2}{2} \times 40 = 322.9 \text{ kN}$$



## Određivanje poprečne armature

Potrebna površina armature  $A_{a,k}$  povijene pod uglom  $\alpha_k$  u odnosu na osu nosača:

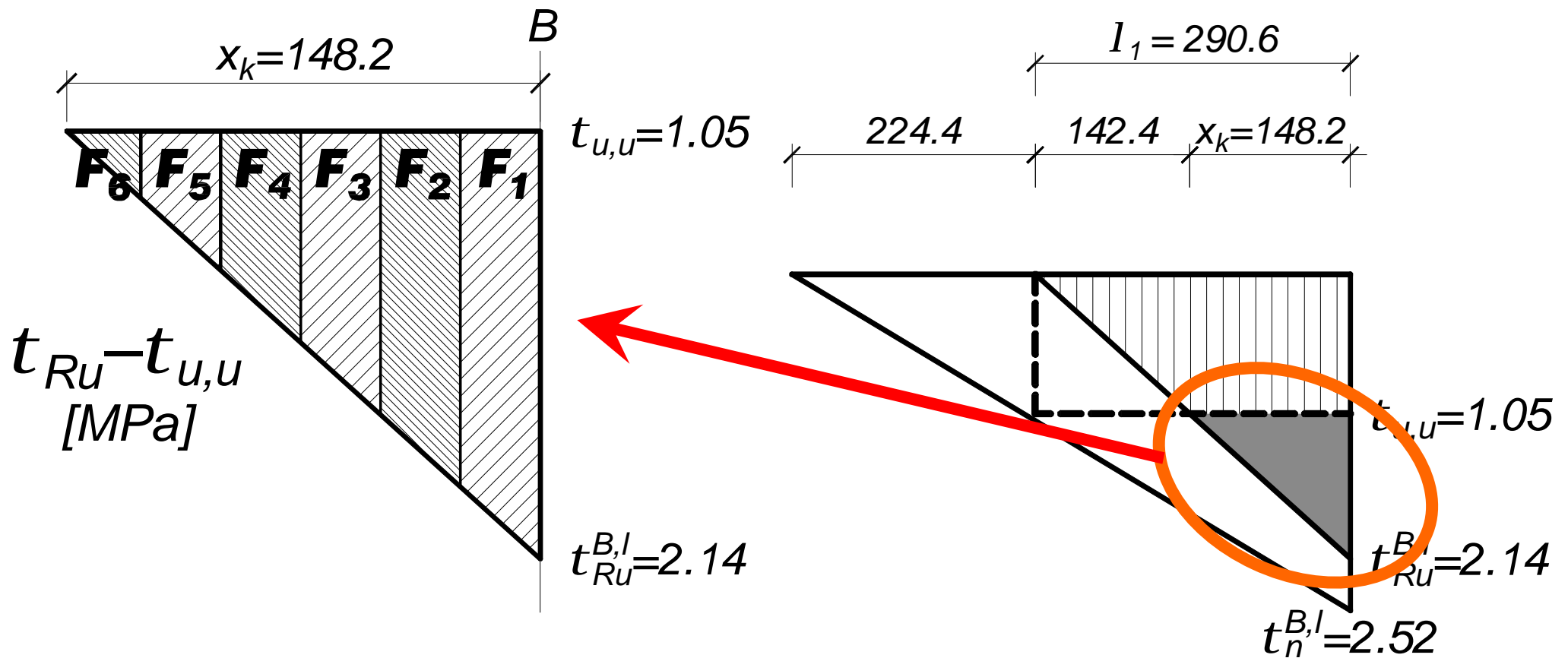
$$A_{a,k} = \frac{H_{vu,k}}{\sigma_v \cdot (\cos \alpha_k + \sin \alpha_k \cdot \operatorname{ctg} \theta)}$$

$$\left. \begin{array}{l} \alpha_k = 45^\circ \\ \theta = 45^\circ \end{array} \right\} \Rightarrow \cos \alpha_k + \sin \alpha_k \cdot \operatorname{ctg} \theta = \sqrt{2}$$

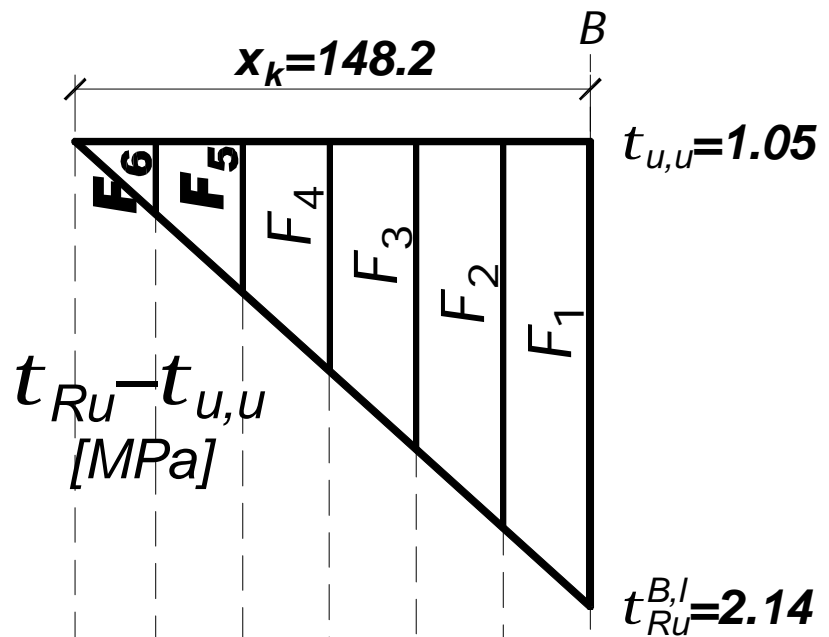
$$A_{a,k} = \frac{322.9}{40 \times \sqrt{2}} = 5.71 \text{ cm}^2$$

usvojeno: **2RØ25 (9.82 cm<sup>2</sup>)**

# Određivanje mesta povijanja kosih profila

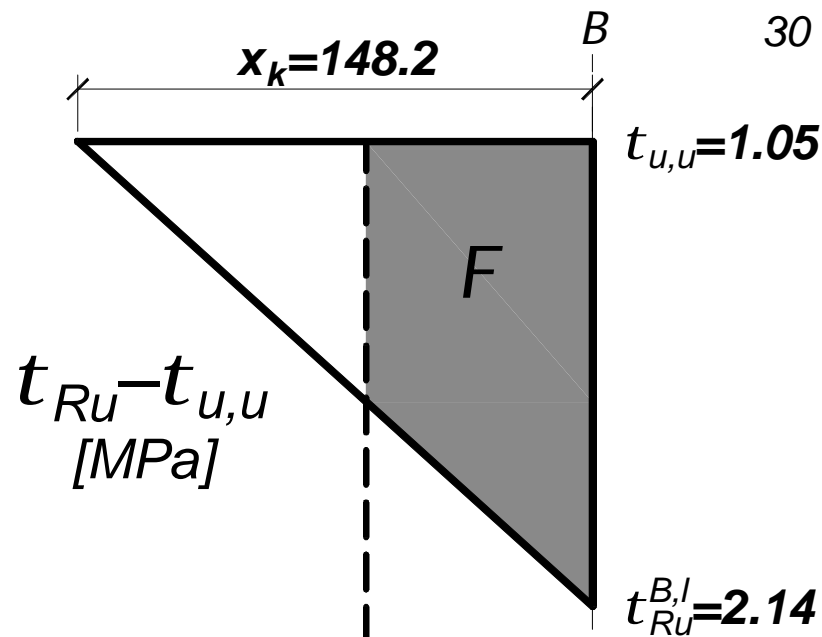


$H_{vu,k}$



KV. PARABOLA

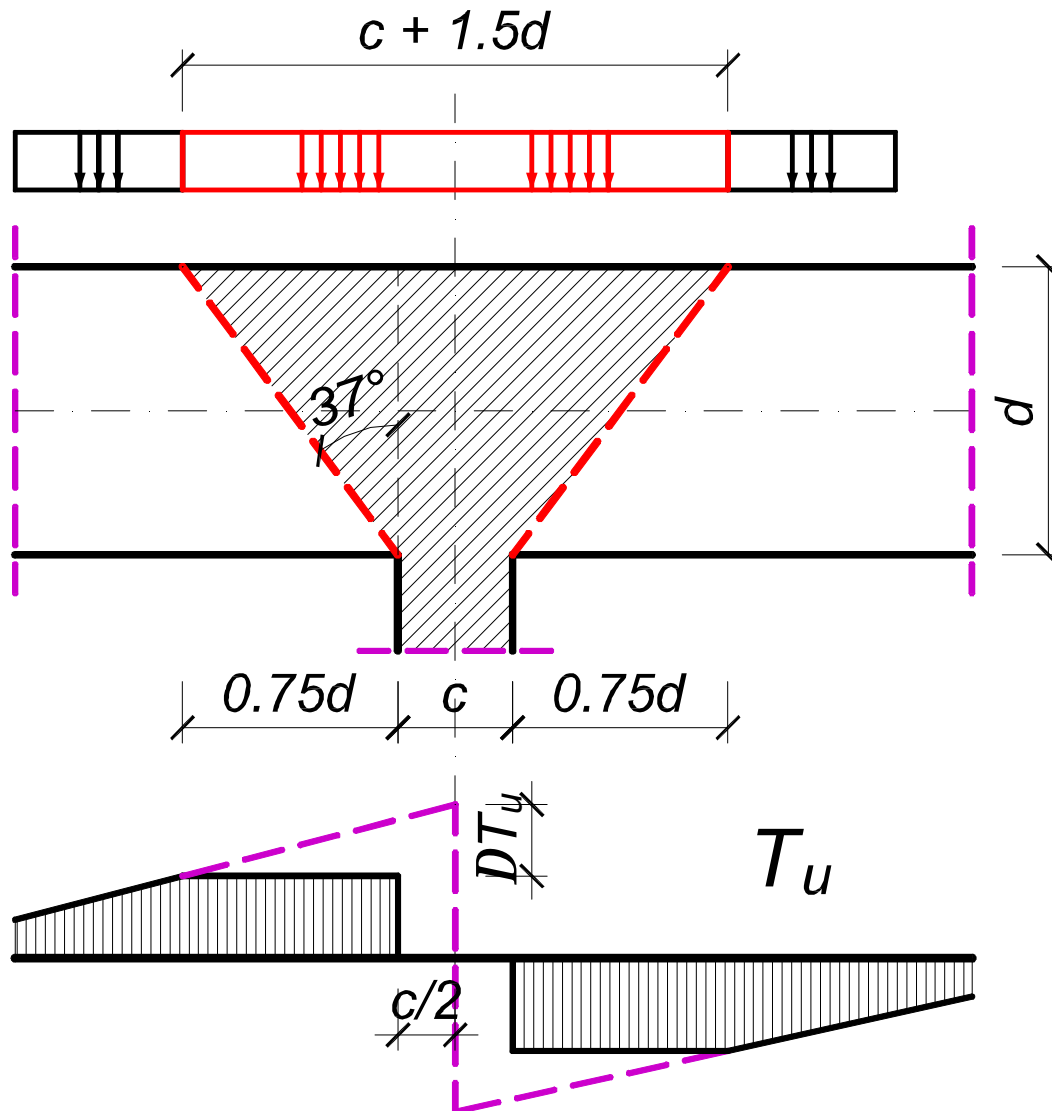
$F_6$   
 $F_4$   
 $F_5$   
 $F_2$   
 $F_1$   
 $F_3$



KV. PARABOLA

$F$   
 $F_6$   
 $F_4$   
 $F_5$   
 $F_2$   
 $F_1$   
 $F_3$

# Redukcija transverzalnih sila u zoni oslonaca



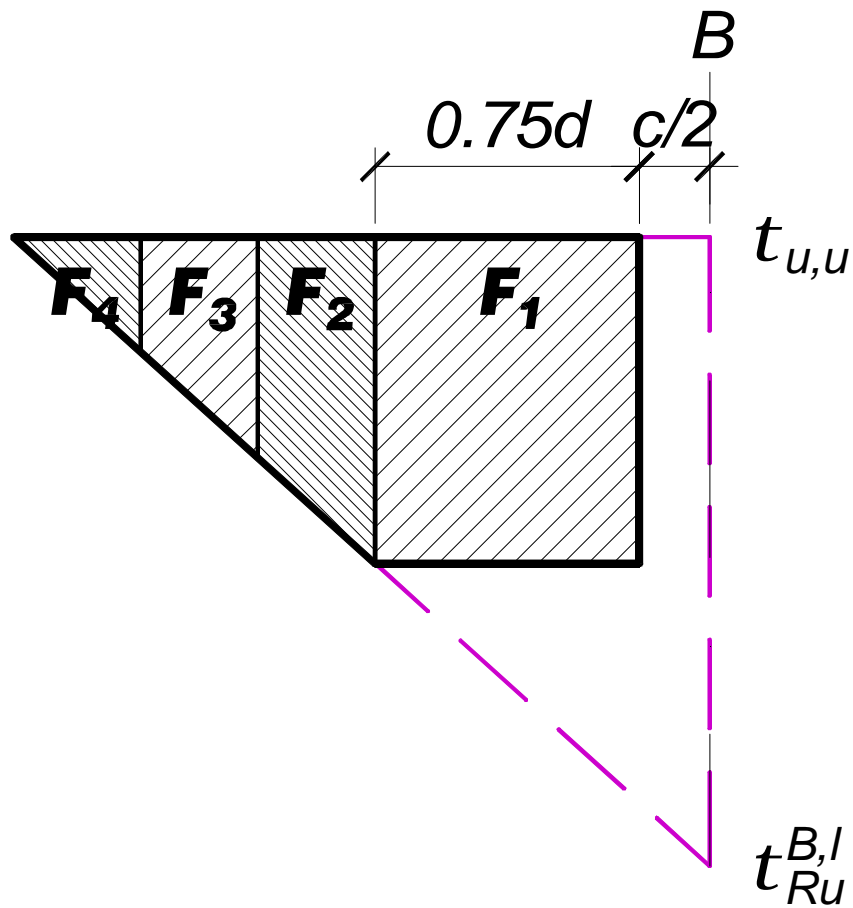
Može se vršiti ublažavanje uticaja transverzalnih sila  $T_u$  u zoni oslonca a prema skici levo

Ovu redukciju dopuštaju:

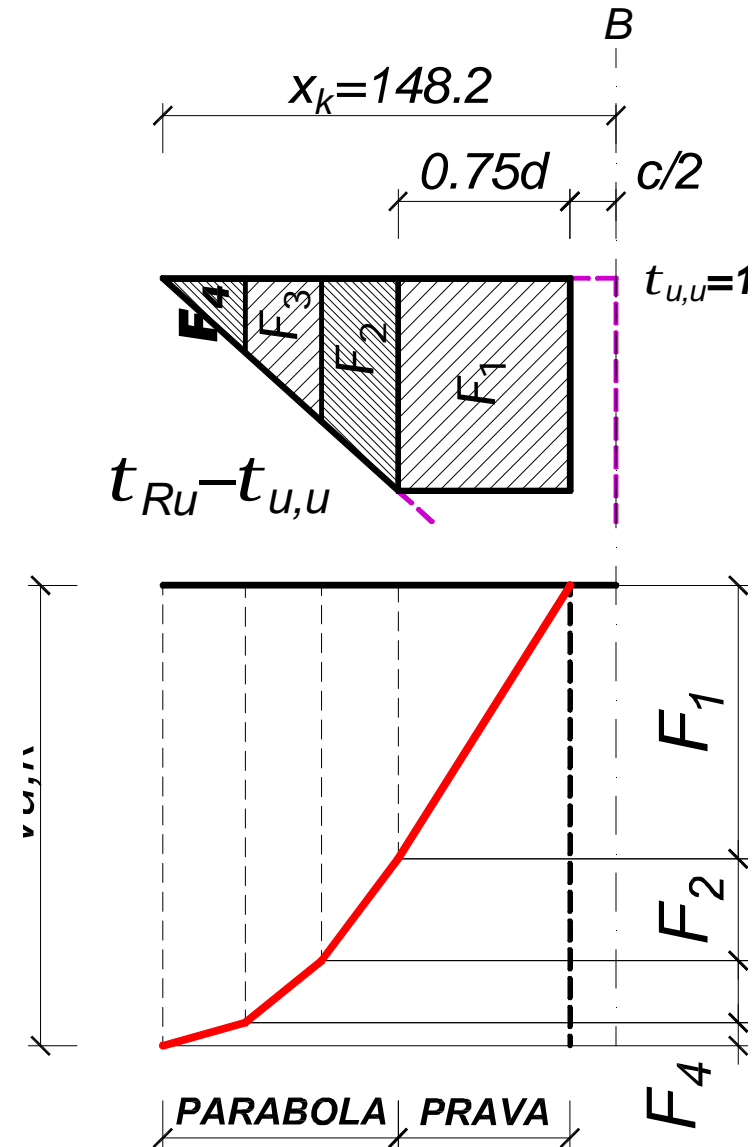
**Član 95. PBAB 87**  
(proračun po graničnoj nosivosti)

**Član 132. PBAB 87**  
(proračun po dopuštenim naponima)

# Određivanje mesta povijanja kosih profila



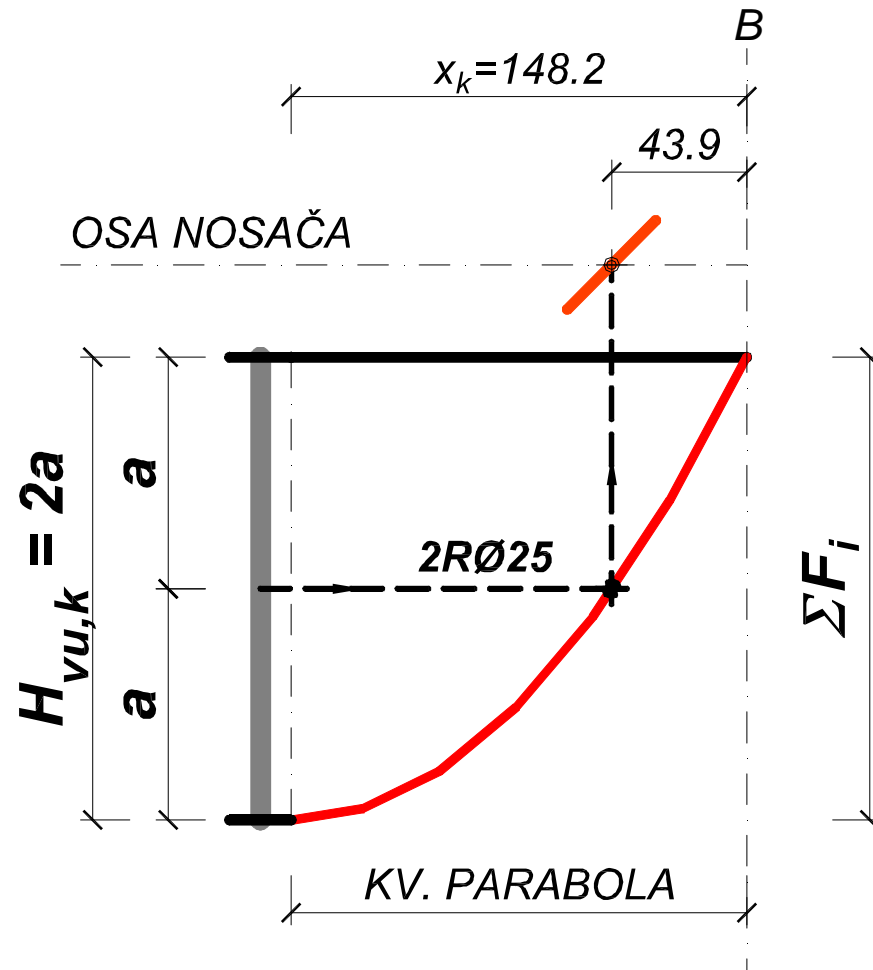
$c = 30 \text{ cm}$  - širina oslonca  
 $d = 75 \text{ cm}$  - visina grede



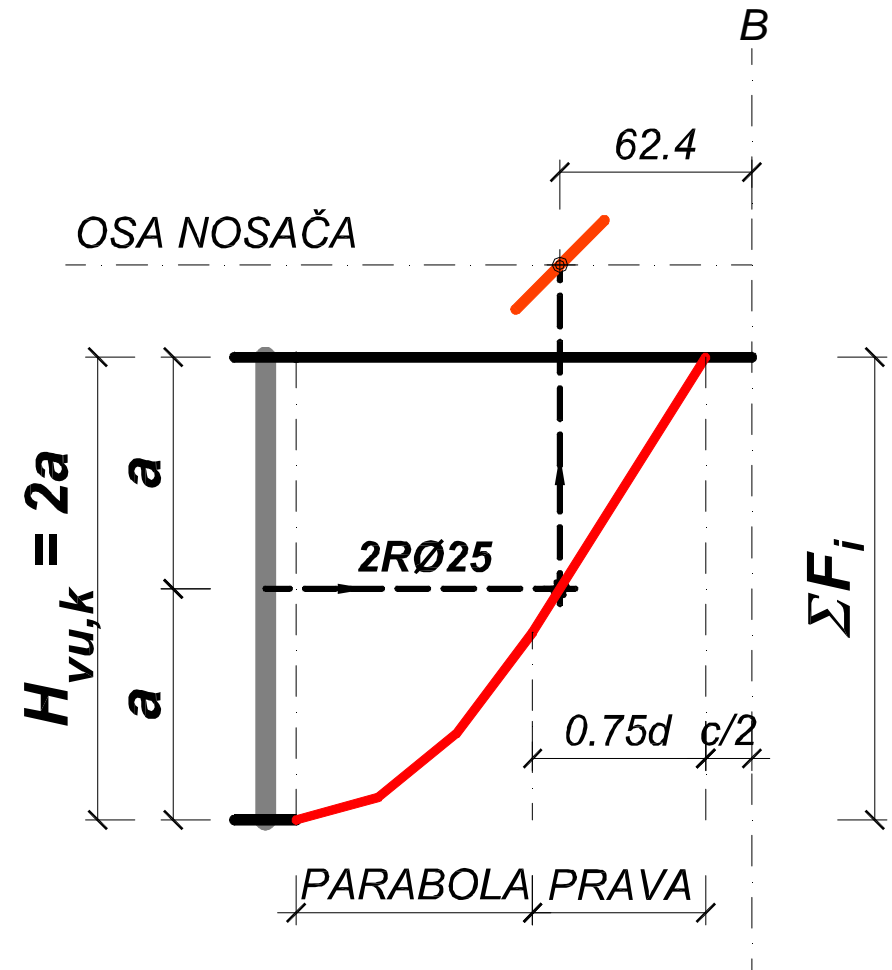


# Određivanje mesta povijanja kosih profila

**BEZ REDUKCIJE T-SILA NA OSLONCU**

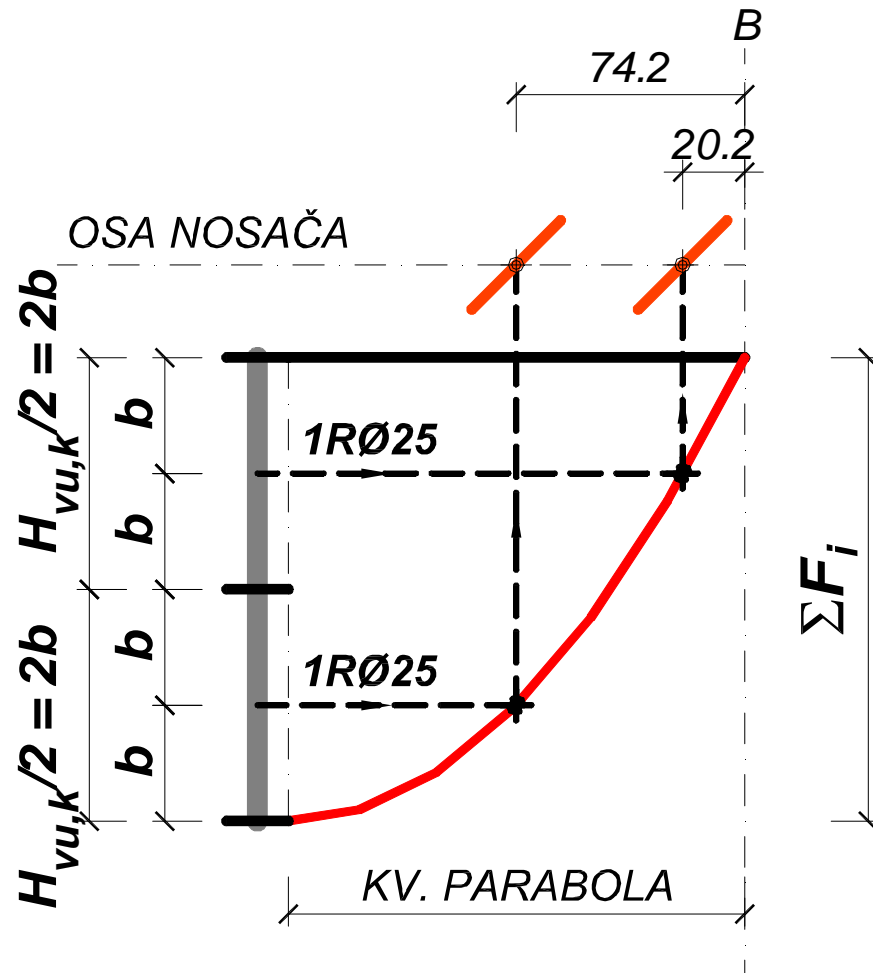


**SA REDUKCIJOM T-SILA NA OSLONCU**

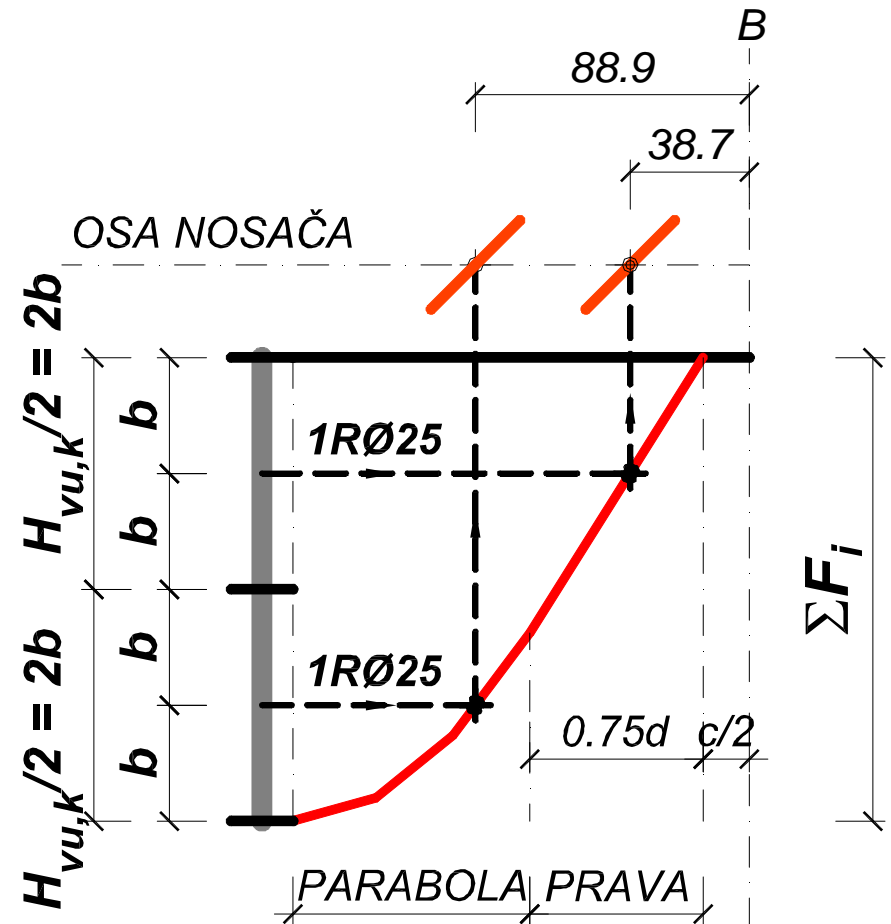


# Određivanje mesta povijanja kosih profila

**BEZ REDUKCIJE T-SILA NA OSLONCU**

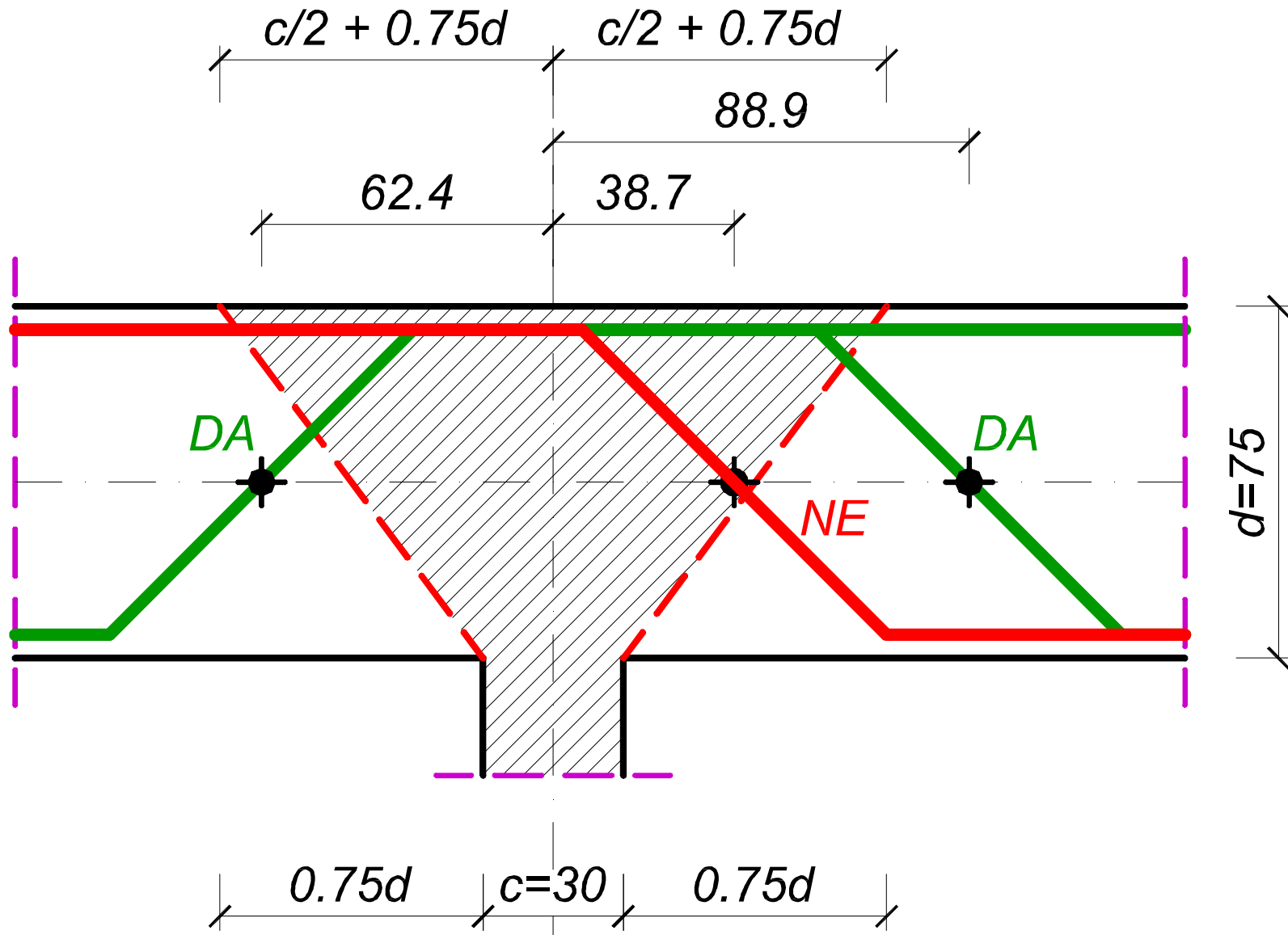


**SA REDUKCIJOM T-SILA NA OSLONCU**





# Određivanje mesta povijanja kosih profila



## Određivanje podužne armature

Potrebna površina dodatne podužne armature  $DA_a$  usled dejstva  $T$  sila, koja se sabira sa postojećom podužnom armaturom sračunatom za  $M$ , dobija se iz izraza (**član 93 PBAB 87**):

$$\Delta A_a = \frac{T_{mu}}{2\sigma_v} \times (\text{ctg}\theta - \text{ctg}\alpha)$$

**U oblastima momentnih špiceva** (npr. srednji oslonci kontinualnih nosača) **nema potrebe za armaturom  $DA_a$**  usled  $T$  sila (str. 268. BAB 87, Tom 1 - Priručnik)

**Pomeranje linije zatežućih sila:**

$$v = \frac{z}{2} \times (\text{ctg}\theta - \text{ctg}\alpha) \geq \begin{cases} 0.75 \times h & (U\emptyset) \\ 0.50 \times h & (U\emptyset + A_{ak}) \end{cases}$$

# Dijagram transverzalnih sila - anvelopa

